An Integrated EPQ Inventory Model for Delayed Deteriorating Items with Time and Price Dependent Demand with Inflation under Discount Policy

Monika Rani
Department of Mathematics C.C.S University Meerut

Abstract:- In this article, we will study an integrated EPQ inventory model for decaying items with time and price dependent demand under discount for both deteriorated and fresh units. In EPQ model, we have used the discount to moderate the inventory and growth the profit or rate of sale. In this concept, the salvage value is merged to the failed units. We establish a model that gives the best maximizes annual profit. We also discover the relationship between demand depending factors and discount policy. A mathematical preparation for the model by using differential equations is developed. Numerical example and sensitivity have been used in this model.

Keywords:- *EPQ, decaying items, Inflation, Time and price dependent demand, discount.*

I. INTRODUCTION

Deterioration is an active area used in our life and can be defined as decay, damage, decomposition and loss of quality from the original condition. The study of decaying inventory continuing with Ghare and Schrader [1], they established the traditional no-shortage inventory model with a continuous rate of decaying items. Nahmias [2], Goyal and Giri [3] provided a complete and updated review of available works for the decaying inventory models. Baker and Urban [4] represented an EOQ model with a control form inventory level demand. Teng and Chang [5] have established an EPQ model for decaying items when the demand rate depends on amount and on-display inventory level. Wee [6] established a model for shared pricing and renewal policy for decaying inventory with price variable demand rate that declines ended time. Singh et al. (7) resultant an inventory model for decaying items with inventory level demand with acceptable shortages.

Nowadays inflation has become a regular feature. Inflation follows due to an inequality between demand and supply of money, fluctuations in production and delivery cost or growth in taxes on products. When the economy involvements inflation, i.e. when the price level of goods and facilities rises, the value of money decreases. Buzacott [8] first represented an EOQ model with inflation issue to changed types of pricing rules. Hou [9] represented an inventory model for decaying items with inventory dependent depletion rate and shortages under inflation and time disregarding. Singh and Jain [10] considerate supplier credits in an inflationary atmosphere when backup money is obtainable. Singh and Jain [11] established understanding supplier credits in an inflationary atmosphere when backup money is offered.

Discount policy are regular price discounts such that once the price of a product is clear down, it may not be brought up to the same price level again in the same selling season. Urban and Baker [12] represented optimum ordering and pricing plans in a single-period atmosphere with multivariate demand and discounts. Srivastava and Gupta [13] an EPQ model for decaying items with time and price dependent demand under markdowns. Kamaruzaman and Omar [14] an EPQ model of delayed decaying items with price and inventory level dependent demand under reduction plan.

The salvage value is merged to the failed units. In this model we have used inflation and discount policy. This is very important so we studied effect of inflation and discount policy in this paper. The public perception about Inflation is the overall increase in the goods prices which creates the most continuous effect on the price level of goods prices. They established a model that gives the best discount time and at the same time maximizes annual profit.

- *Notations:*

I(t)= Inventory level with respect to time t.

T= Length of replenishment cycle.
Q_1= Inventory smooth after timet_1.
Q_2= Inventory smooth after time $(t_1 + t_2)$
h= Holding cost.
C =Initial price.
A =Setup cost.
θ = constant deterioration rate.
P= Production rate.
k =Production cost.
i= Discount rate
γ= Discount percentage.
ω=Production percentage.
r=inflation rate.
$(t_1 + t_2)$=Discount offering time.

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II. ASSUMPTIONS

- Demand is changeable with respect to time. For particular period of time it is exponentially

Reducing function of time and after that it is a linear function of selling price.

