

# Performance of a Tapered Horizontal Well in an Infinite-Acting Reservoir

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**Abstract:-** A new form of horizontal well completion technology is the main objective of this research. This paper presents a mathematical model to evaluate the performance of a horizontal well during infinite-acting flow period with tapered completion strategy.

The dimensionless pressure solution for a horizontal well exhibiting infinite acting flow was used to compute total wells pressure drop per unit rate function as a single tubing component and then tapered tubing component. Three cases were on spotlight which are single completions, two step tapered completions and three step tapered completions to appositely evaluate their comparative performance. Sensitivity analysis was carried out by investigating the influences of wellbore properties assuming a homogenous reservoir that produces a single-phase fluid at a constant rate.

The results show that the tapered horizontal well outperforms that of the conventional on the basis of total pressure drop per unit flow rate per completion. The longer the length of the tapered horizontal well sequentially with corresponding increase in wellbore radii, the lower the pressure drop per unit flow rate obtained and also longer time to end infinite-activity. The maximum well length for any completion strategy is dictated chiefly by wellbore skin and reservoir anisotropy.

An excellent performance is achieved when the horizontal well is tapered yielding even much more by varying the parameters of length and radii. It is therefore evident that tapered horizontal well completions presents a lot more capabilities in extending the period of oil production in spite of reservoir size and nature of external boundaries.

**Keywords:-** Infinite-Acting, Horizontal Well, Performance, Tapered, Completion Efficiency.

## I. INTRODUCTION

Horizontal wells have continually exhibited a sustainable proficiency in oil and gas recovery owing to its well property which has a better contact with the bedding plane in its affixed reservoir with a promising performance profile in comparison to vertical wells. Amongst the key governing factors that affect the horizontal well performance is the well property but more even completion scheme is of overlying significance.

All wells, production or interference, in systems surrounded by no-flow boundaries have four distinct flow periods in their pressure/time history: wellbore-storage dominated, infinite-acting, transient, and pseudo-steady state. The infinite-acting period starts after the wellbore-storage effects have diminished and lasts until boundary effects become significant at the point of interest. At the end of the infinite-acting period, a brief transition occurs followed by the pseudo-steady-state period (Gobran et.al 1986).

Considering a single-phase flow, it is assumed that the pressure differential between the reservoir and the wellbore is directly proportional to the production. Thus, a better understanding of what factors contribute to the total pressure drop in horizontal wellbore has the potential of improving its well completion designs. The pressure drop not only has impact on well performance, but also is one of the key factors to determine the completion scheme of the well (Joshi, 1991).

However, the capabilities of the horizontal well have not yet been fully digested in terms of maximizing recovery. This study thus, focuses on idealizing the necessity to transcend into a new form of horizontal well completion technology by varying the well's cross section and analyzing its performance taking a slightly compressible fluid as our case study for a reservoir masked by infinite-activity. A change in the configuration of the conventional horizontal well system is to be explored and analyzed to ascertain its effect on overall completion efficiency. The dimensionless pressure approach provides a way to calculate pressure response and to develop techniques for analyzing transient tests in a variety of systems (Chaudhry, 2004). In this work, equations that enable us to determine the different at different tapered completion section involve developing a mathematical model.

The preliminary findings unveil the effect of varying the cross section of the producing well length of a horizontal well on its performance evaluations, so as its efficiency.

**II. AIMS AND OBJECTIVES**

This work aims to establish the extensive impact of tapering on a horizontal well performance as the reservoir exhibits an unsteady state flow regime. Thus, in that light, the specific objectives are:

- To determine the effect of tapered well completion on production performance.
- To investigate the effect of varying well length on total pressure drop to flow rate.
- To investigate the effect of varying radii on the completion efficiency of the well.
- To analyze its completion efficiency.
- To compare performance of the tapered horizontal well completion to that of the conventional horizontal well.

**III. SCOPE OF STUDY**

This paper will focus on evaluating the efficacy of tapering the horizontal well cross section on its performance, limited to an early radial flow period with

slightly compressible fluid as our case study. Due to the fact that this form of horizontal well technology has not yet been adopted in any field across the globe, this work will consider using general data in its analysis.

**IV. METHODOLOGY**

➤ *Tapered Horizontal Well Model Development*

The following assumptions were made

- The reservoir is homogeneous, isotropic and has a constant permeability and porosity.
- The production occurs through a well of different radius and length represented in the model.
- A single-phase fluid, of small and constant compressibility, constant viscosity, and formation volume factor  $B_0$ , flows from the reservoir to the well.
- It is assumed that the whole tapered horizontal sections are open to production (open hole production or slotted liner) and fluid flow is that of a slightly compressible single phase (oil) with uniform pressure distributions across the different sections.
- The reservoir has constant properties along the producing well length (homogenous reservoir) and in an infinite acting flow period.

