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The Effect of Varying Magnetic Number, Reynolds Number and Pressure Gradient on Velocity Profiles in an MHD Flow

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Abstract:- Magnetohydrodynamic ow of a hot viscous electrically conducting incompressible uid through parallel plates is studied. In the study, the e ect of Hartmann number (M), pressure gradient and Reynolds number (Re) on the velocity eld is investigated. The Navier-stokes equations were coupled with Ohms law and then solved using nite di erence method (FDM). The velocity eld was computed for var-ious values of the physical parameters and shown graphically. It was found that as the Hartmann number M increases, the velocity pro les decreased due to increased Lorents force while an increase in Reynolds number causes an increase in the velocity of the uid. All these analysis was done using MATLAB program and the results were presented in tables and graphs.

***Keywords:-** Magneto hydrodynamic uid ow, Hartmann number, Reynolds number, pressure gradient, skin friction.*

I. INTRODUCTION

The magneto hydrodynamic (MHD) ow between two parallel walls has many applications in MHD power generators, MHD pumps, ac-celerators etc. Particulary, the MHD ows a liating with heat trans-fer have acquired considerable attention so far. The ability to control the ow of uid metal at high temperature through magnetic eld without mechanical in uence has many applications.[13]

The MHD principle is used for pumping uids that are hard to pump by convectional pumps. MHD molten metal pump is a replace-ment to conventional pumps because their moving parts cannot stand molten metal temperature. The need for MHD pumps is increasing due to its advantages and wide applications.[10] Systems with very high temperature like molten metal or liquid can be driven using MHD force. The pump is silent and reliable since there are no moving parts hence requires minimal maintenance. This MHD pumps are applied in pumping sea water, molten salt, molten metal and nano uid.

When a unidirectional current is established through electrically con-ducting uid and then a high intensity magnetic eld perpendicular to the current is imposed through the uid, this combination of or-thogonal magnetic eld and electric eld and a relative motion of ions results in a Lorentz force with direction de ned by the cross product of current and magnetic eld vectors.[10]

The equations which describe MHD ow are a combination of con-tinuity equation and Navier-stokes equation of uid dynamics and Maxwell's equation of electromagnetism.[11] The study of electric conducting uid ows eg plasmas, liquid metals, salt water and air has gained popularity in our world today. This has attracted many researchers to carry out research in the same eld since it has found its application in many areas that involves study of electrically con-ductinguids.

Je-Ee (2007) focused on the prediction of pumping performance in MHD ow. He used an analytical model based on steady state, incompressible and fully developed lamina ow theory to analyze the ow characteristics with di erent scalar dimensions in the rectangu-lar duct.[4] Daoud and Kandev (2008) did a study on DC electromag-netic pump for liquid metal (aluminum) at large Reynolds number under externally imposed non-uniform magnetic eld.[1]

Sim and Choi (2008) used nite di erence method in solving the velocity pro le of the working uid across the micro channel under various operation current and magnetic ux densities. They used a commercial CFD code called CFE-ACE for simulating the MHD pump.[2]

Kandev, Kagan and Daoud (2010) considered an electromagnetic pump for both lamina and turbulent metal ow under an externally imposed strongly non-uniform magnetic eld. Die rent cases were simulated using nite element method.[3] Kuiry and Bahadur (2011) investigated the e ect of external uniform transverse magnetic eld, pressure gradient on the ow and the temperature. They used - nite di erence method to solve the momentum transfer and energy equation. [12]

Idowu and Olabode (2014) discussed the e ects of magnetic in-clination to velocity and skin friction ignoring the pressure gradient. The lower plate was considered porous. Momentum equation was solved by variable separable technique.[8]

Mburu, Kwanza and Onyango (2016) considered the e ect of Hartmann number, the angle of magnetic inclination, pressure gradient and Reynolds number on the ow.[6]. Chitia (2016) investigated the e ects of various parameters such as Hartmanns number, Grashof number, Prandtl number, Eckert number and angle of inclination on the velocity and temperature distribution.[5]