D(t) = $\{ae^{-bt}, 0\leq t \leq (t_1+t_2)\}$
$\{ (\varepsilon_1 - \varepsilon_2 C)\ 0\leq t \leq t_3\}$

Where a, ε_1, b and ε_2 are positive constants and a>0, ε_1 >0 and b, ε_2 both lies between 0 and 1. ε_2=1- i

- Only one item is measured over a given period of T units of time.
- The time distance of the inventory system is unlimited.
- After a unit of the product is manufactured, it is required to be sold.
- Shortages are not permitted.
- Rate of deterioration, θ is constant any time, where $0\leq \theta <1$.
- Single one time discount price at one planning period is considered.
- Discount price is known.
- After the production is accomplished, the product starts decaying and discount is offered after some time of product decaying.
- Inflation rate is applied

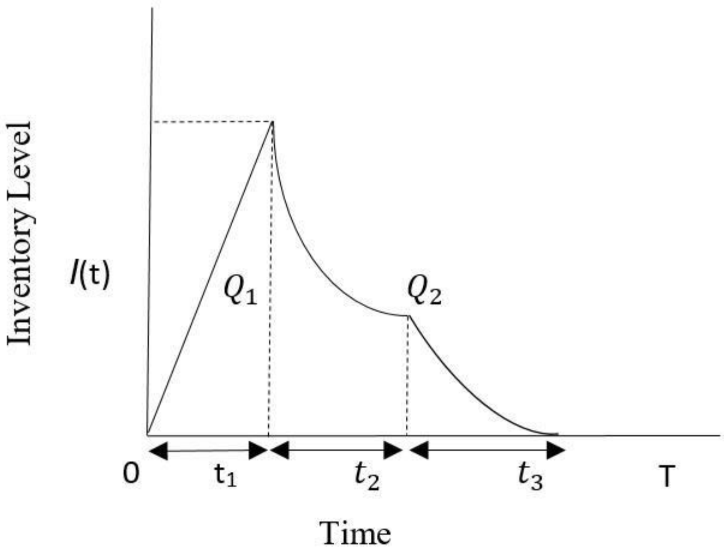

Fig 1

III. MATHEMATICAL FORMULATION

From diagram, t=0 the inventory level is zero. And after that production and supply start instantaneously and production stop t=t_1. Q_1 is maximum inventory level reached. Interval (0, t_1), there is no decaying. Interval (t, (t_1+t_2) production start decaying and demand start decreasing nonstop with time. and now, discount is offered at time (t_1+t_2). The position of the inventory at any instantaneous of time t over a period (0, T) is directed by the following differential equation.

$\frac{dI(t)}{dt} = P - ae^{-bt}$ $\quad 0\leq t \leq t_1$ (1)

$\frac{dI(t)}{dt} + \theta I(t) = -ae^{-bt}$ $\quad 0 \leq t \leq t_2$ (2)

$\frac{dI(t)}{dt} + \theta I(t) = -(\varepsilon_1 - \varepsilon_2 C)$ $\quad 0 \leq t \leq t_3$ (3)

With boundary condition t=0, I(t_1)= 0

The explanation of differential equation (1)

I(t) $= Pt + \frac{a}{b}(e^{-bt} - 1)$ (4)

Inventory level Q_1 at point t= t_1

$Q_1 = Pt_1 + \frac{a}{b}(e^{-bt_1} - 1)$ (5)

The solution of differential equation (2)

t=0 I(t) =Q_1

I(t) = $Q_1 e^{-\theta t} + \frac{a}{b-\theta}(e^{-bt} - e^{-\theta t})$ (6)

Inventory level Q_2 at t=t_2

$Q_2 = Q_1 e^{-\theta t_2} + \frac{a}{b-\theta}(e^{-bt_2} - e^{-\theta t_2})$ (7)

And finally, differential equation explanation having boundary condition
t=0, I(t) =Q_2

I(t) = $Q_2 e^{-\theta t}$+$\frac{(\varepsilon_1-\varepsilon_2 C)}{\theta}(e^{-\theta t_3}-1)$ (8)

at t=t_3 , I(t) = 0

Therefore,

$Q_2 = \frac{(\varepsilon_1-\varepsilon_2 C)}{\theta}(1-e^{-\theta t_3})$ (9)

Sales revenue cost =$S_R[\int_0^{t_1} D(t)e^{-rt}$dt+$\int_0^{t_2} D(t)e^{-rt}$dt+$\int_0^{t_3} D(t)e^{-rt}$dt]

