

Application of Fuzzy Set Theory in Obtaining Optimal Preference Ordering in Multi-Person Decision Making

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Abstract:- This paper presents the application of fuzzy set theory in decision making called Fuzzy decision making. Decision making in Fuzzy environment is done by the extending the application of Fuzzy set theory to each of the influential factors involved in making the decision.

Many at times decision(s) is/are made by more than one person (multi-person decision making) and over many alternatives, as such this research work considered a group of 10 graduates on a choice preference ordering over 5 alternatives, which are; Politics, Civil Service, Marriage, Farming and Business. These alternatives are considered because of the fact that they are the immediate possibilities for our graduates and also, finding the opinions of our youths, with regard to them, will help in determining the direction in thoughts and attitudes of our graduates.

In this paper, the opinion/goal of the individuals involved in the group is equally considered and based on Shiruma approach, the preference ordering of each individual is obtained and then, later, by Blin approach, the optimal preference ordering of the decision for the group is given.

This provided a better avenue for group decision making be it by human beings or machines as it always gives the best decision.

Keywords:- Fuzzy set; fuzzy Decision; preference ordering; membership function; Optimal Decision.

I. INTRODUCTION

Making decisions is undoubtedly one of the most fundamental activities of human beings. We all are faced in our daily life with varieties of alternative actions available to us and, at least in some instances, we have to decide which of the available actions to take. The beginnings of decision making, as a subject of study, can be traced, presumably, to the late 18th century, when various studies were made in France regarding methods of election and social choice [14].

The subject of decision making is, as the name suggests, the study of how decisions are actually made and how they can be made better or more successfully. Since these initial studies, decision making has evolved into a

respectable and rich field of study that resulted in developing many theories and methods.

Applications of fuzzy sets within the field of decision making have, for the most part, consisted of fuzzifications of the classical theories of decision making [21]. While decision making under conditions of risk have been modeled by probabilistic decision theories and game theories, fuzzy decision theories attempt to deal with the vagueness and nonspecificity inherent in human formulation of preferences, constraints, and goals [4]. In this paper, the application of fuzzy set theory to the multi-person decision making problems is processed and a best decision is obtained for the group in consideration with an appreciating degree of group acceptance.

II. LITERATURE REVIEW

The attempts to justify whether all valid propositions can only be assigned the logical values true or false by the introduction of fuzzy sets by Zadeh [14] brought about fuzzy concepts. In 1965, Lotfi Zadeh introduced his seminal idea in a continuous-valued logic that he called *fuzzy set theory*.

As its name implies, the theory of Fuzzy Set is basically, a theory in which everything is a matter of degree. A question that was frequently raised is: Are there, in fact any significant problem – areas in which the use of the theory of Fuzzy Sets leads to results that could not be obtained by classical methods [16].

Professor Zimmermann's treatise [22],[23],[24] provides an affirmative answer to this question. His comprehensive exposition of both the theory and its applications explains in clear terms the basic concepts underline the theory and how they relate to their classical counterparts.

Professor Zimmermann's treatise relates to the distinction between the concepts of Probability and possibility, with latter concept having a close connection with that of membership in a Fuzzy Set. The concept of possibility plays a particularly important role in the representation of meaning, in the management in expert system, and in applications of the theory of Fuzzy Sets to decision analysis [23].

As an extension to fuzzy sets, intuitionistic fuzzy set (IFS) was proposed by Atanassov [13] to handle the issues of membership, non-memberships and hesitation degrees of decision problems. The IFS was intended to be an extension of single membership of fuzzy set theory. However, the two memberships of IFS have some limitations particularly on the arithmetic addition of two memberships and also hesitation degree. To improve hesitation degree, author such as Zeng et al. [20], [21] proposed intervalvalued hesitant fuzzy sets and its arithmetic operations. For the two memberships of IFS, the focal point is on its arithmetic addition. It is known that the sum of two memberships of IFS is limited to one. In response to this limitation, Yager [19] introduced Pythagorean fuzzy set (PFS) where the limitation in IFS has been modified. The sum of two memberships in IFS is now substituted with squares of each membership where the sum of these two squares is less or equal to one. In other words, the PFS is characterized by a membership degree and non-membership degree where the square sum of its membership degree and non-membership degree is less than or equal to one. The PFS is one of the most successful sets, in terms of representing comprehensively uncertain and vague information [6]. The PFS also was successfully integrated with the concepts of confidence level [9], and decision making with probabilities [11]. The new linguistic and exponential operational laws with PFS were also proposed [10], [12]. About similar with other sets, the PFS was also extended to interval-valued PFS and hesitant PFS. These two sets were successfully used in developing new aggregation operators such as Maclaurin Symmetric Mean Operator [8], [18], averaging and geometric aggregation operators [7]. Sa'idu gives the effect of Shiruma approach in collecting individual's opinion in making fuzzy group decision over the direct approach [1].

