Hyperspectral Image Restoration via Tensor-Based Preconditioner and Iterative Filter

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Abstract:- Hyperspectral images (HSIs) are often corrupted by noise during an acquisition process, e.g., Gaussian noise, salt and pepper noise, deadlines, strip, and many others. This project proposes an image restoration algorithm based on Higher Order Singular Value Decomposition algorithm (HOSVD). The HOSVD acts as fast preconditioner. Instead of dealing as pixels the image is processed as tensor. This project reduces the noises that occur in remote sensing satellite images. The satellite images are degraded by the noise such as Gaussian noise, Impulse noise, Strip noise etc. The tensor of the degraded image is added with tensor of Gaussian, Impulse, Strip noise. The Singular values are extracted from the degraded image which controls intensity of Gaussian, Impulse and Strip noise tensor. This process gets repeated iteratively to get the restored image. The higher order singular value decomposition (HOSVD) of the degraded tensor is obtained very fast and so could be used as a preconditioner. Iterative median filtering for restoration of images corrupted by mixed noise is proposed. The boundary condition for the iteration is based on minimum distance between any two successive iterations is less than a threshold value. Experimental results show that proposed system has higher convergence speed. The complexity of an image restoration process reduces highly further we measures Peak Signal Noise Ratio (PSNR) and Mean Square Error (MSE). The PSNR values appear to be high while the MSE values appear to be low.

***Keywords**:- Image Restoration, Hyperspectral Image (HSI), Mixed Noise, HOSVD, Iterative Median Filter.*

I. INTRODUCTION

Image restoration is one of the important process in an image processing in which the focus is on extracting an original image from a noise mixed image [1], [2]. Image restoration is defined as the process of removal or reduction of noise in an image through linear or non-linear filtering. Usually, iterative methods are used to solve restoration problems [1], [2], [5], [6], [7]. Semi-convergency method is used for iterative process which is known as CGLS (Conjugate Gradient for Least Square) [8], [9]. Interpolation technique is used for restore sample of an image. Most important technique used are median filter, bilinear interpolation and low pass filter that satisfied Nyquist rate. This method can be done approximation process for noisy matrix [12], [13].

Hyperspectral imaging employs an imaging spectrometer to collect hundreds of spectral bands for the same area on the surface of an Earth ranging. It has a wide range of applications including environmental monitoring, military surveillance, mineral exploration, among numerous others [3], [4]. The truncated version of eigenvalue decomposition method is used for reconstruct an original matrix from the blurring matrix. This will prevent propagation of noise for target detection [4]. But, it consumes more time for computation. Due to various factors hyperspectral images (HSIs) are corrupted by different noises during the acquisition process. This greatly degrades the visual quality of the HSIs. Hence, the task of removing the noise in hyperspectral imagery is a valuable research topic.

The discrete version of the blurring process can be defined as the following model,

$y = Ax, y = b + e$

Where A is a blurred matrix and degradation is usually caused during the acquisition process of an image itself. Image enhancement is an ultimate goal in an image restoration. Enhancement is a subjective process while restoration is an objective process. A filtering method can be applied to reduce the noise in a needed image. Nonlinear filters have different behavior as compared to linear filters. Digital images are mainly corrupted by pixels from malfunctioning camera sensors, faults in memory location of hardware systems or an image transmission in a noisy channel and some other problems also and that can be destroys an image quality. It also affects the accuracy of an image processing applications such as edge extraction, image compression, segmentation, and image classification.

The restoration of image depends upon how much researchers know about an original image, reasons behind the degradations and how much an image is degraded. Here, we demonstrate that image restoration can be modeled as tensorization process, which its matricization is equal to matrix modeling. An experiment shows that HOSVD of the degrading tensor can be obtained very fast.

The median filter is a nonlinear technique that takes the center value of the pixels inside a sliding window [17], [18]. Median Filtering is efficient in reducing impulse noise [19] that is commonly caused by errors in transmission channel. Median filtering technique is mostly used for extracting an original images that caused by malfunctiong of pixel elements in the camera sensors, fault in memory locations, time based error in conversion process [20]. Median filter can also used for preserving edges in restored images [19], but its works falter when the probability of an impulse noise becomes higher [21].