=$S_R[\int_0^{t_1} ae^{-bt}e^{-rt}$dt+$\int_0^{t_2} ae^{-bt}e^{-rt}$dt+$\int_0^{t_3}(\varepsilon_1-\varepsilon_2 C)e^{-rt}$dt]

=S_R[a($\frac{e^{lt_1}}{-(b+r)}$+$\frac{1}{(b+r)}$)+ ($\frac{e^{t_2}}{-(b+r)}$+$\frac{1}{(b+r)}$)+ $(\varepsilon_1-\varepsilon_2 C)(\frac{e^{rt_3}}{-r}+\frac{1}{r})$] (10)

Holding cost of the inventory (HC) =h[$\int_0^{t_1} e^{-rt}I(t)$dt+$\int_0^{t_2} e^{-rt}I(t)$dt+$\int_0^{t_3} e^{-rt}I(t)$dt]

=h[$\int_0^{t_1}(Pt+\frac{a}{b}(e^{-bt}-1))e^{-rt}$dt+$\int_0^{t_2}(Q_1 e^{-\theta t}+\frac{a}{b-\theta}(e^{-bt}-e^{-\theta t}))e^{-rt}$dt+$\int_0^{t_3}(Q_2 e^{-\theta t}+\frac{(\varepsilon_1-\varepsilon_2 C)}{\theta}(e^{-\theta t_3}-1))e^{-rt}$dt]

=h[$P\left(\frac{t_1 e^{-rt_1)}}{-r}-\frac{e^{-rt_1}}{r^2}+\frac{1}{r^2}\right)+\frac{a}{b}\left(\frac{e^{-t_1(b+r)}}{-(b+r)}+\frac{e^{-rt_1}}{r}+\frac{1}{(b+r)}-\frac{1}{r}\right)+(Q_1(\frac{e^{-t_2(\theta+r)}}{-(\theta+r)}+\frac{1}{(\theta+r)})+\frac{a}{b-\theta}(\frac{e^{-t_2(b+r)}}{-(b+r)}+\frac{e^{-t_2(\theta+r)}}{(\theta+r)}+\frac{1}{(b+r)}-\frac{1}{(\theta+r)})+(Q_2((\frac{e^{-t_3(\theta+r)}}{-(\theta+r)}+\frac{1}{(\theta+r)})+\frac{(\varepsilon_1-\varepsilon_2 C)}{\theta}(\frac{e^{-t_3(\theta+r)}}{-r}+\frac{e^{-rt_3}}{r}+\frac{e^{-\theta t_3}}{r}-\frac{1}{r})$ (11)

Deterioration cost (DC) = $C_d[\int_0^{t_1} e^{-rt}\theta I(t)$dt+$\int_0^{t_2} e^{-rt}\theta I(t)$dt+$\int_0^{t_3} e^{-rt}\theta I(t)$dt]

=$C_d\theta[P\left(\frac{t_1 e^{-rt_1)}}{-r}-\frac{e^{-rt_1}}{r^2}+\frac{1}{r^2}\right)+\frac{a}{b}\left(\frac{e^{-t_1(b+r)}}{-(b+r)}+\frac{e^{-rt_1}}{r}+\frac{1}{(b+r)}-\frac{1}{r}\right)+(Q_1(\frac{e^{-t_2(\theta+r)}}{-(\theta+r)}+\frac{1}{(\theta+r)})+\frac{a}{b-\theta}(\frac{e^{-t_2(b+r)}}{-(b+r)}+\frac{e^{-t_2(\theta+r)}}{(\theta+r)}+\frac{1}{(b+r)}-\frac{1}{(\theta+r)})+(Q_2((\frac{e^{-t_3(\theta+r)}}{-(\theta+r)}+\frac{1}{(\theta+r)})+\frac{(\varepsilon_1-\varepsilon_2 C)}{\theta}(\frac{e^{-t_3(\theta+r)}}{-r}+\frac{e^{-rt_3}}{r}+\frac{e^{-\theta t_3}}{r}-\frac{1}{r})$ (12)

Production cost (PC) = $\frac{Pkt_1}{T}$ (13)