In this paper, the primary approach is considered whereby only the degree of truth is taken into consideration and the process considered the normal aggregation operator by comparing one alternative over the other in order to obtained the preference ordering which resulted in determining the optimal preference ordering for the group of the decision makers.

III. DEFINITIONS OF SOME BASIC TERMINOLOGIES

Below are descriptions of some important terms that are used in this paper;

A. Decision Theory

Decision theory deals with methods for determining the optimal course of action when a number of alternatives are available and their consequences cannot be forecast with certainty. According to John Dewey [4], decision making is a problem-solving process that consists of five consecutive stages:

- a felt difficulty,
- the definition of the character of that difficulty,
- suggestion of possible solutions,
- evaluation of the suggestion, and
- further observation and experiment leading to acceptance or rejection of the suggestion.

B. Classical Set Theory

Mathematicians defined set as a collection of objects having one or more common characteristics. The objects that belong to a set are called members/elements of the set. Classical set theory is built on the fundamental concept of “set” of which an individual is either a member or not a member. A sharp, crisp, and unambiguous distinction exists between a member and a non-member for any well-defined “set” of entities in this theory.

C. Fuzzy Set Theory

A fuzzy set theory is an extension of the classical set theory where elements have varying degrees of membership. A fuzzy set is any set that allows its members to have different degree of membership in the interval [0, 1].

A fuzzy set is described by a membership function $\mu_A(x)$ of A . This membership function associates each $x \in X$ a number $\mu_A(x)$ in the closed interval [0, 1]. Below is a graphical descriptions of membership functions in classical case as well as in fuzzy case;

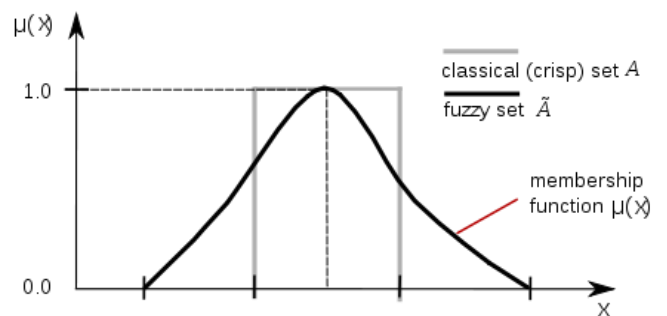


Fig 1:- Graphical representations of membership function in crisp and fuzzy sets

➤ *Definition of Fuzzy Set*

Let X be a universal space (objects) with a generic element of X denoted by x (i.e. $X = \{x\}$) and A be a fuzzy set in X , then a fuzzy set (class) A in X is a function defined in X characterized by a membership function $\mu_A(x)$ which assigns to each point in X a real number in the interval [0,1], that is;

$$A: X \rightarrow [0, 1]$$

A fuzzy set A is defined by the membership function $\mu_A(x) : X \rightarrow [0, 1]$

in addition, the value $\mu_A(x)$ represents the grade of membership of $x \in A$ and it is called membership value of $x \in X$.

NOTE: Each fuzzy set is defined by a unique membership function.

➤ Representation of a Fuzzy Set

A fuzzy set A consisting of elements $x_i \in X$, for $i = 1, 2, 3, \dots, n$ can be expressed as

$$1. \quad A = \{(x_i, \mu_A(x_i)) / x_i \in X, \mu_A(x_i) : X \rightarrow [0, 1]\}$$

$$2. \quad A = \left\{ \frac{\mu_A(x_1)}{x_1} + \frac{\mu_A(x_2)}{x_2} + \dots + \frac{\mu_A(x_n)}{x_n} \right\}$$

$$= \sum_{i=1}^n \frac{\mu_A(x_i)}{x_i} \text{ and}$$

3. by describing the nature/character of the elements of the set as in crisp case.

D. Fuzzy Relation

A fuzzy relation is a fuzzy set defined on the Cartesian product of crisp sets $A_1, A_2, A_3, \dots, A_n$ where the n -tuples $(x_1, x_2, x_3, \dots, x_n)$ may have varying degrees of membership within the relation.

The strength of the relation between the elements of the tuple is expressed by the membership function

$$\mu_R: A_1 \times A_2 \times A_3 \times \dots \times A_n \rightarrow [0, 1]$$

$$\therefore R = \{(x_1 \times x_2 \times x_3 \times \dots \times x_n), \mu_R\} / \mu_R(x_1, x_2, x_3, \dots, x_n) \geq 0, x_1 \in A_1, x_2 \in A_2, \dots, x_n \in A_n\}$$

➤ α -Cut of Fuzzy Relation

Assume $R \subseteq A \times B$, and α_R is a α -cut relation then $\alpha_R = \{(x, y) / \mu_R(x, y), \mu_R(x, y) \geq \alpha, x \in A, y \in B\}, \alpha \in [0, 1]$

Therefore α -cut of a fuzzy relation is always a crisp relation.