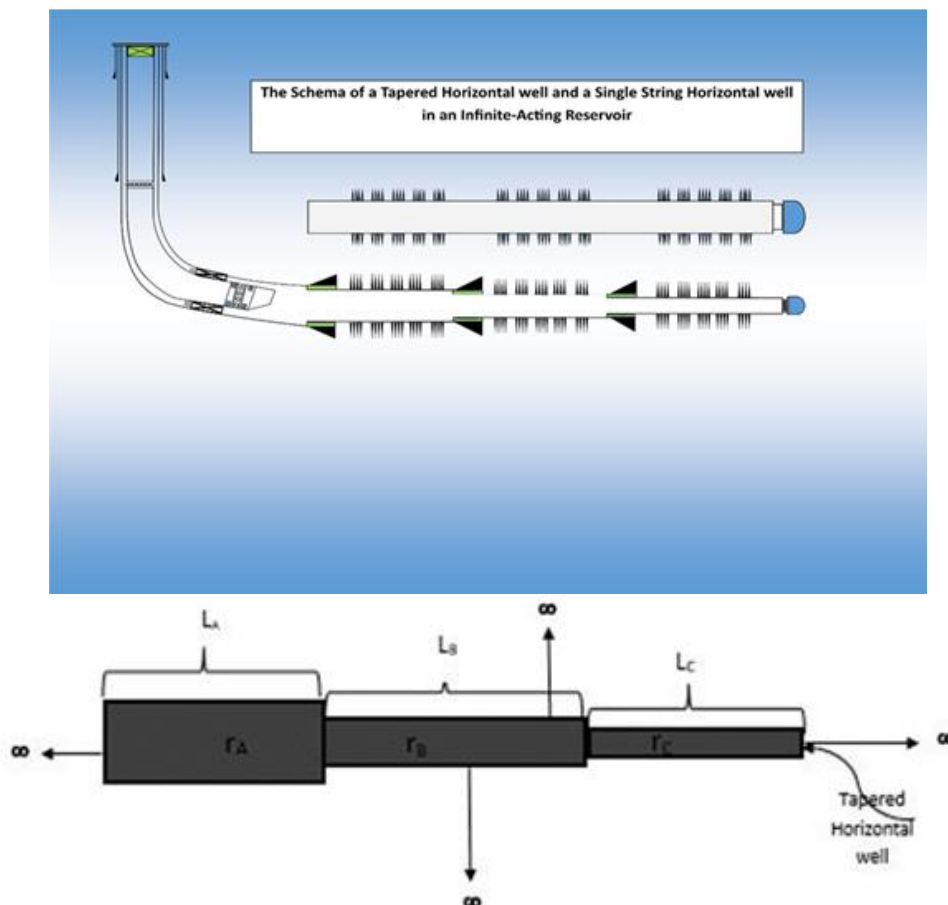


Fig 1:- Tapered Horizontal Well in an Infinite-Acting Reservoir

**V. MATHEMATICAL IDEALIZATION**

The dimensionless pressure solution to a horizontal well in an infinite acting reservoir flow (eqn.1) was used to derive the wells’ total pressure drop per unit flow rate per completion (eqn.2). By the addition of the source strength functions of the different ith sections either as two steps tapered (eqn.3) or three steps tapered (eqn.4), the total wells’ pressure drop per unit flow rate can be computed.

$$P_D(x_D, y_D, z_D, z_{wD}, L_D, t_D) = -\frac{\beta}{4L_D} Ei \left\{ -\frac{(y_D - y_{wD})^2 + (z_D - z_{wD})^2}{4\tau_D} \right\} \dots\dots 1$$

$$\left(\frac{\Delta P}{q}\right)_i = -\frac{162.6B\mu}{kL_i} \left\{ \log \left( \frac{1688\phi\mu C_t r_w^2}{kt} \right) - 0.896s \right\} \dots\dots 2$$

$$\left(\frac{\Delta P}{q}\right)_T = \left(\frac{\Delta P}{q}\right)_A + \left(\frac{\Delta P}{q}\right)_B \dots\dots\dots 3$$

$$\left(\frac{\Delta P}{q}\right)_T = \left(\frac{\Delta P}{q}\right)_A + \left(\frac{\Delta P}{q}\right)_B + \left(\frac{\Delta P}{q}\right)_C \dots\dots 4$$

More so, the completion efficiency is a dimensionless parameter which exemplifies how best a well is completed and would fare in terms of performance evaluation can be computed using eqn.6 below.

$$C.E = \frac{\left(\frac{\Delta P}{q}\right)_{Tapered}}{\left(\frac{\Delta P}{q}\right)_{Conventional}} \dots\dots 5$$

**VI. APPLICATION**

The addition of source strength functions was applied to computing the total pressure drop per unit rate function per completion using the general data set given below. A physical description of the problem is illustrated in Figure 1. Sensitivity analysis was carried out to appositely estimate the influence of well length and wellbore radii on the three

cases under study: conventional single string, two steps tapered, three steps tapered horizontal well.

The total pressure drop per unit rate is computed for the different cases under study are as follows, for the varying range of length and radii with corresponding change in times with the methodology as shown above.

➤ *Case One (Conventional Single String)*

Using the model eqn.1:

$$\left(\frac{\Delta P}{q}\right) = -\frac{162.6B\mu}{kL} \left\{ \log \left( \frac{1688\phi\mu C_t r_w^2}{kt} \right) - 0.896s \right\}$$

➤ *Case Two (Two Steps Tapered)*

To appositely understand and analyze the two steps tapered completion strategy and how it affects performance, the producing well length was split in the ratios of 0.25:0.75, 0.5:0.5 and 0.75:0.25.