Set up cost = $\frac{A}{T}$ (14)

Annual Profit = Sales revenue cost – holding cost – deterioration cost – production cost – set up cost

=$\frac{1}{T}$ [S_R(a($\frac{e^{t_1}}{-(b+r)}$+$\frac{1}{(b+r)}$)+ ($\frac{e^{t_2}}{-(b+r)}$+$\frac{1}{(b+r)}$)+ $(\varepsilon_1-\varepsilon_2 C)(\frac{e^{rt_3}}{-r}+\frac{1}{r})$)-h($P\left(\frac{t_1 e^{-rt_1)}}{-r}-\frac{e^{-rt_1}}{r^2}+\frac{1}{r^2}\right)+\frac{a}{b}\left(\frac{e^{-t_1(b+r)}}{-(b+r)}+\frac{e^{-rt_1}}{r}+\frac{1}{(b+r)}-\frac{1}{r}\right)+(Q_1(\frac{e^{-t_2(\theta+r)}}{-(\theta+r)}+\frac{1}{(\theta+r)})+\frac{a}{b-\theta}(\frac{e^{-t_2(b+r)}}{-(b+r)}+\frac{e^{-t_2(\theta+r)}}{(\theta+r)}+\frac{1}{(b+r)}-\frac{1}{(\theta+r)})+(Q_2((\frac{e^{-t_3(\theta+r)}}{-(\theta+r)}+\frac{1}{(\theta+r)})+\frac{(\varepsilon_1-\varepsilon_2 C)}{\theta}(\frac{e^{-t_3(\theta+r)}}{-r}+\frac{e^{-rt_3}}{r}+\frac{e^{-\theta t_3}}{r}-\frac{1}{r})$ $-C_d\theta$ $(p\left(\frac{t_1 e^{-rt_1)}}{-r}-\frac{e^{-rt_1}}{r^2}+\frac{1}{r^2}\right)+\frac{a}{b}\left(\frac{e^{-t_1(b+r)}}{-(b+r)}+\frac{e^{-rt_1}}{r}+\frac{1}{(b+r)}-\frac{1}{r}\right)+(Q_1(\frac{e^{-t_2(\theta+r)}}{-(\theta+r)}+\frac{1}{(\theta+r)})+\frac{a}{b-\theta}(\frac{e^{-t_2(b+r)}}{-(b+r)}+\frac{e^{-t_2(\theta+r)}}{(\theta+r)}+\frac{1}{(b+r)}-\frac{1}{(\theta+r)})+(Q_2((\frac{e^{-t_3(\theta+r)}}{-(\theta+r)}+\frac{1}{(\theta+r)})+\frac{(\varepsilon_1-\varepsilon_2 C)}{\theta}(\frac{e^{-t_3(\theta+r)}}{-r}+\frac{e^{-rt_3}}{r}+\frac{e^{-\theta t_3}}{r}-\frac{1}{r}))-Pkt_1-A)]$ (15)

Solution Procedure: In our case we are find a better solution by varying T and γ. The main aim is to maximize the Annual profit and Substitute t_1, t_2 and $t_3$into equation (15). Then we will find optimum solution.

Numerical example: In this sector, a numerical example is deliberated to show the model resulting values of parameter are given below which used in example.

A =180, C=10, P=200, h=8, C_d=.02, S_R=2, ε_1=150, ε_2 =.1, ω=.2, r=.5, θ=.7, a=120, b=.2,k=120