A fast tensor-based preconditioner can be used in some 2D and 3D image restoration problems. So, using HOSVD of the blurring operator as preconditioner is reasonable. The proposed iterative median filtering scheme provides number of iterations as less than 50 and lower computational complexity.

This paper is organized as follows. Section II includes some notations and preliminaries of proposed method. In Section III, the proposed system and its motivations are introduced. We then develop algorithm for solving the proposed model. Section IV presents some experimental results. Finally, we conclude this paper with some discussions on future research in Section V.

II. NOTATION AND PRELIMINARIES

A real-valued tensor of order N is denoted by $X \in R^{I_1 x I_2 x \ldots x I_N}$. It is known that a tensor can be seen as a multi-index numerical array, and its order is defined as the number of its modes or dimensions. Different "dimensions" of tensors as represented as

$$A(i,j,k) = a_{ijk} \quad \text{(II.1)}$$

The above equation as referred to as modes. Tensor can be obtained by performing tensorization process by using gradient function along x-axis or y-axis that will find relative difference between all pixels in an image. In this experiment, gradient along y-axis as choosen to performed tensorization. To show the results of preconditioner to a viewers the reverse operation of tensorization as performed called matricization.

A fiber is a sub-tensor, where all indices but one are fixed. For example mode-2 fibers of A, have following form

$$A(i,:,j) \in R^{I_2} \quad \text{(II.2)}$$

All mode-n fibers of A are multiplied by the matrix X. A is the same as $A \times nX$ in that system. The Frobenius norm of the order-M tensor A can be defined as

$$||A|| = (\textstyle\sum_{i1,\ldots\ldots iM} a^2_{i1\ldots\ldots\ldots iM})^{\frac{1}{2}} \quad \text{(II.3)}$$

The discrete N-dimensional exact and degraded images, now by definition of contraction product in the image restoration model could be represented as the following tensor equation

$$Y = \langle A, X \rangle_{N+1:2N;1:N} \quad \text{(II.4)}$$

The tensor equation can be reformulated as the following linear equation

$$y = Ax \quad \text{(II.5)}$$

Its least squares (LS) should be considered,

$$\min_x ||Ax - y|| \quad \text{(II.6)}$$

Median filter replaces a pixel by the median, instead of average all pixels in a neighborhood *w*.

$$y[m,n] = median\{x[i,j], (i,j) \in w\} \quad \text{(II.7)}$$

Where w represents a neighborhood defined by the user, centered around location $[m, n]$ in the image. The important parameter in using median filter is size of the window. Choice of the window size depends on the context.

III. PROPOSED METHOD

The proposed system uses a tensor modeling of an image restoration technique. This framework enables us to deal with 2D or 3D images directly. Also, based on the structure of the obtained tensor, for the first time, we introduce a tensor-based preconditioner. The performance of the proposed strategy and number of iterations required to restoration as comparable by proposed system with iterative filtering and without filtering.

A. Motivation

Digital images are corrupted by noise during an image acquisition process or transmission in a noisy channel. In such case, image restoration is an important technique for noise suppression or reduction while preserving the detail of image. In many situations, the observed hyperspectral images are contaminated by several noises. As a result, a noise HSI cube denoted by a tensor $X=\{X^1,X^2,\ldots..X^B\}$, where B denotes the number of bands, can be described as

$$Y = X + E \quad \text{(III.1)}$$

Where X and E are with the same size of Y, which represent an original image and the mixed noise, respectively. Now the objective of HSI restoration is to estimate X from the observed Y. We divide the noise term E into two sub-terms as Gaussian noise term N and the sparse noise term S including strip noise, salt and pepper noise, and deadlines are leading to the following degradation model:

$$Y = X + N + S \quad \text{(III.2)}$$

As such, two noise terms N and S can be modeled by Frobenius norm and the `L norm respectively.