➤ Characteristics of Relation

For a relation $R(X, Y)$ we have some characteristics as follows;

- Reflexive: $R(X, X)$ is reflexive if $x \in A, \Rightarrow (x, x) \in R, \forall x \in A$.
- Antisymmetric: $R(X, Y)$ is antisymmetric if $\forall x, y \in A, (x, y) \in R$ and $(y, x) \notin R$
- Transitive: $R(X, Y)$ is said to be transitive if $\forall x, y, z \in A, (x, y) \in R, (y, z) \in R$ then $(x, z) \in R$.

➤ Order Relation

A Relation $R \subset X \times X$ is said to be order relation if it is Reflexive, Antisymmetric and Transitive.

IV. FUZZY MULTIPERSON DECISION MAKING

When decisions made by more than one person are modeled, two cases can be considered: first, the goals of individual decision makers may differ such that each places a different ordering on the alternative; second, the individual decision makers may have access to different information upon which to base their decision. It can easily be seen that for the decision maker to passed a better decision, in this case, he/she must consider the uncertainty involved and the best way to, appropriately, capture such uncertainties is by using fuzzy decision making approach.

V. METHODOLOGY

The applications of fuzzy decision making approach, in this paper, is demonstrated by considering two important established methods; Shiruma [17] and Blin [2] on a group of ten graduates that were to take a common acceptable decision to obtained an optimal preference ordering over five alternatives.

A. Shiruma Approach

Shiruma approach is designed to construct an ordering of all given alternatives on the basis of their pairwise comparisons instead of direct preferring. This approach gives the strength of the preference of one option over the other. In this approach, $f(x_i, x_j)$ denotes the desirability grade given by the individual to x_i with respect to x_j . These evaluations, which are expressed by positive numbers in a given range, are made by the individual for all pairs of alternatives in the given set X . They are then converted to relative preference grades, $F(x_i, x_j)$, by the formula

$$F(x_i, x_j) = \frac{f(x_i, x_j)}{\max[f(x_i, x_j), f(x_j, x_i)]} = \min\left[1, \frac{f(x_i, x_j)}{f(x_j, x_i)}\right] \quad (1)$$

For each pair $(x_i, x_j) \in X^2$. Clearly, $F(x_i, x_j) \in [0, 1]$ for all pairs $(x_i, x_j) \in X^2$. When $f(x_i, x_j) = 1$, x_i is considered at least as desirable as x_j . Functions F , which stand as the membership function of a fuzzy relation on X , has for each pair the property that; $\max[f(x_i, x_j), f(x_j, x_i)] = 1$.

Then, for each $x_i \in X$, the overall relative preference grades $P(x_i)$, of x_i with respect to all other alternatives in X is calculated by the formula:

$$P(x_i) = \min_{x_j \in X} F(x_i, x_j) \quad (2)$$

The preference ordering of each of the alternatives in X is then induced by the numerical ordering of these grades $P(x_i)$. This gives the preference ordering for each of the members in the group [17].

B. Blin Approach

A fuzzy model group decision was proposed by Blin [2] and Blin and Whinstone [3]. Here, each member of a group of n individual decision makers is assumed to have a reflexive, antisymmetric and transitive preferences ordering $P_k, k \in N$ which totally or partially ordered a set X of alternatives. A "social choice" function is then found which, given the individual preferences orderings, produces the most acceptable overall group preference orderings. In order to deal with the multiplicity of opinion evidenced in the group, the social preference S is defined as a fuzzy binary relation with membership grade function

$$S: X \times X \rightarrow [0, 1]$$

which assigns the membership grade $S(x_i, x_j)$, indicating the degree of group preference of alternative x_i over x_j . The appropriate means of aggregating the individual preferences of this group requires computing the relative popularity of alternative x_i over x_j by dividing the number of persons preferring x_i to x_j , denoted by $N(x_i, x_j)$, by the total number of decision makers, n . Thus,

$$S(x_i, x_j) = \frac{N(x_i, x_j)}{n} \tag{3}$$

Once the fuzzy relationship S has been defined, the final nonfuzzy group preference can be determined by converting S into its resolution form

$$S = \bigcup_{\alpha \in [0,1]} \alpha^{\circ} S \tag{4}$$

which is the union of the crisp relations $\alpha^{\circ} S$ comprising the α - cuts of the fuzzy relation S , each scaled by α . Each value α essentially represents the level of agreements between the individual concerning the particular crisp ordering $\alpha^{\circ} S$, to maximize the final agreement level, the intersection of the classes of crisp total ordering that are compatible with the pairs in the α -cuts $\alpha^{\circ} S$ is considered for increasingly smaller values of α until a single crisp total ordering is achieved. In this process, any pairs (x_i, x_j) that lead to an intransitivity are removed. The largest value α , for which the unique compatible ordering on $X \times X$ is found, represents the maximized agreement level of the group, and the crisp ordering itself represents the group decision.