Using eqn.2:  $\left(\frac{\Delta P}{q}\right)_T = \left(\frac{\Delta P}{q}\right)_A + \left(\frac{\Delta P}{q}\right)_B$

➤ *Case Three (Three Steps Tapered)*

The same procedure would be used to compute the well’s total pressure drop per unit flow rate, splitting the total producing well length in the ratio 1:3 for the three sections. Inputting the values into the eqn.4

$$\left(\frac{\Delta P}{q}\right)_T = \left(\frac{\Delta P}{q}\right)_A + \left(\frac{\Delta P}{q}\right)_B + \left(\frac{\Delta P}{q}\right)_C$$

**VII. RESULT AND ARGUMENT**

The results obtained are properly laid out in tables but spotlight of analysis will be on the first 10 hours of activity period for the different case studies as similar trends would be observed for the plot that will be generated from the other tables of results as shown in the appendix section.

Well Length Ft.	Wellbore Radius Ft.				
	0.00001	0.0001	0.001	0.01	0.1
1000	0.166614	0.136933	0.107252	0.077571	0.04789
2000	0.083307	0.068466	0.053626	0.038785	0.023945
3000	0.055538	0.045644	0.035751	0.025857	0.015963
4000	0.041653	0.034233	0.026813	0.019393	0.011972
5000	0.033323	0.027387	0.02145	0.015514	0.009578

Table 1:- Results of the Conventional Single String Variation with Length and Radii for t= 10<sup>1</sup> hours

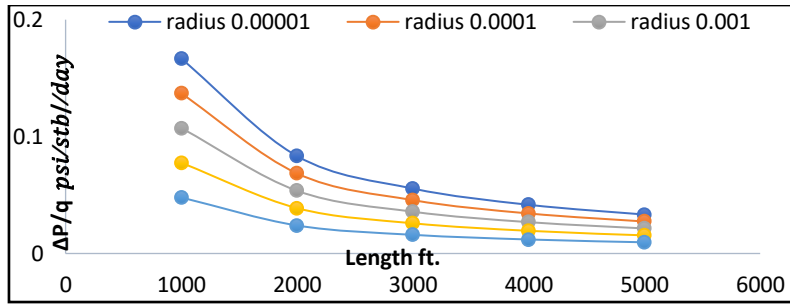


Fig 2:- Influence of Pressure Drop per Unit Flow Rate on Well Length

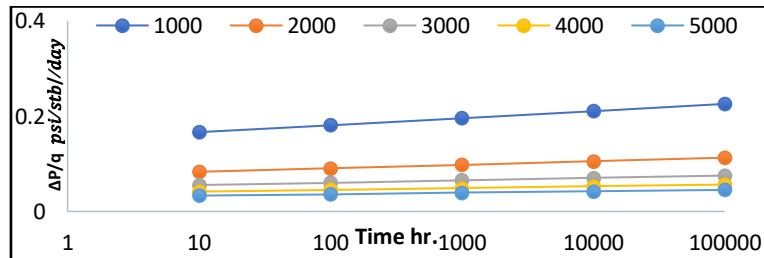


Fig 3:- Semi-Log Plot of Flowing Well Pressures and Time

Tables 1 through 5 presents results for the pressure drop to unit rate function of a conventional single string horizontal well completion. The radius is held constant for different well lengths and vice versa for different time ranges. As shown from these tables, the pressure drops to unit rate function for each radius considered decreases with increasing well length.

The Figure 2, implies that general performance exponentially declines with an increasingly producing well length and thus registers a clear indication that to drain the

reservoir rapidly would require the use of shorter lengths. Also, the impact of wellbore radii is evidently manifest as an incremental gain would result in a reduced total pressure drop per unit flow rate. But will increase for a particular well length and well bore radii configuration at every new time step as can be seen in Figure 3. It is seen therefore, that by splitting the producing well length in the ratio 0.25:0.75 with a sequential reduction in wellbore radii for the tapered sections results in a higher total pressure drop per unit flow rate for each well length in Table 6 through 10.

Length Ft (0.25:0.75)	Radius Ft.			
	0.00001 and 0.0001	0.0001 and 0.001	0.001 and 0.01	0.01 and 0.1
1000	0.769882	0.611583	0.453285	0.294987
2000	0.384941	0.305792	0.226642	0.147493
3000	0.256627	0.203861	0.151095	0.098329
4000	0.19247	0.152896	0.113321	0.073747
5000	0.153976	0.122317	0.090657	0.058997

Table 6:- Results of the Two Steps Tapered Variation with Length and Radii for t=10<sup>1</sup> hours

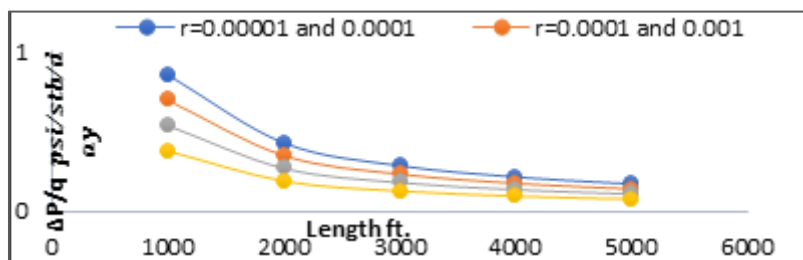


Fig 4:- Influence of pressure drop per unit flow rate on Well Length

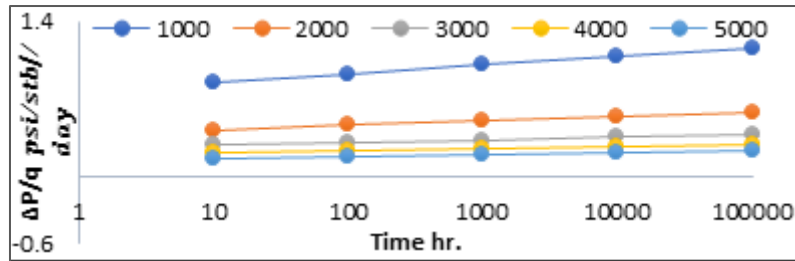


Fig 5:- Semi-Log plot of flowing well pressures and Time

It was observed that much less pressure drop to flow rate is achieved when compared to that of 0.75:0.25 but yet better than that of 0.5:0.5 and the single string. In the plot from Figure 4, a better performance profile is recorded. The Figure 5, shows that the total pressure per unit flow will increase with incremental gain in time considering how short the activity period is: which is strongly dependent on