The purpose of image restoration is to "compensate for" or "undo" defects which degrade an image. Degradation comes in many forms such as motion blur, noise, and camera misfocus. Gaussian noise is statistical noise having a probability density function (PDF) equal to that of the normal distribution, which is also known as Gaussian distribution. In other words, the values that the noise can take on are Gaussian-distributed. The probability

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density function p of a Gaussian random variable z is given by

$$P(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \qquad \text{(III.3)}$$

Where z represents the grey level, μ the mean value and σ the standard deviation. Compared with Gaussian noise, the stripe has significantly structural characteristics. Moreover, these line structures exhibit directional characteristic.

Impulse noise is a category of (acoustic) noise, which includes unwanted, almost instantaneous sharp and sudden disturbances. It presents itself as sparsely occurring white and black pixels. Noises of the kind are usually caused by electromagnetic interference. An effective noise reduction method for this type of noise is a median filter or morphological filter.

B. Higher Order Singular Value Decomposition

Different types of preconditioners have been proposed for image restoration in 2D and 3D. All of these preconditioners have been executed based on blurring matrix. In this section, we demonstrate that based on tensor modeling framework used in, one can obtain a HOSVD preconditioner based on an approximation of the blurring tensor. Now consider the HOSVD of the blurring tensor A as follows

$$A = (U^{(1)}, \dots, U^{(2N)})_{1:2N}.S \qquad \text{(III.4)}$$

In remote sensing, satellite captured an exact image of an area with different resolutions. For our purpose, captured images are transmission from satellite to earth station. Due to presents of noise in atmosphere transmission images to be degraded. That degraded HSI images are taken as to restoration problems. The degraded HSIs are of size large and that matrix format has large number of pixels, that's leads to computational complexity. To avoid this complexity performed tensorization process to received degraded images. The first columns of singular matrices $U^{(i)}$ in every mode are smoother than the last ones. So, for an appropriate index

$$K_i, U_{ki}^{(i)} = [u_1^{(i)}, \dots, u_{ki}^{(i)}] \in R^{ni\times ki} v \qquad \text{(III.5)}$$

Which denotes the first ki columns of $U^{(i)}$ has an important role in the reconstruction of the exact image. This means that if S_k denotes the first part of tensor. By these properties, the HOSVD of blurring operator A defined as

$$M = (U_{k1}^{(1)}, \dots, U_{k2}^{(2N)})1:2N.^{S_k} \qquad \text{(III.6)}$$

Also, for small values of k_i in k,

$$||A - M||^2 = ||S||^2 - ||S_k||^2 \qquad \text{(III.7)}$$

The matrix representation of this preconditioner is

$$M = U_K S_K V_K^T \qquad \text{(III.8)}$$

Therefore, at each step of preconditioned iterative method finding the solution with minimum length of the following least squares problem

$$\min_x ||M_q - Z|| \qquad \text{(III.9)}$$

The tensor based Image restoration where the image is converted into tensor by the process of the tensorization. Similarly the tensor data is converted to matrix form by the process of matricization.

The degraded image is first converted into tensor by using gradient function. The gradient is rate of change of a function. The term "gradient" is typically used for functions with several inputs and a single output. Image gradients can be used to extract information from images. The Gaussian noise tensor, and sparse noise tensor is created. This noise tensor is added to create a mixture noise tensor. Using this noise tensor, the degraded image is restored to get back the restored image. From the input HIS degraded image singular matrix to be found by using HOSVD preconditioner. Usually, iterative methods are used to solve this restoration peoblems.

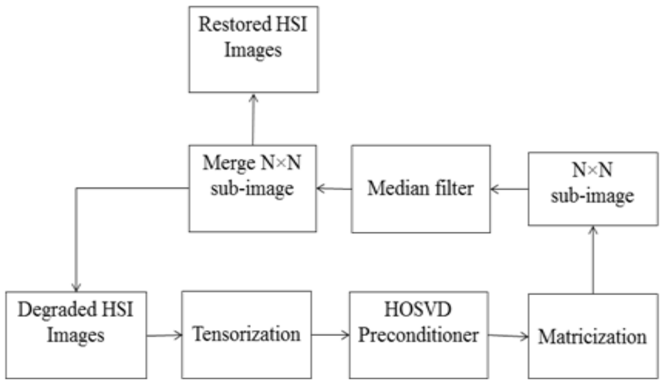

Fig 1:- Block diagram of proposed method for HSI restoration.