Alireza et al. (2018) investigated the e ect of Deborah numbers Hartman electric number, Reynolds number and prandtl number on the velocity and temperature elds of MHD ow. [10] Aruna (2019) did a study on the e ect of applied gradient on MHD ow between parallel plates under in uence of inclined magnetic eld by di eren-tial transform method. The upper plate was moving at a constant velocity while the lower plate held stationary.[7]

From the previous research done none of the researchers has con-sidered solving the governing equations using nite di erence method and computing the velocity pro les using the MATLAB software. In this research we considered a two dimensional ow between two par-allel horizontal plates in the x-direction. The incompressible viscous electrically conducting uid inside the plates is subjected to exter-nally applied magnetic eld perpendicular to uid ow. The problem is to solve the equations governing the MHD ow when the pressure gradient is applied in the x-direction. We investigated the e ect magnetic parameter, Reynolds number and pressure gradent on the velocity pro les. The gorverning equations were solved using the - nite di erence method. This will help in identifying the best working condition in the MHD pumps that will give the uid highest velocity.

II. METHOD OF SOLUTION

We considered 2 dimensional ow with coordinates (x; y), time t, density , pressure p, velocity components (u; v) and Prandtl Pr.

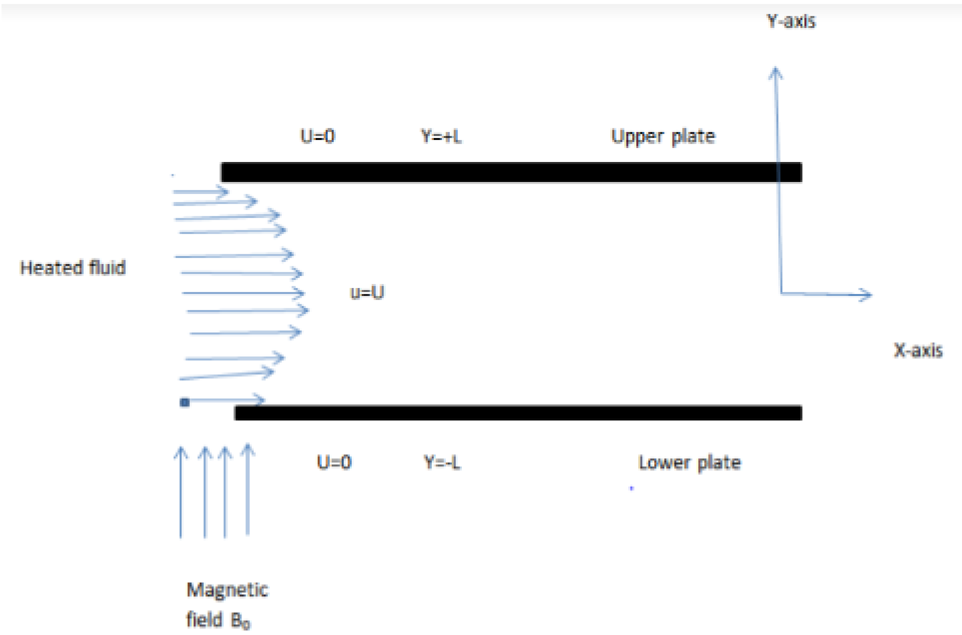

Fig 1:- Flow of a hot uid between two parallel plates in a transverse magnetic Feld.

The equations that govern the ow are;

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] + \frac{1}{\rho}\left(\vec{J} \times \vec{B}\right) \quad (1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] + \frac{1}{\rho}\left(\vec{J} \times \vec{B}\right) \quad (2)$$

The fluid is considered incompressible, steady and two dimensional. the non slip condition is satisfied and there is a constant pressure gradient $\frac{dp}{dx}$ applied to the fluid.