wellbore configuration. Thus, a longer horizontal well will prolong the delay of infinite activity resulting from its low pressure drop per unit flow rate obtained and also due to the fact that a large portion of the reservoir is in contact and the total pressure drop per unit flow rate along the wellbore is reduced.

Length Ft. (0.5:0.5)	Radius Ft.			
	0.00001 and 0.0001	0.0001 and 0.001	0.001 and 0.01	0.01 and 0.1
1000	0.607092	0.488368	0.369645	0.250921
2000	0.303546	0.244184	0.184822	0.12546
3000	0.202364	0.162789	0.123215	0.08364
4000	0.151773	0.122092	0.092411	0.06273
5000	0.121418	0.097674	0.073929	0.050184

Table 11:- Results of the Two Steps Tapered Variation with Length and Radii for t=10<sup>1</sup> hours

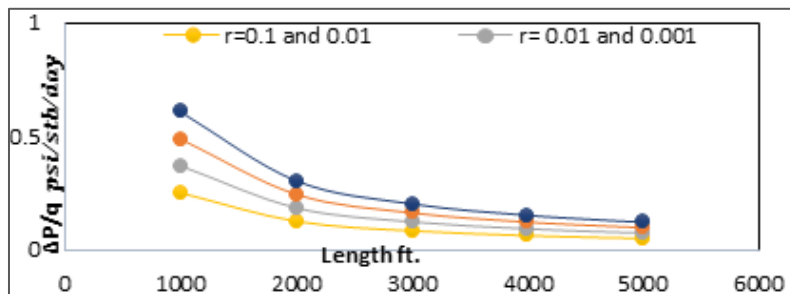


Fig 6:- Influence of Pressure Drop per Unit Flow Rate on Well Length

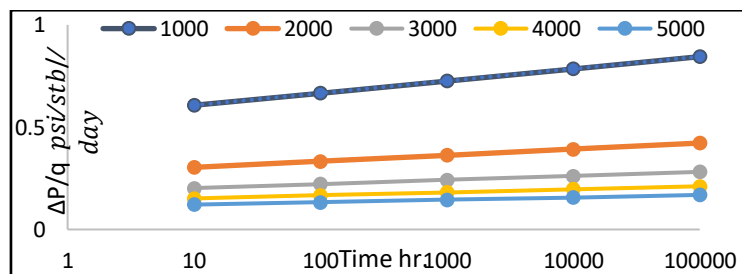


Fig 7:- Semi-Log Plot of Flowing Well Pressures and Time

Table 11 through 15 presents the effect of change of a two-step tapered horizontal well length ratio 0.5:0.5 on the total pressure drop per unit flow rate. It is interesting to note that the  $\Delta P/q$  for this case study increased more pronouncedly when compared with the conventional single string horizontal well completion and decreases with a corresponding increase in length also. The larger the

subsequent radii considered at a constant length, the smaller the  $\Delta P/q$ .

Therefore, plotting the results as shown in Figure 6, shows an exponential decay for every length increment but increase for the different time step analyzed as made manifest in Figure 7.

Length Ft. (0.75:0.25)	Radius Ft.			
	0.00001 and 0.0001	0.0001 and 0.001	0.001 and 0.01	0.01 and 0.1
1000	0.849031	0.690733	0.532434	0.374136
2000	0.424516	0.345366	0.266217	0.187068
3000	0.28301	0.230244	0.177478	0.124712
4000	0.212258	0.172683	0.133109	0.093534
5000	0.169806	0.138147	0.106487	0.074827

Table 16:- Results of the Two Steps Tapered Variation with Length and Radii for t=10<sup>1</sup> hours

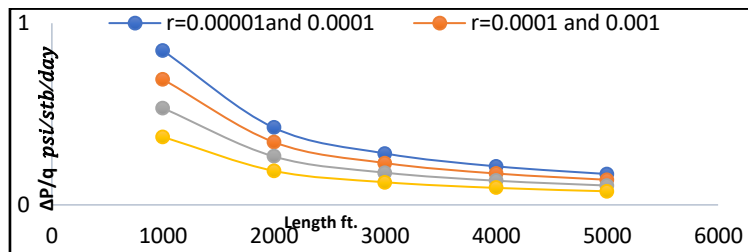


Fig 8:- Influence of pressure drop per unit flow rate on Well Length

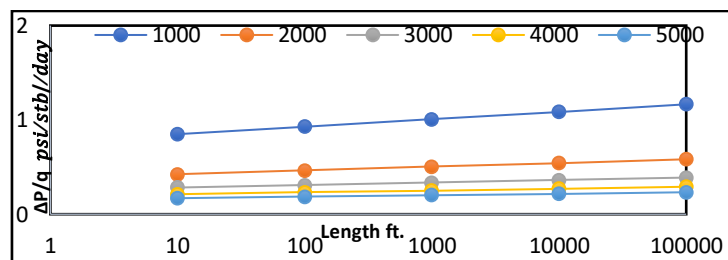


Fig 9:- Semi-Log plot of flowing well pressures and Time

Table 16 through 20 presents results for a two-step tapered horizontal well completion with a producing well length ratio 0.75:0.25. Comparing the latter and the former a longer horizontal well length offers a great deal of advantage than the former.