The proposed system restores the degraded images that are degraded in remote sensing application. The satellite captures an exact image, that will degraded when transfer from satellite to earth station. The degraded HSI are first tensorized to obtain the HSI tensor. Similarly a Gaussian tensor is created from the Gaussian matrix and sparse tensor is created from the sparse matrix. This Gaussian tensor and sparse tensor are added with the HSI tensor to obtain the restored image. The HOSVD is most commonly applied to the extraction of relevant information from multi-way arrays. The singular matrix can be used adjust the intensities of noise pixels in an image by performed masking operation. The iteration stops if the mean square error between any two successive iterations is less than a threshold value. The obtained images are partially restored. To improves the performance of proposed system filtering technique to be used.

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C. Iterative Median Filter

Median Filtering is a type of nonlinear filtering technique, often used to reduce noise from degraded image. Such noise reduction is a typical pre-processing step to improve the results of later processing. In this type of filtering all of the pixels in an *n* by *n* square mask of the image are selected, where *n* is an odd number usually 3 or 5. The center of the mask is a lost pixel that is to be restored. Median filter process which estimates by sorting all the pixel values from the surrounding neighborhood and replacing the pixel with center pixel value. This procedure is performed for all the lost pixels in the image. The main idea of the median filter is to run through the pixel entry by entry, replacing each entry with the median of neighbors is called "window", which slides, entry by entry over the entire pixels. Note that if the window has an odd numbers of entries, then the median is simple to define: it is just the middle value after all entries in the window are sorted numerically. If the neighborhood under consideration contains an even number of pixel, then average of the two pixel values is used. The detection of noisy and noise-free pixels can be found by intensity of particular pixel lies between the maximum (max) and minimum (min) value for a selected window. If the value of the particular pixel denoted by $p(x, y)$, is within the range ($0<p(x, y)<255$), then it is an noise-free pixel and left unchanged. If the value does not lie within this range, then it is a noisy pixel and is replaced by median value of selected window. A median filtering that can be applied iteratively is proposed. The degraded HSI are first divided into NxN sub-images. The median filtering is applied to all sub-image, that will replaces a pixel by median value instead of average of all pixels in neighborhood and the resulting filtered outputs are merged. This process to be performs until the iterations get completed.

If MSE between any two successive iteration is greater than 0.2, then subdivide an image into NxN sub-images to improve computational efficiency of median filter using non-overlapping method. For a small size of blocks, an efficiency of iterative median filter is improved. The sub-image size selection is one of the important factors. In most of the applications, the sub-image size is selected as *n* x *n* such that *n* is an integer power of 2. The level of computation increases as the sub-image size increases. The experiment have been conducted by sub-image size *nxn* for *n*=16, or 32 and apply filtering to all sub-images. The median filtering to be applied iteratively until MSE between successive iteration is less than 0.2. The proposed system has lower computational complexity, high convergence speed and improved performance of system when compared to other previous methods. The steps of proposed systems are:

Step1: Get the degraded Hyper Spectral images.
Step2: Convert an input Hyper Spectral images to Tensor cube using Tensorization process.
Step3: Estimate the Gaussian and sparse coefficients from the degraded input image using HOSVD preconditioner.
Step4: Convert the Gaussian and sparse coefficients to Tensor cube using Tensorization process.
Step5: Add the Gaussian and sparse tensor obtained in step 4 with the degraded image tensor obtained in step 2.
Step6: Convert the Tensor obtained in step 5 to matrix using matricization process.
Step7: If the MSE between two successive iteration is greater than 0.2,

- Sub-divide an image into NxN sub-images.
- Apply median filtering to all sub-images.
- Merge all sub-images into a single image.

Step8: Repeat step 1 to 7 until the Mean Square Error (MSE) between any two successive iteration is less than 0.2.