The optimum value is AP = 4254.65, γ =0.804090, T= 20.0

$t_1 = \omega T$

$t_2 = \gamma(1-\omega)T$

$t_3 = (1-\gamma)(1-\omega)T$

t_1 = 4.0, t_2 = 9.6, t_3 = 2.4, Q_1= 469.597, Q_2=211.479

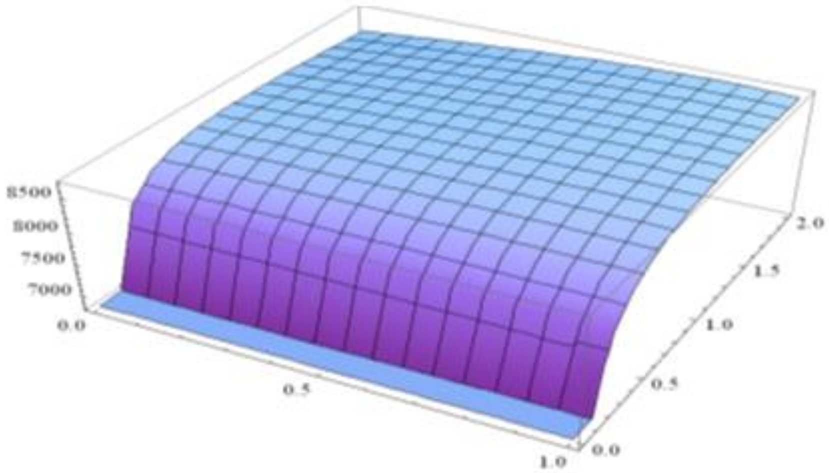

Fig 2

This Fig show the concavity of the optimal profit.

- *Sensitivity Analysis:*
  Sensitivity w.r.to θ, r, ω and C_d

θ	AP	γ	T	Q_1	Q_2
+30%	4238.37	.787535	20.0	469.597	163.503
+20%	4250.71	.793516	20.0	469.597	176.962
+10%	4258.09	.798892	20.0	469.597	192.962
-10%	4233.84	.809404	20.0	469.597	233.975
-20%	4187.21	.815077	20.0	469.597	261.352
-30%	4102.64	.821386	20.0	469.597	295.153
R					
+30%	3230.79	.756955	19.2739	469.597	211.479
+20%	4105.92	.781855	20.0	469.597	211.479
+10%	4105.92	.781855	20.0	469.597	211.479
-10%	4326.69	.829469	20.0	469.597	211.479
-20%	4390.85	.858247	20.0	469.597	211.479
-30%	4509.11	.890952	20.0	469.597	211.479
ω					
+30%	16874.20	.833521	20.0	469.597	211.479
+20%	11456.44	.823352	20.0	469.597	211.479
+10%	7364.440	.813544	20.0	469.597	211.479
-10%	1907.52	.819339	18.734	469.597	211.479
-20%	1030.82	1.0	12.1141	469.597	211.479
-30%	687.062	1.0	10.5975	469.597	211.479
C_d					
+30%	3915.13	.803313	20.0	469.597	211.479
+20%	4028.29	.803573	20.0	469.597	211.479
+10%	4141.46	.803832	20.0	469.597	211.479
-10%	4367.85	.804348	20.0	469.597	211.479
-20%	4418.07	.804852	20.0	469.597	211.479
-30%	4594.31	.804852	20.0	469.597	211.479

Table 1

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IV. OBSERVATION

- In case of θ, we will θ increase then, AP, γ and Q_2 increase and Q_1 and T constant and after that we will θ decrease then AP, γ decrease and T and Q_1 constant.
- In case of r, we will r increase then AP, γ and T decrease and Q_1, Q_2 constant after that if we will r decrease then AP, γ increase and Q_1, Q_2 and T constant.
- In case of ω , we will ω increase then AP, γ increase and Q_1, Q_2 and T constant after that we will ω decrease AP decrease and γ increase and T decrease and Q_1, Q_2 and T constant.
- In case of C_d, we will C_d increase then AP increase γ decrease and Q_1, Q_2 and T constant and after that we will C_d decrease then AP, γ increase and Q_1, Q_2 and T constant.

V. CONCLUSION

In this concept, we will study about an integrated EPQ inventory model for delayed deteriorating items with time and price dependent demand under discount policy. It is clear that the inventory and maximize the profit, discount policy is introduced. Discount policy and inflation both are vital role play in the inventory. The optimum replenishment time, optimum amounts, optimum discount time as well as optimal annual profit have been derived. The result prove offering time and discount rate give significant contribute improve the total profit and a policy maker must be very alert to set discount rate. Maximize the profit which discount in production cycle time for a given discount rate and discount percentage.

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