0.1 Mathematical analysis

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] + \frac{1}{\rho}\left(\vec{J}\times\vec{B}\right) \tag{3}$$

Given that the flow is steady, and is in x-direction only then v=0 then

$$\frac{\partial u}{\partial t} = 0 \tag{4}$$

Equation (3.7) reduces to

$$u\frac{\partial u}{\partial x} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] + \frac{1}{\rho}\left(\vec{J}\times\vec{B}\right) \tag{5}$$

since v=0 , the equation of continuity also reduces to

$$\frac{\partial u}{\partial x} = 0 \tag{6}$$

Using equation (3.10) in equation (3.9), it simplifies to

$$0 = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left[\frac{\partial^2 u}{\partial y^2}\right] + \frac{1}{\rho}\left(\vec{J}\times\vec{B}\right) \tag{7}$$

simplifying $\left(\vec{J}\times\vec{B}\right)$

given that $\vec{J} = \sigma(\vec{V}\times\vec{B})$

$$\vec{B} = \begin{bmatrix} 0 \\ B_0 \\ 0 \end{bmatrix} \text{ and } \vec{V} = \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{V}\times\vec{B} = \begin{bmatrix} i & j & k \\ u & 0 & 0 \\ 0 & B_0 & 0 \end{bmatrix}$$

Which simplifies to $uB_0\vec{k}$.
Hence $\vec{J} = \sigma u B_0$

$$\text{Now } \vec{J}\times\vec{B} = \begin{bmatrix} i & j & k \\ 0 & 0 & \sigma u B_0 \\ 0 & B_0 & 0 \end{bmatrix}$$

which reduces to $-\sigma u B^2{}_0$
Substituting into equation [3.11] we obtain

$$0 = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left[\frac{\partial^2 u}{\partial y^2}\right] - \frac{1}{\rho}\sigma u B^2{}_0 \tag{8}$$

Non-dimensionalising equation (3.12) using the following

$$x^* = \frac{x}{L}, y^* = \frac{y}{L}, u^* = \frac{u}{U}, \nabla^* = \frac{\nabla}{L}, \nabla^2 = \frac{\nabla*^2}{L^2}, p^* = \frac{p}{\rho U^2} \qquad (9)$$

Equation (3.12) reduces to

$$0 = \frac{-\rho U^2}{\rho L}\left[\frac{\partial p^*}{\partial x^*}\right] + \frac{\mu U}{\rho L^2}\left[\frac{\partial^2 u^*}{\partial y^{*2}}\right] - U\frac{1}{\rho}\sigma u^* B^2{}_0 \qquad (10)$$

multiplying both sides by $\frac{\rho L^2}{\mu U}$

$$0 = \frac{-U\rho L}{\mu}\left[\frac{\partial p^*}{\partial x^*}\right] + \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B^2{}_0 L^2 u^*}{\mu} \qquad (11)$$

using $\frac{U\rho L}{\mu}$ as the Renolds number Re

and $\frac{\sigma B_0^2 L^2}{\mu}$ as the square of the Hartmann number $(Ha)^2$ or M^2

equation (3.15 reduces to

$$0 = -Re\left[\frac{\partial p^*}{\partial x^*}\right] + \frac{\partial^2 u^*}{\partial y^{*2}} - M^2 u^* \qquad (12)$$

dropping the star in equation (3.16)

$$0 = -Re\left[\frac{\partial p}{\partial x}\right] + \frac{\partial^2 u}{\partial y^2} - M^2 u \qquad (13)$$

seting a constant pressure gradient $-\frac{\partial p}{\partial x} = P$ then

$0 = PRe + \frac{\partial^2 u}{\partial y^2} - M^2 u$ is then solved using the finite diffence method

0.1.1 Method of solution

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_{j+1} - 2u_j + u_{j-1}}{h^2} \qquad (14)$$

and $u = u_j$ then we have

$$0 = PRe + \frac{u_{j+1} - 2u_j + u_{j-1}}{h^2} - M^2 u_j \qquad (15)$$

$$0 = h^2 PRe + u_{j+1} - 2u_j + u_{j-1} - h^2 M^2 u_j$$

which can be written as

$$0 = h^2PRe + u_{j+1} - [2 + h^2M^2]u_j + u_{j-1}$$

where h is the step size, Re is the reynolds number and P is the applied pressure gradient.

$$u_j = \frac{u_{j+1}}{2 + h^2M^2} + \frac{u_{j-1}}{2 + h^2M^2} + \frac{h^2PRe}{2 + h^2M^2} \tag{16}$$