VI. THE APPLICATION

To illustrate the application of the described approaches above, in this research work a group of ten graduates, who intend to take a common decision over five (selected) jobs/carrier, is considered. They considered the alternatives: Politics (P), Civil Service (CS), Business (B), Marriage (M) and Farming (F). These alternatives are the most available opportunities for any graduate in our dear country, Nigeria, and that is the main reason why they formed the basis of this work.

A. Calculating the Individual Preference Ordering

In order to apply the above description to construct an ordering of all alternatives in consideration, on the basis of their pairwise comparisons, a questionnaire was designed and the data was obtained and processed. Let refer each of the n individuals involved in making this decision by P_k , $k \in N$. As the number of the graduate is 10, then $n = 10$. The table below shows the descriptions used to obtained the data;

$f(x_i, x_j)$	Desirability of x_i over x_j	$f(x_i, x_j)$	Desirability of x_i over x_j
1	Little Desirable	6	Between 5 and 7
2	Between 1 and 3	7	Desirable Very strongly
3	Desirable Moderately	8	Between 7 and 9
4	Between 3 and 5	9	Extremely Desirable
5	Desirably Strongly		

Table 1:- Suggested numbers for desirability grading

Each of the individuals provided his desirability preference by pairwise comparisons of one alternative over the other on a table using the numbers suggested in the table 1 for specifying the desirability grades, the evaluating prepared by P_1 (person number 1) is given in table 2 below. The corresponding relative preference grades (calculate by (1)) and the overall relative preference grades calculated by (calculated (2)) are given in the table 3. The overall relative preference induced the preference ordering of the alternatives.

$f(x_i, x_j)$	P	CS	B	M	F
P	0	1	3	4	8
CS	8	0	6	6	6
B	1	4	0	8	5
M	2	1	5	0	7
F	5	9	6	6	0

Table 2:- The desirability grades prepared by P_1

Calculating the Relative Preference Grades for P_1 ;

For $i = 1$ and $j = 1, 2, 3, 4, \text{ or } 5$

$$F(x_1, x_1) = \frac{1}{\max[1,1]} = 1, F(x_1, x_2) = \frac{1}{\max[1,8]} = 0.125$$

$$F(x_1, x_3) = \frac{3}{\max[3,1]} = 1, F(x_1, x_4) = \frac{4}{\max[4,2]} = 1,$$

$$F(x_1, x_5) = \frac{8}{\max[8,5]} = 1$$

For $i = 2$ and $j = 1, 2, 3, 4, \text{ or } 5$

$$F(x_2, x_1) = \frac{8}{\max[8,1]} = 1, F(x_2, x_2) = 1, F(x_2, x_3) =$$

$$\frac{6}{\max[6,4]} = 1$$

$$F(x_2, x_4) = \frac{6}{\max[6,1]} = 1, F(x_2, x_5) = \frac{6}{\max[6,9]} = 0.667$$

For $i = 3$ and $j = 1, 2, 3, 4, \text{ or } 5$

$$F(x_3, x_1) = \frac{1}{\max[1,3]} = 0.333, F(x_3, x_2) = \frac{4}{\max[4,6]} =$$

$$0.667, F(x_3, x_3) = 1$$

$$F(x_3, x_4) = \frac{8}{\max[8,5]} = 1, F(x_3, x_5) = \frac{5}{\max[5,6]} = 0.833$$

For $i = 4$ and $j = 1, 2, 3, 4, \text{ or } 5$

$$F(x_4, x_1) = \frac{2}{\max[2,4]} = 0.5, F(x_4, x_2) = \frac{1}{\max[1,6]} = 0.167$$

$$F(x_4, x_3) = \frac{5}{\max[5,8]} = 0.625, F(x_4, x_4) = 1, F(x_4, x_5) =$$

$$\frac{7}{\max[7,6]} = 1$$

For $i = 5$ and $j = 1, 2, 3, 4, \text{ or } 5$

$$F(x_5, x_1) = \frac{5}{\max[5,8]} = 0.625, F(x_5, x_2) = \frac{9}{\max[9,6]} = 1$$

$$F(x_5, x_3) = \frac{6}{\max[6,5]} = 1, F(x_5, x_4) = \frac{6}{\max[6,7]} = 0.857,$$

$$F(x_5, x_5) = 1$$

The relative preference grades are obtained as in the table below;