The restored NxN sub-images from median filter can be merged into a single large sized image by merge neighboring blocks. The small square blocks are then merged if they are adjacent. The merging operations eliminate false boundaries and spurious pixels by merging adjacent blocks. The process terminates when no further merges are possible.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

The proposed an image restoration technique is implemented in the working platform of MATLAB with machine configuration. In our proposed method, the degraded images are given to an image restoration process by using the techniques HOSVD and median filter. The original images or clean data captured by a remote sensing satellite (Remote sensing 1) on same area with different resolution are given Fig. 2. The image that was transmitted by the satellite was degraded because of the noise such as Gaussian noise, salt and pepper noise and strip noise. Therefore the received signal was degraded as shown in Fig. 3. The output restored images from HOSVD preconditioner for 4 noisy inputs such as Gaussian noise, salt and pepper noise, strip noise and combination of all the three noises as shown in Fig. 4., that will reduces some amount of noise. The restored images from iterative median filter for different noise case as shown in Fig. 5. From figure shows that, the restoration with median filter result is better than without median filter. The proposed system performed in iterative process and less number of iterations only required for restoration.

Fig 2:- An Original Image (Remote Sensing 1)

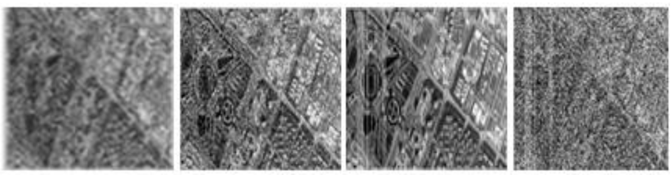

Fig 3:- Received Degraded Image

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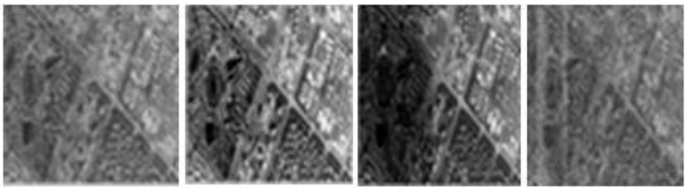

Fig 4:- Restored Images from HOSVD

Fig 5:- Restored Images from Median Filter

Similarly, Fig. 6. shows the original images captured by a remote sensing satellite (Remote sensing 2) and degraded images as shown in Fig. 7. Fig. 8. shows the restored images from HOSVD for an input images remote sensing 2. The restored image appears to be good for salt and pepper noise and strip noise, when compared to other two restorations. The restored images from iterative median filter for different noise case as shown in Fig. 9. The restoration result with median filter appears to be better for all noisy images than restoration without median filter.

Fig 6:- Original Image (Remote Sensing 2)

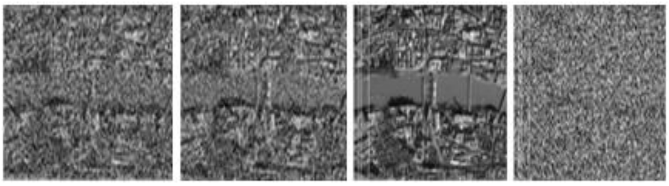

Fig 7:- Received Degraded Image

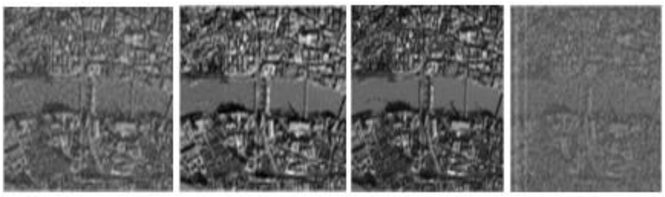

Fig 8:- Restored Images from HOSVD

Fig 9:- Restored Images from Median Filter

A. Performance Analysis

The performance evaluation of the proposed system is carried out by calculating the metrics such as Peak Signal to Noise Ratio (PSNR), Mean Square Error (MSE), Maximum Error, root Mean Square Error. The performance of proposed system can be evaluated by comparing the restored HSI with satellite captured original images.