The equation shall be solved subject to the following boundary conditions governing the flow.
$y = 0, u = U$, $y = L, u = 0$ and $y = -L, u = 0$
In dimensionless form ,
$y = 0, u = 1$, $y = 1, u = 0$ and $y = -1, u = 0$
For the values of $j = 1, 2, 3 - - - - - n$ equation (3.20) can be written as

$$\left.\begin{array}{lll}
j = 1 & : & u_1 = \frac{u_2}{2 + h^2M^2} + \frac{u_0}{2 + h^2M^2} + \frac{h^2PRe}{2 + h^2M^2} \\
j = 2 & : & u_2 = \frac{u_3}{2 + h^2M^2} + \frac{u_1}{2 + h^2M^2} + \frac{h^2PRe}{2 + h^2M^2} \\
j = 3 & : & u_3 = \frac{u_4}{2 + h^2M^2} + \frac{u_2}{2 + h^2M^2} + \frac{h^2PRe}{2 + h^2M^2} \\
\vdots & & \vdots \\
j = n-1 & : & u_{n-1} = \frac{u_n}{2 + h^2M^2} + \frac{u_{n-2}}{2 + h^2M^2} + \frac{h^2PRe}{2 + h^2M^2} \\
j = n & : & u_n = \frac{u_{n+1}}{2 + h^2M^2} + \frac{u_{n-1}}{2 + h^2M^2} + \frac{h^2PRe}{2 + h^2M^2}
\end{array}\right\} \tag{17}$$

These system of equations are solved using Gauss Siedel iteration.

The Skin friction is given as $cf = \left.\frac{du}{dy}\right|_{y=0}$

$$\left.\frac{du}{dy}\right|_{y=0} = \left.\frac{u_{j+1} - u_{j-1}}{2h}\right|_{y=0} \tag{18}$$

III. RESULTS

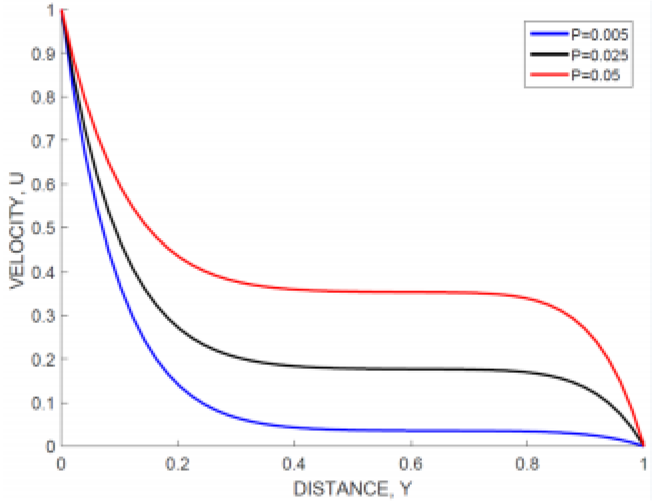

Fig 2:- Velocity profiles for different values of pressure gradient.