$F(x_i, x_j)$	P	CS	B	M	F	$P(x_i)$
P	1.000	0.125	1.000	1.000	1.000	0.125
CS	1.000	1.000	1.000	1.000	0.667	0.667
B	0.333	0.667	1.000	1.000	1.000	0.333
M	0.500	0.167	0.625	1.000	0.833	0.167
F	0.625	1.000	1.000	0.857	1.000	0.625

Table 3:- Relatives preference grades and overall relative preference grades of P1

Now, from the above table the fuzzy set below is obtained;

$$P_1 = \{CS/0.667, F/0.625, B/0.333, M/0.167, P/0.125\}$$

The ordering expressions by the 9 other members of the group are determined in similar way and the following results/fuzzy sets are obtained:

The ordering expressions by the 9 other members of the group are determined in similar way and the following results/fuzzy sets are obtained:

$f(x_i, x_j)$	P	CS	B	M	F
P	0	2	3	5	6
CS	9	0	6	7	6
B	7	5	0	5	9
M	8	2	4	0	5
F	5	7	6	6	0

Table 4:- The desirability grades prepared by P2

$F(x_i, x_j)$	P	CS	B	M	F	$P(x_i)$
P	1.000	0.222	0.429	0.625	1.000	0.222
CS	1.000	1.000	1.000	1.000	0.857	0.857
B	1.000	0.833	1.000	1.000	1.000	0.833
M	1.000	0.286	0.800	1.000	0.833	0.286
F	0.833	1.000	0.667	1.000	1.000	0.667

$$P_2 = \{CS/0.857, B/0.833, F/0.667, M/0.286, P/0.222\}$$

Table 5:- Relatives preference grades and overall relative preference grades of P2

$f(x_i, x_j)$	P	CS	B	M	F
P	0	5	5	6	7
CS	8	0	6	9	8
B	6	5	0	7	7
M	4	3	4	0	7
F	4	4	5	8	0

Table 6:- The desirability grades prepared by P3

$F(x_i, x_j)$	P	CS	B	M	F	$P(x_i)$
P	1.000	0.625	0.833	1.000	1.000	0.625
CS	1.000	1.000	1.000	1.000	1.000	1.000
B	1.000	0.833	1.000	1.000	1.000	0.833
M	0.667	0.333	0.571	1.000	0.875	0.333
F	0.571	0.500	0.714	1.000	1.000	0.500

$$P_3 = \{CS/1.000, B/0.833, P/0.625, F/0.500, M/0.333\}$$

Table 7:- Relatives preference grades and overall relative preference grades of P3

$f(x_i, x_j)$	P	CS	B	M	F
P	0	1	3	9	8
CS	2	0	3	4	8
B	7	9	0	9	8
M	4	5	7	0	8
F	5	6	7	7	0

Table 8:-The desirability grades prepared by P4

$F(x_i, x_j)$	P	CS	B	M	F	$P(x_i)$
P	1.000	0.500	0.429	1.000	1.000	0.429
CS	1.000	1.000	0.333	0.800	1.000	0.333
B	1.000	1.000	1.000	1.000	1.000	1.000
M	0.444	1.000	0.778	1.000	1.000	0.444
F	0.625	0.750	0.875	0.875	1.000	0.625

$$P_4 = \{B/1.000, F/0.625, M/0.444, P/0.429, C.S/0.333\}$$

Table 9:- Relatives preference grades and overall relative preference grades of P4

$f(x_i, x_j)$	P	CS	B	M	F
P	0	3	4	5	3
CS	6	0	4	6	7
B	3	5	0	8	5
M	2	3	6	0	5
F	5	4	4	4	0

Table 10:- The desirability grades prepared by P5

$F(x_i, x_j)$	P	CS	B	M	F	$P(x_i)$
P	1.000	0.500	1.000	1.000	0.600	0.500
CS	1.000	1.000	0.800	1.000	1.000	0.800
B	0.750	1.000	1.000	1.000	1.000	0.750
M	0.400	0.500	0.750	1.000	1.000	0.400
F	1.000	0.571	0.800	0.800	1.000	0.571

$P_5 = \{CS/0.800, B/0.750, F/0.571, P/0.500, M/0.400\}$

Table 11:- Relatives preference grades and overall relative preference grades of P5

$f(x_i, x_j)$	P	CS	B	M	F
P	0	8	5	7	4
CS	2	0	4	4	6
B	8	8	0	6	7
M	3	5	4	0	4
F	9	8	5	5	0

Table 12:- The desirability grades prepared by P6

$F(x_i, x_j)$	P	CS	B	M	F	$P(x_i)$
P	1.000	1.000	0.625	1.000	0.444	0.444
CS	0.250	1.000	0.500	0.800	0.750	0.250
B	1.000	1.000	1.000	1.000	1.000	1.000
M	0.429	0.833	0.667	1.000	0.800	0.429
F	1.000	1.000	0.714	1.000	1.000	0.714