Let *f(x, y)* be the original image with size MxN and *f'(x, y)* be the restored image with the same size. The MSE measures the average of squares of errors that is, the difference between the restored and an original image is expressed as in

$$\text{MSE} = \frac{1}{MN} \sum_{xy} \left(f'(x,y) - f(x,y)\right)^2 \qquad \text{(IV.1)}$$

And the PSNR is the ratio of maximum possible power and the power of corrupting noise in decibel is expressed as in

$$\text{PSNR} = 10 \log_{10}\left(\frac{255^2}{MSE}\right) \qquad \text{(IV.2)}$$

The Maximum Error is the difference between an original and restored value as expressed as

$$E = z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \qquad \text{(IV.3)}$$

Root Mean Square Error (RMSE) measures how much error there is between two data sets. In other words, it compares a predicted value and an observed value as expressed as in

$$\text{RMSE} = \sqrt{\frac{1}{MN} \sum_{xy} \left(f'(x,y) - f(x,y)\right)^2} \qquad \text{(IV.4)}$$

Table 1 shows the performance of proposed system for Remote sensing 1. From this table, the PSNR value is high for Image 3, that was degraded with strip noise and the PSNR value is low for image 4, which is the combination of all three noise. The number of iterations is less than 50, which is a less value. The performance of proposed system is best because it has high PSNR and less Mean square error.

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Images	PSNR	MSE	Maximum Error	Root mean square Error	Iteration
Image 1	9.6088	7115	118	84.35	41
Image 2	10.2783	6098	127	78.09	
Image 3	12.5164	3642	127	60.35	
Image 4	7.6860	11079	127	105.25	

Table 1:- Performance of Proposed System for Remote Sensing 1

Table 2 shows the performance of the proposed system for the Remote sensing 2. It shows that the PSNR results for different noise models that have been implemented with different noise intensities of an image. The PSNR is high for Salt and pepper noise image restoration and strip noise restoration, the number of iterations is found to be 37, which is less than the conventions methods. It could be observed that the proposed method provide a far better performance when compared with other well known methods.

Images	PSNR	MSE	Maximum Error	Root mean square Error	Iteration
Image 1	15.14	1987	98	44.57	37
Image 2	21.14	499	127	22.34	
Image 3	20.50	579	90	24.06	
Image 4	14.66	2220	106	47.12	

Table 2:- Performance of Proposed System for Remote Sensing 2

Fig. 10. shows the MSE between adjacent iterations. From the graph, black color resembles the MSE of Gaussian noise image restoration, red color resembles the MSE of salt and pepper noise image restoration, blue color resembles the MSE of strip noise image restoration and green color resembles the MSE of all noise image restoration. Fig. 10. graphically illustrate the MSE value of proposed system for an image corrupted by different noise.

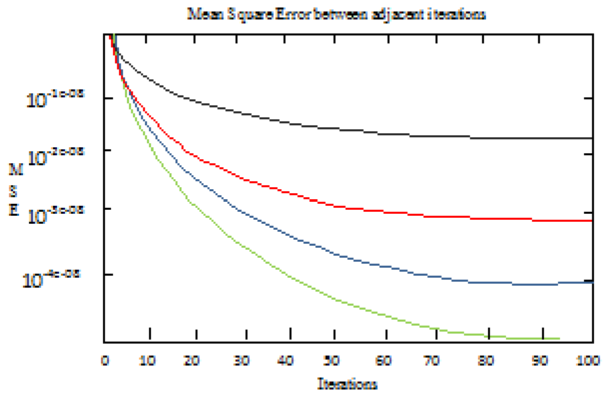

Fig 10:- MSE between adjacent iterations

Table 3 shows the performance of system with and without median filter for an input image Remote sensing 1. From this table, the performance is worst for combination of all the three noise, when using system without filtering and the PSNR value is high for mixed noise, when using system with filtering. The performance of proposed system with median filter is improved when compared to system without median filter. The number of iterations is found to be 39 which is less than the preconditioner. Table IV shows the performance of the system with and without median filter for an input image Remote sensing 2. From this table, the PSNR is high for Salt and pepper noise image restoration and strip noise restoration and the number of iterations is found to be 38, when system without iterative filtering. The PSNR value is high for mixed noise and the number of iterations is found to be 36, when system with iterative filtering. Table 1 shows that, the performance of proposed system