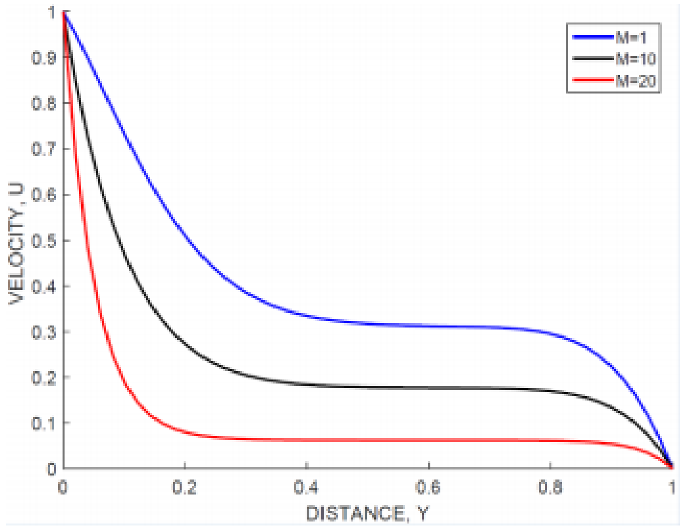

Fig 3:- Velocity profiles for different values of Hartmann number M

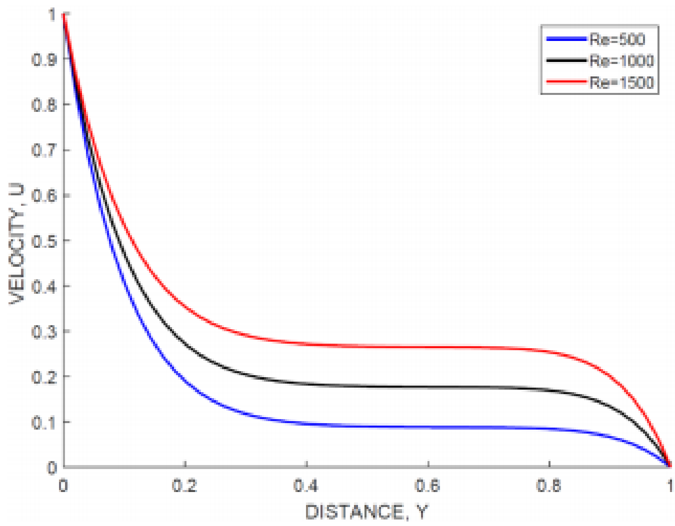

Fig 4:- Velocity profiles for different values of Reynolds number Re

		skin friction	
x[0]	M_1=1	M_2=10	M_3=20
0.02	-2.5226	-6.7937	-12.766
0.14	-2.5226	-6.7937	-12.766
0.27	-2.5226	-6.7937	-12.766
0.39	-2.5226	-6.7937	-12.766
0.51	-2.5226	-6.7937	-12.766
0.63	-2.5226	-6.7937	-12.766
0.76	-2.5226	-6.7937	-12.766
0.88	-2.5226	-6.7937	-12.766

Table 1: Variation of the Skin friction for values of M

		skin friction	
x[0]	Re_1 = 500	Re_2 = 1000	Re_3 = 1500
0.02	-7.6432	-6.7937	-5.9442
0.14	-7.6432	-6.7937	-5.9442
0.27	-7.6432	-6.7937	-5.9442
0.39	-7.6432	-6.7937	-5.9442
0.51	-7.6432	-6.7937	-5.9442
0.63	-7.6432	-6.7937	-5.9442
0.76	-7.6432	-6.7937	-5.9442
0.88	-7.6432	-6.7937	-5.9442

Table 2: Variation of the Skin friction for values of Reynolds number

IV. DISCUSSION

From gure (1), An increase in the pressure gradient leads to an in-crease in in the velocity pro le. Since $p = \frac{-\partial p}{\partial x}$ when p is positive, the pressure gradient is negative. This nega-tive pressure gradient indicates that pressure is decreasing in the x-direction. Due to this pressure gradient, there is a force that acts on the uid hence leading to an increase in the uid velocity.

In gure (2), Magnetic parameter was varied while all other param-eters were held constant. An increase in the Magnetic parameter M leads to decrease in the velocity pro les of the uid. This is because Hartmann number is the ratio of magnetic forces to viscous forces so the larger the Hartmann number the stronger the magnetic forces hence there will be high Lorents force which reduces the velocity pro les.

From gure (3), As the Reynolds number increases, the velocity of the uid also increase. When p is positive, the pressure gradient is negative indicating a decreasing pressure in the x direction. Because of increased Reynolds number, there is reduced viscous forces which causes an increase in the velocity pro les.

Table 1 and 2 shows variation of the skin friction cf for values of Hart-mann number M and Reynolds number Re. As the Hartmann num-ber M increases the skin friction cf decrease while the skin friction increases for increasing values of the Reynolds number Re. Increase in Reynolds number means there is reduced viscous forces. This leads to increase in the velocity of the uid thus increase in the velocity gradient. As the Hartmann number increase, the Lorents force also increase causing a retarding e ect on the velocity and hence a de-crease in the velocity gradient.

V. CONCLUSION

An increase in the pressure gradient and Reynolds number Re leads to to an increase in the velocity pro les. While an increase in mag-netic parameter M leads to an increase in the lorents force which has a retarding e ect on the velocity pro les. Surface shear stress increases with increase in magnetic parameter m while it reduces with increasing values of Reynolds number. The permormance of the MHD pump can be predicted by varying these parameters.

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