$P_6 = \{B/1.000, F/0.714, P/0.444, M/0.429, CS/0.250\}$

Table 13:- Relatives preference grades and overall relative preference grades of P6

$f(x_i, x_j)$	P	CS	B	M	F
P	0	3	4	5	8
CS	2	0	3	4	6
B	7	9	0	6	8
M	6	7	4	0	7
F	5	7	3	6	0

Table 14:- The desirability grades prepared by P7

$F(x_i, x_j)$	P	CS	B	M	F	$P(x_i)$
P	1.000	1.000	0.571	0.833	1.000	0.571
CS	0.667	1.000	0.333	0.571	0.857	0.333
B	1.000	1.000	1.000	1.000	1.000	1.000
M	1.000	1.000	0.667	1.000	1.000	0.667
F	0.625	1.000	0.375	0.857	1.000	0.375

$P_7 = \{B/1.000, M/0.667, P/0.571, F/0.375, CS/0.333\}$

Table 15:- Relatives preference grades and overall relative preference grades of P7

$f(x_i, x_j)$	P	CS	B	M	F
P	0	2	1	5	4
CS	4	0	1	3	3
B	5	2	0	3	3
M	4	2	1	0	3
F	3	4	2	5	0

Table 16:- The desirability grades prepared by P8

$F(x_i, x_j)$	P	CS	B	M	F	$P(x_i)$
P	1.000	0.500	0.200	1.000	1.000	0.200
CS	1.000	1.000	0.500	1.000	0.750	0.500
B	1.000	1.000	1.000	1.000	1.000	1.000
M	0.800	0.667	0.333	1.000	0.600	0.333
F	0.750	1.000	0.667	1.000	1.000	0.667

$P_8 = \{B/1.000, F/0.667, CS/0.500, M/0.333, P/0.200\}$

Table 17:- Relatives preference grades and overall relative preference grades of P8

$f(x_i, x_j)$	P	CS	B	M	F
P	0	8	5	7	4
CS	1	0	4	5	5
B	8	8	0	6	7
M	3	5	4	0	5
F	9	8	5	5	0

Table 18:- The desirability grades prepared by P9

$F(x_i, x_j)$	P	CS	B	M	F	$P(x_i)$
P	1.000	1.000	0.625	1.000	0.444	0.444
CS	0.125	1.000	0.500	1.000	0.625	0.125
B	1.000	1.000	1.000	1.000	1.000	1.000
M	0.429	1.000	0.667	1.000	1.000	0.429
F	1.000	1.000	0.714	1.000	1.000	0.714

$P_9 = \{B/1.000, F/0.714, P/0.444, M/0.429, CS/0.125\}$

Table 19:- Relatives preference grades and overall relative preference grades of P9

$f(x_i, x_j)$	P	CS	B	M	F
P	0	1	3	4	2
CS	7	0	9	5	4
B	9	7	0	5	5
M	8	2	4	0	4
F	7	6	7	5	0

Table 20:- The desirability grades prepared by P10

$F(x_i, x_j)$	P	CS	B	M	F	$P(x_i)$
P	1.000	0.143	0.333	0.500	0.286	0.143
CS	1.000	1.000	1.000	1.000	0.667	0.667
B	1.000	0.778	1.000	1.000	0.714	0.714
M	1.000	0.400	0.800	1.000	0.800	0.400
F	1.000	1.000	1.000	1.000	1.000	1.000

$$P_{10} = \{F/1.000, B/0.714, CS/0.667, M/0.400, P/0.143\}$$

Table 21:- Relatives preference grades and overall relative preference grades of P10

For simplicity, we let $P = a_1, CS = a_2, B = a_3, M = a_4$ and $F = a_5$,

Then, by this approach the following fuzzy sets as the preference ordering of all the ten graduates were obtained:

$$P_1 = \{a_2, a_5, a_3, a_4, a_1\}, P_2 = \{a_2, a_3, a_5, a_4, a_1\},$$

$$P_3 = \{a_2, a_3, a_1, a_5, a_4\}, P_4 = \{a_3, a_5, a_4, a_1, a_2\},$$

$$P_5 = \{a_2, a_3, a_5, a_1, a_4\}, P_6 = \{a_3, a_5, a_1, a_4, a_2\},$$

$$P_7 = \{a_3, a_4, a_1, a_5, a_2\}, P_8 = \{a_3, a_5, a_2, a_4, a_1\},$$

$$P_9 = \{a_3, a_5, a_1, a_4, a_2\}, P_{10} = \{a_5, a_3, a_2, a_4, a_1\}$$

B. Calculating the Overall Optimal Preference Ordering

Having the ten ordered sets above, Blin approach is applied for the calculation of the overall optimal preference ordering for the group. Using the membership function given in (3) for fuzzy group preferences ordering relation S (where $n = 10$), the following fuzzy social preferences relation is arrived at:

	a_1	a_2	a_3	a_4	a_5
a_1	0.0	0.4	0.0	0.4	0.2
a_2	0.6	0.0	0.4	0.6	0.4
a_3	1.0	0.6	0.0	1.0	0.8
a_4	0.6	0.4	0.0	0.0	0.1
a_5	0.8	0.6	0.2	0.9	0.0

