

A Generalization of κ -Separable Metric Spaces

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Abstract:- In this we noticed about the generalized factor of kappa separable metric space we also given detail description about the kappa separable metric space is normal. This deals with the relationship between algebra and topology.

Keyword:- Metric space, Separable, Dense, Countable, Clopen, Admissible.

I. INTRODUCTION

Some paper already described about the metric space and their specification. Also few years before one article described about the kappa metrizable space. Now we worked in the topic of generalized factor of separable metric space which is normal and countable. This will expand the relationship between algebra and Topological space.

II. PRELIMINARIES

A. Definition:

If the set A is Dense in X such that closure of A is equal to the whole set X .

B. Definition:

A function $F: X \rightarrow Y$ is said to be continuous if it satisfies the condition $f(x)=y$ for all $x \in X$ and $y \in Y$.

C. Definition:

A subset K of a metric space X is said to be Compact if every open cover of K contains a finite subcover.

D. Proposition:

Compact Subsets of metric space are closed.

E. Definition:

A metrizable space is a topological space that is homeomorphic to a metric space. That is, a topological space (X, \mathcal{T}) is said to be metrizable if there is a metric.

F. Definition:

If X is said to be κ -separable metric space if it satisfy the following condition

- $\varrho(x, C)=0$ if and only if $x \in C$ for any $x \in X$ and $C \in RC(X)$
- If $C \subseteq D$ then $\varrho(x, C) \geq \varrho(x, D)$ for any $x \in X$ and $C, D \in RC(X)$
- $\varrho(., C)$ is a continuous function.
- $\varrho(x, cl(\bigcup_{\alpha < \lambda} C_\alpha)) = \inf_{\alpha < \lambda} \varrho(x, C_\alpha)$ for any non-decreasing totally ordered sequence
- $\{c_\alpha: \alpha < \lambda\} \subseteq RC(X)$ and any $x \in X$.

➤ Remark:

Every countable kappa (κ) metric is kappa metric.

III. MAIN RESULT

A. Definition:

A topological space X is kappa separable metric space if it satisfies the following condition

- X is kappa metric space
- X is dense
- X is countable

B. Theorem:

The product of the kappa separable metric space is kappa separable metric space.

➤ Proof:

Let us consider two function f and g is defined as $f: X \times RC(X) \rightarrow [0, \infty)$ and $g: Y \times RC(Y) \rightarrow [0, \infty)$.

Then,

$F(x, c) = 0$ if and only if $x \in C$ and $C \in RC(X)$.

Also, $g(y, d) = 0$ if and only if $y \in D$ and $D \in RC(Y)$

Then it is product $h((x, y), (c, d)) = 0$ if and only if $(x, y) \in E$ and $E \in RC(Z)$ where $(x, y) = Z$

- $h(Z, E) \geq f(x, c)$ and $h(Z, E) \geq g(y, d)$ for any $x, y, z \in X$ and $C, D, E \in RC(X)$.
- Also the product of the continuous function is continuous.
- The product of the countable metric space is countable.
- It is also dense in Z where $z \in X \times Y$
- If $h(z, cl(\bigcup_{\alpha < \lambda} E_\alpha)) = \inf_{\alpha < \lambda} h(z, E_\alpha)$ for any non-decreasing totally ordered sequence for any $z \in X$.

So, it is kappa separable metric space.

C. Lemma:

Every separable metric space is normal.

➤ Proof:

Let x be a neighbourhood of X Which is disjoint from Y and

y be the neighbourhood of Y , Which is disjoint from X such that $x \cap y = \emptyset$ if it is empty

Since by the condition of kappa separable metric space which is also continuous

So the set which has the neighbourhood are also disjoint.

Hence X is normal.

D. Theorem:

Let X is a completely regular space. If $A \in RC(\mu X)$ then $A \cap X \in RC(X)$.

➤ **Proof:**

Let us assume that X be a topological space and Consider that X to be pseudo compact and countably metrizable with the function f .

So each continuous function $f(\star, A \cap X) : X \rightarrow [0, \infty)$ is bounded and the bounded function is named as

$d_A \in [0, \infty)$ for each regular closed subset $A \subseteq \mu X$.

So we brief our proof by define the map $f(\star, A \cap X) : \mu X \rightarrow [0, d_A)$

We have to prove that the function $\delta : \mu X \times RC(\mu X) \rightarrow \mathbb{R}$ by defining

$\delta(q, A) = f(q, A \cap X)$ is countable kappa metric so $A \cap X \in RC(X)$

E. Proposition

Let $z : A \rightarrow B$ be a continuous function then h is a d -open map if and only if there exist a base $C_b \subseteq R_b$ such that

$q = \{f^{-1}(Q) : Q \in C_b\} \subseteq R_b$

i.e. for any $r \subset q$ and $y \notin cl_B \cup r$ there exist $E \in q$ such that $a \in E$ and $E \cap q = \phi$.

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