In same regards analyzing the sensitivities for this ratio, the results plotted as shown in Figures 8 and 9,

displays a better performance than the ratios of 0.5:0.5 in all respects. Once more for the two steps tapered completion, it can be seen from the total pressure drop per unit flow rate computed is of a higher value than the other counterparts. This shows therefore that this form will allow for an improved eased of flow and correspondingly maximize recovery for the flow period. Since one has control on well properties such as well length and radii.

Length Ft.	Radius Ft.		
	0.00001 and 0.0001 and 0.001	0.0001 and 0.001 and 0.01	0.001 and 0.01 and 0.1
1000	1.253048	0.984506	0.71596389
2000	0.626524	0.492253	0.35798195
3000	0.410798	0.321755	0.23271209
4000	0.309041	0.242238	0.17543493
5000	0.247771	0.194302	0.14083254

Table 21:- Results of the Three Steps Tapered Variation with Length and Radii for t=10<sup>1</sup> hours

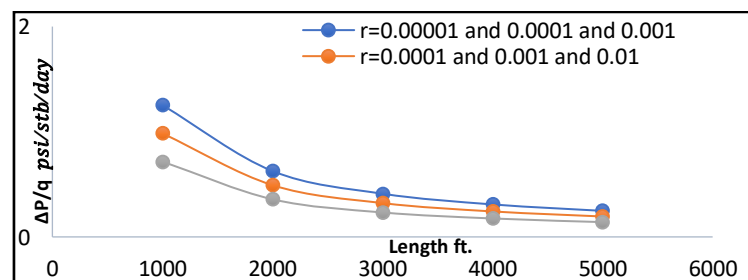


Fig 10:- Influence of Pressure Drop per Unit Flow Rate on Well Length



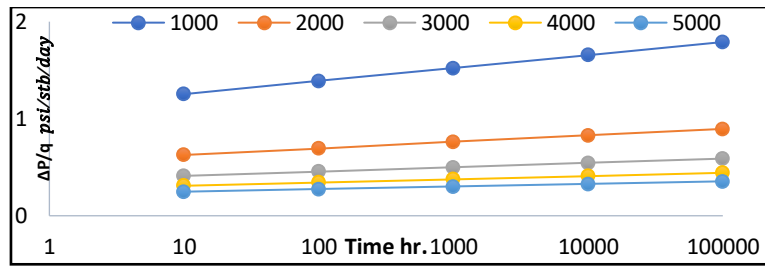


Fig 11:- Semi-Log Plot of Flowing Well Pressures and Time

Lastly Table 21 through 25 presents results for a three-steps tapered horizontal well and as can be seen clearly that it unit rate functions outperforms either of the other two forms of horizontal well completions. Thus, it goes to show that better measure of productivity can be attained by having more sections across the entire producing well length. The plots of this measure of productivity versus variables which are lengths, radii and time evidently shows the aforementioned analysis true as can be shown in the Figure 10 and 11 respectively.

**VIII. GENERAL DISCUSSION**

Differential pressure is a very strong reservoir property that results in the transport of fluid from the reservoir to the wellbore. This therefore implies that an increased drawdown indicates the improved ease with which a reservoir or a

wellbore releases it fluid. For every total pressure drop per unit flow rate evaluation for the different cases under study, we see a veritable outperformance of the conventional single string horizontal well completion by the tapered horizontal well completion as can be seen in Figure 12 below. The incremental recovery of oil decreases with increasing well length. Thus, to prolong infinite-activity use longer lengths as it yields a lower total pressure drop per unit flow rate per completions. More so, to appositely evaluate the performance of a two-steps tapered horizontal well in terms of length and radii variation, the producing well length was split in the ratios of 0.25:0.75, 0.5:0.5, and 0.75:0.25. It was therefore manifest that a better flow assurance would be achieved with the latter. There is correspondingly no flow experienced as the incremental gain in the producing well length becomes distinct for a single string completions as lower total pressure drop per unit flow rate results.

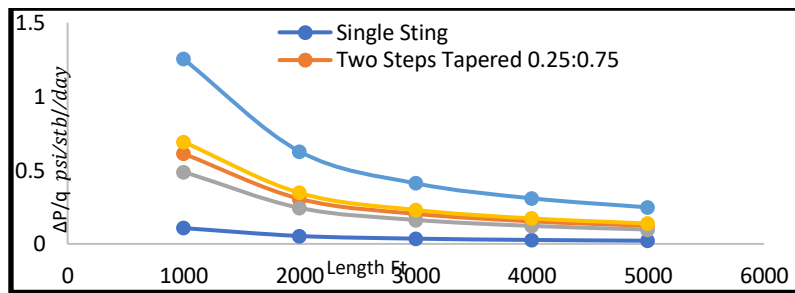


Fig 12:- Comparative Analysis of Three forms of Completions

The results presented shows that there is an increase in pressure drop to unit rate as the taperedness become distinct, this in a nut shell exemplifies the improved assurance of the reservoir deliverability. Since it is known that increased production offtake reduces infinite acting flow period. From the trend lines of  $\Delta P/q$  against the logarithm of time, decrease in pressure drop for a unit flowrate is noticeable for every stepwise increment of wellbore radius combination.