The α -cuts of this fuzzy relation S are:

$$^1S = \{(a_3, a_1), (a_3, a_4)\}$$

$$^{0.9}S = \{^1S, (a_5, a_4)\}$$

$$^{0.8}S = \{^{0.9}S, (a_3, a_5), (a_5, a_1)\}$$

$$^{0.6}S = \{^{0.8}S, (a_2, a_1), (a_2, a_4), (a_3, a_2), (a_4, a_1), (a_5, a_2)\}$$

$$^{0.4}S = \{^{0.6}S, (a_1, a_2), (a_1, a_4), (a_2, a_3), (a_2, a_4), (a_4, a_2)\}$$

$$^{0.2}S = \{^{0.4}S, (a_1, a_5), (a_5, a_3)\} \text{ and } ^{0.1}S = \{^{0.1}S, (a_4, a_5)\}$$

Where (a_i, a_j) means a_i precedes a_j . Then, the intersection of the classes of crisp total ordering that are compatible with the pairs in the α -cuts $^\alpha S$ in order to arrive at unique crisp ordering that constitutes the group choice is taken.

The total ordering, $^T O$, on $X \times X$, which is the set of all the five-elements sets that can be generated from $X = \{a_1, a_2, a_3, a_4, a_5\}$, which by permutation are 120 different ordered sets, were obtained and 40 out of those 120 sets are compatible with the set of 1S and they are:

$$^1O = \{\{a_5, a_3, a_4, a_1, a_2\}, \{a_5, a_3, a_4, a_2, a_1\}, \{a_5, a_3, a_1, a_4, a_2\},$$

$$\{a_5, a_3, a_1, a_2, a_4\}, \{a_5, a_3, a_2, a_1, a_4\}, \{a_5, a_3, a_2, a_4, a_1\},$$

$$\{a_5, a_2, a_3, a_1, a_4\}, \{a_5, a_2, a_3, a_4, a_1\}, \{a_3, a_4, a_5, a_1, a_2\},$$

$$\{a_3, a_4, a_5, a_2, a_1\}, \{a_3, a_4, a_1, a_5, a_2\}, \{a_3, a_4, a_1, a_2, a_5\},$$

$$\{a_3, a_4, a_2, a_1, a_5\}, \{a_3, a_4, a_2, a_5, a_1\}, \{a_3, a_5, a_4, a_1, a_2\},$$

$$\{a_3, a_5, a_4, a_2, a_1\}, \{a_3, a_5, a_1, a_4, a_2\}, \{a_3, a_5, a_1, a_2, a_4\},$$

$$\{a_3, a_5, a_2, a_1, a_4\}, \{a_3, a_5, a_2, a_4, a_1\}, \{a_3, a_1, a_5, a_4, a_2\},$$

$$\{a_3, a_1, a_5, a_2, a_4\}, \{a_3, a_1, a_4, a_5, a_2\}, \{a_3, a_1, a_4, a_2, a_5\},$$

$$\{a_3, a_1, a_2, a_4, a_5\}, \{a_3, a_1, a_2, a_5, a_4\}, \{a_3, a_2, a_5, a_1, a_4\},$$

$$\{a_3, a_2, a_5, a_4, a_1\}, \{a_3, a_2, a_1, a_5, a_4\}, \{a_3, a_2, a_1, a_4, a_5\},$$

$$\{a_3, a_2, a_4, a_1, a_5\}, \{a_3, a_2, a_4, a_5, a_1\}, \{a_2, a_3, a_4, a_1, a_5\},$$

$$\{a_2, a_3, a_4, a_5, a_1\}, \{a_2, a_3, a_1, a_4, a_5\}, \{a_2, a_3, a_1, a_5, a_4\},$$

$$\{a_2, a_3, a_5, a_1, a_4\}, \{a_2, a_3, a_5, a_4, a_1\}, \{a_2, a_5, a_3, a_1, a_4\},$$

$$\{a_2, a_5, a_3, a_4, a_1\}\}$$

Thus, $^T O \cap ^1O = ^1O$.