Performance analysis without iterative filtering					
Noise case	**PSNR**	**MSE**	**Maximum Error**	**Root mean square Error**	**Iteration**
Gaussian	14.6836	2212	105	47.0285	44
Impulse	16.4182	1483	127	38.5149	
Strip	13.3326	3019	127	54.9430	
Mixed	13.3411	3013	108	54.8893	
Performance analysis with iterative filtering					
Gaussian	16.1788	1567	63	39.5915	39
Impulse	15.1375	1992	64	44.6341	
Strip	15.4596	1850	68	43.0090	
Mixed	16.8510	1343	63	36.6427	

Table 3:- Comparative Performance of Proposed System for Remote Sensing 1

Performance analysis without iterative filtering					
Noise case	**PSNR**	**MSE**	**Maximum Error**	**Root mean square Error**	**Iteration**
Gaussian	15.1488	1987	96	44.5759	38
Impulse	21.1463	499	127	22.3473	
Strip	20.4994	580	127	24.0753	
Mixed	14.6661	2221	106	47.1235	
Performance analysis with iterative filtering					
Gaussian	17.8594	1064	63	32.6265	36
Impulse	18.6925	878	64	29.6426	
Strip	19.0122	816	63	28.5712	
Mixed	19.2381	775	63	27.8378	

Table 4:- Comparative Performance of Proposed System for Remote Sensing 2

With median filter is better than the system without median filter.

From this project work, it can be easily observe that proposed method yield more satisfying results and can applied to different type of images. In future this method can be applied to degraded images from scanner and also uses different filtering methods to restore blurring images.

V. CONCLUSION

In this paper, we demonstrated that image restoration could be modeled with the tensor framework. Based on this context, we proposed a tensor-based preconditioner and iterative filtering for removing mixed noise in HSIs based on using an approximation of HOSVD. Instead of dealing as pixels the image is processed as tensor. This project reduces the noises that occur in remote sensing satellite images. The satellite images are degraded by the noise such as Gaussian noise, Impulse noise, Strip noise etc. The tensor of the degraded image is added with tensor of Gaussian, Impulse, Strip noise. The Singular values are extracted from the degraded image which controls intensity of Gaussian, Impulse and Strip noise tensor. This process gets repeated iteratively to get the restored image. The boundary condition for the iteration is based on minimum distance between any two successive iteration is less than a threshold value. Also, experimental results confirm the high quality of the proposed preconditioner in speeding up the convergence rate of iterative restoration methods. We suggested to apply median filtering on the images with mixed noise iteratively. The suggested method has a low implementation complexity. The Performance analysis of the median filter shows that an image with much lower pixel loss. The results shows that the complexity of the image restoration process reduces highly because the proposed system restores the image only on less number of iterations further we measures Peak Signal Noise Ratio (PSNR) and Mean Square Error (MSE). The PSNR values appear to be HIGH for the restored images while the MSE values appear to be LOW.

REFERENCES

[1]. M. Rezghi, S. M. Hosseini, and L. Elden, "Best Kronecker product approximation of The blurring operator in three dimensional image restoration problems", SIAM J. Matrix Anal. Appl., vol. 35, pp. 1086-1104, 2014.

[2]. C. He, C. Hu, X. Li, and W. Zhang, "A parallel primal- dual splitting method for image restoration", Information Science, vol. 358, pp. 73-91, 2016.

[3]. A. F. H. Goetz, "Three decades of hyperspectral remote sensing of the earth: a personal view", Remote Sensing of an Environment, vol. 113, pp. S5–S16, 2009.

[4]. R. Willett, M. Duarte, M. Davenport, and et al, "Sparsity and structure in hyperspectral imaging: Sensing, reconstruction, and target detection", IEEE Signal Process. Mag., vol. 31, no. 1, pp. 116–26, 2014.

[5]. M. Rezghi, S. M. Hosseini, "Lanczos based preconditioner for discrete ill-posed problems Computing", vol. 88, pp. 79-96, 2010.