**IX. COMPLETION EFFICIENCY**

Using  $(\Delta P/q)_T$  values for wellbore radii of 0.001ft.and length 1000ft through 5000ft. at time equal to 10hours as a snapshot to adequately represent the whole, the C.E is computed using Eqn.6, the results are tabulated below and analyzed using a bar chart.

C.E for Two Step Tapered			C.E Three Step Tapered
0.25:0.75	0.5:0.5	0.75:0.25	
5.684388	4.553478	4.553478	11.683270
5.684388	4.553478	4.553478	11.683177
5.684388	4.553496	4.553496	11.490732
4.743237	4.553478	4.553478	11.524299
4.763982	4.55331	4.553310	11.552269

Table 26:- Completion Efficiency for the different Case Studies

The table 26 above and the Figure 13 below validates that a better completion efficiency would be achieved by tapering the horizontal well sections with the two steps

tapered completion having a C.E of averagely 4.5 while that of the three steps tapered completion 11 times more efficient than the conventional single string well completion.

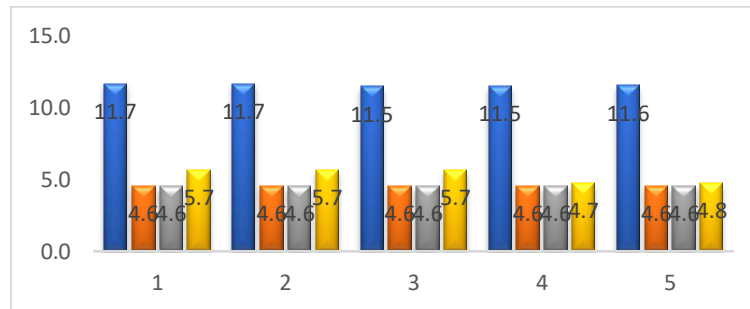


Fig 13:- Completion Efficiency for the different Case Studies

## X. CONCLUSION

Pressure drop to flow rate functions ( $\Delta P/q$ ) for a tapered horizontal well completion was investigated analytically in this study. This research is a novel approach which has for the first time, suggested the use of a new concept and mathematical model to efficiently analyze performance of this new form of horizontal well completion technology. Solution to the problem of varying the well properties of an active horizontal well in an infinite acting reservoir is used to generate results for various combinations of well and reservoir properties. The following relevant conclusions are therefore made:

- Results have shown that it will be gainful to taper the producing well in an infinite acting reservoir condition since its  $\Delta P/q$  is of a better value than that of the conventional single string with better assurance of flow for an increased drawdown.
- The decrease in  $\Delta P/q$  ratio as well length increases for a tapered horizontal well completion, would recover more oil and even reducing the potential for external fluid breakthroughs.
- In order to obtain high drawdown, larger wellbore radius and sequentially smaller radius should be used.
- The factors which affects the total pressure drop per unit flow rate between reservoir and the wellbore such as well length, permeability, fluid viscosity and wellbore radii are also factors affecting the performance of a tapered horizontal well.
- Performance evaluation of a horizontal well does not only depend on the well length and well bore radii, but also on the type of completion used and the efficiency of the completion work done.
- From the completion efficiencies evaluated, the three steps tapered completion offered a more robust capability than that of the other completions.

## RECOMMENDATION

- The industry has not yet totally digested all the possibilities offered by horizontal well, I therefore strongly recommend tapered horizontal well completions for adoption because it has a better performance evaluation than that of the single string completion, with a producing well length of less than 8000ft.
- For the two-steps tapered completion, I will prescribe the 0.75:0.25 type of completion as it effectively drains the reservoir.
- For a more effective performance evaluation of the tapered horizontal well, field data should be use in the stead of the hypothetical data used in the course of this research.
- A continuing study of how this form of well completion would fare or behave in the pseudo steady and steady state flow period.

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$\phi$  Porosity Fractional (%)  
 r Radius (ft.)  
 $k_x, k_y, k_z$  Permeability in the x, y, z directions (md)  
 h Height (ft.)  
 k Permeability (md)  
 $P_D$  Dimensionless Pressure  
 $t_D$  Dimensionless Time  
 $\left(\frac{\Delta P}{q}\right)_T$  Total well Pressure drop per flow rate (psi/bbl/day)  
 $Z_w$  Well location in the z direction (ft.)  
 $Z_{wD}$  Dimensionless well location in the z direction  
 L Horizontal well length (ft.)  
 $L_D$  Dimensionless Length  
 $B_o$  Formation Volume Factor (bbl/stb)  
 $L_D$  Dimensionless Length  
 $r_{wD}$  Dimensionless wellbore radius  
 $r_w$  Wellbore radius (ft.)  
 $\Delta P$  Pressure drop (psi)  
 Ei Exponential Integral  
 C.E Completion Efficiency  
 K Permeability (md)  
 S Skin Factor

**NOMENCLATURE**

$C_t$  Total compressibility ( $psi^{-1}$ )  
 P Pressure ( $psi$ )  
 i Individual Section  
 $L_D$  Dimensionless Length  
 t Time (hours)  
 $t_D$  Dimensionless time  
 q Flow rate ( $bbl/day$ )  
 $\mu$  Viscosity (cP)