The total orderings $^{0.9}O$ that are compatible with the pairs in the crisp relation $^{0.9}S$ are 25 out of the above 40 sets, that is:

$$^{0.9}O = \{\{a_5, a_3, a_4, a_1, a_2\}, \{a_5, a_3, a_4, a_2, a_1\}, \{a_5, a_3, a_1, a_4, a_2\},$$

$$\{a_5, a_3, a_1, a_2, a_4\}, \{a_5, a_3, a_2, a_1, a_4\}, \{a_5, a_3, a_2, a_4, a_1\},$$

$$\{a_5, a_2, a_3, a_1, a_4\}, \{a_5, a_2, a_3, a_4, a_1\}, \{a_3, a_5, a_4, a_1, a_2\},$$

$$\{a_3, a_5, a_4, a_2, a_1\}, \{a_3, a_5, a_1, a_4, a_2\}, \{a_3, a_5, a_1, a_2, a_4\},$$

$$\{a_3, a_5, a_2, a_1, a_4\}, \{a_3, a_5, a_2, a_4, a_1\}, \{a_3, a_1, a_5, a_4, a_2\},$$

$$\{a_3, a_1, a_5, a_2, a_4\}, \{a_3, a_1, a_2, a_5, a_4\}, \{a_3, a_2, a_5, a_1, a_4\},$$

$$\{a_3, a_2, a_5, a_4, a_1\}, \{a_3, a_2, a_1, a_5, a_4\}, \{a_2, a_3, a_1, a_5, a_4\},$$

$$\{a_2, a_3, a_5, a_1, a_4\}, \{a_2, a_3, a_5, a_4, a_1\}, \{a_2, a_5, a_3, a_1, a_4\},$$

$$\{a_2, a_5, a_3, a_4, a_1\}\}$$

Thus, $^1O \cap ^{0.9}O = ^{0.9}O$.

The total orderings $^{0.8}O$ that are compatible with the pairs in the crisp relation $^{0.8}S$ are 12 out of the above 25 sets, that is:

$$^{0.8}O = \{\{a_3, a_5, a_4, a_1, a_2\}, \{a_3, a_5, a_4, a_2, a_1\}, \{a_3, a_5, a_1, a_4, a_2\},$$

$$\{a_3, a_5, a_1, a_2, a_4\}, \{a_3, a_5, a_2, a_1, a_4\}, \{a_3, a_5, a_2, a_4, a_1\},$$

$$\{a_3, a_2, a_5, a_1, a_4\}, \{a_3, a_2, a_5, a_4, a_1\}, \{a_2, a_3, a_5, a_1, a_4\},$$

$$\{a_2, a_3, a_5, a_4, a_1\}, \{a_2, a_5, a_3, a_1, a_4\}, \{a_2, a_5, a_3, a_4, a_1\}\}$$

Thus, $^{0.9}O \cap ^{0.8}O = ^{0.8}O$.

The total orderings $^{0.6}O$ that are compatible with the pairs in the crisp relation $^{0.6}S$ is 1 out of the above 12 sets, that is:

$$^{0.6}O = \{\{a_3, a_5, a_2, a_4, a_1\}\}$$

finally,

$$^1O \cap ^{0.9}O \cap ^{0.8}O \cap ^{0.6}O = ^{0.6}O = \{\{a_3, a_5, a_2, a_4, a_1\}\}$$

VII. RESULT

A single crisp total ordering is achieved as the group optimal preference ordering, that is: $\{a_3, a_5, a_2, a_4, a_1\}$ with 0.6 (60%) level of group agreement.

It can be remembered that $P = a_1, CS = a_2, B = a_3, M = a_4$ and $F = a_5$, then, the ordering means;

The group optimal preference ordering =
 {Business, Farming, Civil Service, Marriage, Politics}

VIII. DISCUSSION

It was explained earlier that, the available jobs/alternatives considered in this work were due to the current situation, economically and politically, in our country, and it can be seen that the final result reflected the collective opinion of the graduates with 60% level of agreement.

Business happened to be the first option for the group while Farming comes as the second alternative leaving Civil Service as the third. This shows how the thinking of our graduates changed due to experiences. We can remember how graduates, usually, consider civil service in the recent years, and now because of economic and political reasons the youth are taking a different direction. Marriage, which comes as the second to the last options, is the basic of maintaining peaceful existence and creating good family life, but the youth considered, showed that dealing first with economic problem determined the level of readiness for one to established family life as it is fund demanding. Politics is one of the most appealing carriers, now adays, but it is highly critical and full of violence and injustice. The decision makers involved in this work showed that they keep Politics as the last option possibly due to the mentioned reasons. The 60% degree of agreement showed how each of the individuals involved is satisfied with the result obtained.

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X. CONCLUSION

Making decision is one of the fundamental parts of living as it is required in executing any plan. The influential factors affecting our decisions are almost all fuzzy in nature as they are vague/imprecise, as such, for us to have a better decision we must employ a better technique to handle these influential factors.

Application of fuzzy approach in making group decision, as shown in this work, is the most appropriate technique in dealing with the vagueness and imprecision affecting our decisions. This work considered group decision making, the research can be furthered by letting the decision be by stages which, collectively, will give one decision for all.

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