[6]. P. C. Hansen, J. Nagy, and D. P. O'leary, Deblurring Images: Matrices, Spectra and Filtering, SIAM, Philadelphia, 2006.

[7]. M. Hanke, J. Nagy, and R. Plemmons, "Preconditioned iterative regularization methods for ill-posed problems", in: Reichel L, Ruttan A, Varga RS (Eds.), Numerical Linear Algebra, 1993, de Gruyter, Berlin, Germany, pp. 141-163.

[8]. M. Hanke, Conjugate Gradient Type Methods for Ill-Posed Problems, Chapman and Hall/CRC, 1995.

[9]. G. Landi, E. Loli Piccolomini, and I. Tomba, "A stopping criterion for iterative regularization methods", Applied Numerical Mathematics, vol. 106, pp. 53–68, 2016.

[10]. P. Dell'Acqua, M. Donatelli, S. Serra-Capizzano, D. Sesana, and C. Tablino-Possio, "Optimal preconditioning for image deblurring with Anti-Reflective boundary conditions", Linear Algebra and its Applications, vol. 502, pp. 159–185, 2016.

[11]. P. Dell'Acqua, M. Donatelli, and C. Estatico, "Preconditioners for image restoration by reblurring techniques", Journal of Computational and Applied Mathematics, vol. 272, pp. 313–333, 2014.

[12]. **F.** *G* Marvasti, C. Liu, and ***G.*** Adams., "Analysis and recovery of multidimensional signals kom irregular samples using non-linear and iterative techniques", in Proc. IEEE ISCAS '92, vol. ***5,*** Miy 1992, pp. 2445-2448.

[13]. R. C. Gonzalez and R **E.** Woods, Digital Image Processing, Prentice Hall Pub. Co., 2nd Ed., 2.002.

[14]. J. Liu, P. Musialski, P. Wonka, and J. Ye, "Tensor completion for estimating missing values in visual data", In ICCV, 2009, pp. 2114–2121.

[15]. Q. Xie, Q. Zhao, D. Meng, Z. Xu, S. Gu, W. Zuo, and L. Zhang, "Multispectral image denoising by intrinstic tensor sparsity regularization", in CVPR, 2016, pp. 1692–1700.

[16]. D. Goldfarb, and Z. Qin, "Robust low-rank tensor recovery: models and algorithms", SIAM J. Matrix Anal. Appl., vol. 35, pp. 225-253, 2014.

[17]. A. Rosenfeld, and A. C. Kak, Digital picture processing, vol. 1, Elsevier, 1982.

[18]. D. T. Quan, A. A. Sawchuck, T. C. Strand, and P. Chavel, "Adaptive noise smoothing filter for image with signal-dependent noise", IEEE Trans. Pattern Anal. Mach. Intell., vol. PAMI-7, pp. 164-177, 1985.

[19]. J. Astola, and P. Kuosmanen, "Fundamentals of nonlinear digital filtering", CRC Press Inc., Boca Raton, 1997, pp. 288, ISBN 0-8493-2570-6.

[20]. I. Pitas, and A. N. Venetsanopoulos, "Nonlinear digital filters", Kluwer Academic Publishers, 1990.

[21]. H. Hwang, and R. A. Haddad, "Adaptive median filters: New algorithms and results", IEEE Trans. Image Process., vol. 4, no. 4, 1995.

[22]. Amir R. Fourouzan, and Babak N. Araabi, "Iterative median filtering for restoration of images with impulsive noise", 2003 IEEE, ICECS 0-7803-8163-7.

[23]. S. Samsad Beagum, and M. Mohamed Sathik, "Improved Adaptive Median Filters Using Nearest 4-Neighbors for Restoration of Images Corrupted with Fixed-Valued Impulse Noise", 2015 IEEE, ISBN 978-1-4799-7849-6.

[24]. J. Jezebel Priestley, and V. Nandhini, "A decision based switching median filter for restoration of images corrupted by high density impulse noise", 2015 IEEE, ISBN 978-81-925974-3-0.