**APPENDIX**

Length Ft.	Radius Ft.				
	0.00001	0.0001	0.001	0.01	0.1
1000	0.181454	0.151773	0.122092	0.092411	0.06273
2000	0.090727	0.075887	0.061046	0.046206	0.031365
3000	0.060485	0.050591	0.040697	0.030804	0.02091
4000	0.045364	0.037943	0.030523	0.023103	0.015683
5000	0.036291	0.030355	0.024418	0.018482	0.012546

Table 2:- Results of the Conventional Single String Variation with Length and Radii for  $t=10^2$  hours

Length Ft.	Radius Ft.				
	0.00001	0.0001	0.001	0.01	0.1
1000	0.196295	0.166614	0.136933	0.107252	0.077571
2000	0.098147	0.083307	0.068466	0.053626	0.038785
3000	0.065432	0.055538	0.045644	0.035751	0.025857
4000	0.049074	0.041653	0.034233	0.026813	0.019393
5000	0.039259	0.033323	0.027387	0.02145	0.015514

Table 3:- Results of the Conventional Single String Variation with Length and Radii for  $t=10^3$  hours

Length Ft.	Radius Ft.				
	0.00001	0.0001	0.001	0.01	0.1
1000	0.211135	0.181454	0.151773	0.122092	0.092411
2000	0.105567	0.090727	0.075887	0.061046	0.046206
3000	0.070378	0.060485	0.050591	0.040697	0.030804
4000	0.052784	0.045364	0.037943	0.030523	0.023103
5000	0.042227	0.036291	0.030355	0.024418	0.018482

Table 4:- Results of the Conventional Single String Variation with Length and Radii for t=104 hours

Length Ft.	Radius Ft.			
	0.00001	0.0001	0.001	0.01
1000	0.225975	0.196295	0.166614	0.136933
2000	0.112988	0.098147	0.083307	0.068466
3000	0.075325	0.065432	0.055538	0.045644
4000	0.056494	0.049074	0.041653	0.034233
5000	0.045195	0.039259	0.033323	0.027387

Table 5:- Results of the Conventional Single String Variation with Length and Radii for t=105 hours

Length Ft. (0.25:0.75)	Radius Ft.			
	0.00001 and 0.0001	0.0001 and 0.001	0.001 and 0.01	0.01 and 0.1
1000	0.849031	0.690733	0.532434	0.374136
2000	0.424516	0.345366	0.266217	0.187068
3000	0.28301	0.230244	0.177478	0.124712
4000	0.212258	0.172683	0.133109	0.093534
5000	0.169806	0.138147	0.106487	0.074827

Table 7:- Results of the Two Steps Tapered Variation with Length and Radii for t=10<sup>2</sup> hours

Length Ft. (0.25:0.75)	Radius Ft.			
	0.00001 and 0.0001	0.0001 and 0.001	0.001 and 0.01	0.01 and 0.1
1000	0.92818	0.769882	0.611583	0.453285
2000	0.46409	0.384941	0.305792	0.226642
3000	0.309393	0.256627	0.203861	0.151095
4000	0.232045	0.19247	0.152896	0.113321
5000	0.185636	0.153976	0.122317	0.090657

Table 8:- Results of the Two Steps Tapered Variation with Length and Radii for t=10<sup>3</sup> hours

Length Ft. (0.25:0.75)	Radius Ft.			
	0.00001 and 0.0001	0.0001 and 0.001	0.001 and 0.01	0.01 and 0.1
1000	1.007329	0.849031	0.690733	0.532434
2000	0.503665	0.424516	0.345366	0.266217
3000	0.335776	0.28301	0.230244	0.177478
4000	0.251832	0.212258	0.172683	0.133109
5000	0.201466	0.169806	0.138147	0.106487

Table 9:- Results of the Two Steps Tapered Variation with Length and Radii for t=10<sup>4</sup> hours

Length Ft. (0.25:0.75)	Radius Ft.			
	0.00001 and 0.0001	0.0001 and 0.001	0.001 and 0.01	0.01 and 0.1
1000	1.086479	0.92818	0.769882	0.611583
2000	0.543239	0.46409	0.384941	0.305792
3000	0.36216	0.309393	0.256627	0.203861
4000	0.27162	0.232045	0.19247	0.152896
5000	0.217296	0.185636	0.153976	0.122317

Table 10:- Results of the Two Steps Tapered Variation with Length and Radii for  $t=10^5$  hours

Length Ft. (0.5:0.5)	Radius Ft.			
	0.00001 and 0.0001	0.0001 and 0.001	0.001 and 0.01	0.01 and 0.1
1000	0.666454	0.54773	0.429007	0.310283
2000	0.333227	0.273865	0.214503	0.155141
3000	0.222151	0.182577	0.143002	0.103428
4000	0.166614	0.136933	0.107252	0.077571
5000	0.133291	0.109546	0.085801	0.062057

Table 12:- Results of the Two Steps Tapered Variation with Length and Radii for  $t=10^2$  hours

Length Ft. (0.5:0.5)	Radius Ft.			
	0.00001 and 0.0001	0.0001 and 0.001	0.001 and 0.01	0.01 and 0.1
1000	0.725816	0.607092	0.488368	0.369645
2000	0.362908	0.303546	0.244184	0.184822
3000	0.241939	0.202364	0.162789	0.123215
4000	0.181454	0.151773	0.122092	0.092411
5000	0.145163	0.121418	0.097674	0.073929

Table 13:- Results of the Two Steps Tapered Variation with Length and Radii for  $t=10^3$  hours

Length Ft. (0.5:0.5)	Radius Ft.			
	0.00001 and 0.0001	0.0001 and 0.001	0.001 and 0.01	0.01 and 0.1
1000	0.785178	0.666454	0.54773	0.429007
2000	0.392589	0.333227	0.273865	0.214503
3000	0.261726	0.222151	0.182577	0.143002
4000	0.196295	0.166614	0.136933	0.107252
5000	0.157036	0.133291	0.109546	0.085801

Table 14:- Results of the Two Steps Tapered Variation with Length and Radii for  $t=10^4$  hours

Length Ft. (0.5:0.5)	Radius Ft.			
	0.00001 and 0.0001	0.0001 and 0.001	0.001 and 0.01	0.01 and 0.1
1000	0.84454	0.725816	0.607092	0.488368
2000	0.42227	0.362908	0.303546	0.244184
3000	0.281513	0.241939	0.202364	0.162789
4000	0.211135	0.181454	0.151773	0.122092
5000	0.168908	0.145163	0.121418	0.097674

Table 15:- Results of the Two Steps Tapered Variation with Length and Radii for  $t=10^5$  hours

Length Ft. (0.75:0.25)	Radius Ft.			
	0.00001 and 0.0001	0.0001 and 0.001	0.001 and 0.01	0.01 and 0.1
1000	0.92818	0.769882	0.611583	0.453285
2000	0.46409	0.384941	0.305792	0.226642
3000	0.309393	0.256627	0.203861	0.151095
4000	0.232045	0.19247	0.152896	0.113321
5000	0.185636	0.153976	0.122317	0.090657

Table 17:- Results of the Two Steps Tapered Variation with Length and Radii for  $t=10^2$  hours

Length Ft. (0.75:0.25)	Radius Ft.			
	0.00001 and 0.0001	0.0001 and 0.001	0.001 and 0.01	0.01 and 0.1
1000	1.007329	0.849031	0.690733	0.532434
2000	0.503665	0.424516	0.345366	0.266217
3000	0.335776	0.28301	0.230244	0.177478
4000	0.251832	0.212258	0.172683	0.133109
5000	0.201466	0.169806	0.138147	0.106487

Table 18:- Results of the Two Steps Tapered Variation with Length and Radii for t=10<sup>3</sup> hours

Length Ft. (0.75:0.25)	Radius Ft.			
	0.00001 and 0.0001	0.0001 and 0.001	0.001 and 0.01	0.01 and 0.1
1000	1.086479	0.92818	0.769882	0.611583
2000	0.543239	0.46409	0.384941	0.305792
3000	0.36216	0.309393	0.256627	0.203861
4000	0.27162	0.232045	0.19247	0.152896
5000	0.217296	0.185636	0.153976	0.122317

Table 19:- Results of the Two Steps Tapered Variation with Length and Radii for t=10<sup>4</sup> hours

Length Ft. (0.75:0.25)	Radius Ft.			
	0.00001 and 0.0001	0.0001 and 0.001	0.001 and 0.01	0.01 and 0.1
1000	1.165628	1.007329	0.849031	0.690733
2000	0.582814	0.503665	0.424516	0.345366
3000	0.388543	0.335776	0.28301	0.230244
4000	0.291407	0.251832	0.212258	0.172683
5000	0.233126	0.201466	0.169806	0.138147

Table 20:- Results of the Two Steps Tapered Variation with Length and Radii for t=10<sup>5</sup> hours

Length Ft	Radius Ft.		
	0.00001 and 0.0001 and 0.001	0.0001 and 0.001 and 0.01	0.001 and 0.01 and 0.1
1000	1.387319	1.118777	0.85023487
2000	0.693659	0.559388	0.42511743
3000	0.455319	0.366276	0.27723352
4000	0.342443	0.27564	0.20883657
5000	0.274506	0.221037	0.16756722

Table 22:- Results of the Three Steps Tapered Variation with Length and Radii for t=10<sup>2</sup> hours

Length Ft.	Radius Ft.		
	0.00001 and 0.0001 and 0.001	0.0001 and 0.001 and 0.01	0.001 and 0.01 and 0.1
1000	1.52159	1.253048	0.984506
2000	0.760795	0.626524	0.492253
3000	0.499841	0.410798	0.321755
4000	0.375845	0.309041	0.242238
5000	0.301241	0.247771	0.194302

Table 23:- Results of the Three Steps Tapered Variation with Length and Radii for t=10<sup>3</sup> hours

Length Ft	Radius Ft.		
	0.00001 and 0.0001 and 0.001	0.0001 and 0.001 and 0.01	0.001 and 0.01 and 0.1
1000	1.655861	1.387319	1.11877682
2000	0.82793	0.693659	0.55938841
3000	0.544362	0.455319	0.36627637
4000	0.409246	0.342443	0.27563985
5000	0.327975	0.274506	0.22103658

Table 24:- Results of the Three Steps Tapered Variation with Length and Radii for t=10<sup>4</sup> hours

Length Ft	Radius Ft.		
	0.00001 and 0.0001 and 0.001	0.0001 and 0.001 and 0.01	0.001 and 0.01 and 0.1
1000	1.790132	1.52159	1.253048
2000	0.895066	0.760795	0.626524
3000	0.588884	0.499841	0.410798
4000	0.442648	0.375845	0.309041
5000	0.35471	0.301241	0.247771

Table 25:- Results of the Three Steps Tapered Variation with Length and Radii for  $t=10^5$  hours