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Exact Solutions for Couple Stress Fluid with Slip Boundary Conditions in Porous Plane-Poiseuille Flow

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Abstract:- In this paper, an exact solution of plane-poiseuille flow of a couple stress fluid with slip boundary conditions between porous parallel plates are obtained. The solution for the liming case as $a \to 0$ or $b \to \infty$ for Newtonian flow in the absenteeism of couple stresses are obtained and compared with viscous Newtonian fluid. It is found that the occurrence of couple stresses has a reducing impact on the velocity of fluid and the volume flow rate.

I. INTRODUCTION

Stokes [1] offered the principle of couple stress fluids and has attracted the attention of many researchers in fluid mechanics. The [2,3] adequately explains the flow behavior of fluids containing a substance such as lubricants with animal blood, liquid crystals and polymer additives. Navier [4] proposed a general boundary condition that presents the possibility of slip at the solid boundary. Neto *et al*. [5] gives an analysis of experimental studies concerning the slip of Newtonian fluids at solid interface. Svetlana *et al*. [6] invented that the Poiseuille flow is one of the creeping flow with numerous applications in polymer processing such as blow molding, extraction and die flow. Sundarammal *et al*.[7] studied about MHD squeeze film features among finite porous parallel rectangular plates with surface roughness. Santhana Krishnan Narayanan *et al*. [8] invented about Squeeze film behavior in porous transversely circular stepped plates with a couple stress fluid. Ellahi *et al*. [9] and Devakar *et al*. [10] invented the precise answers for three fundamental flows namely, Couette, Poiseuille and generalized Couette flow with nonlinear slip conditions. Ferras *et al*. [11] acquired analytical solutions in Couette and Poiseuille flows for both Newtonian and inelastic non-Newtonian with slip boundary conditions. This motivates us to produce exact solutions of plane poiseuille flow of couple stress fluid between parallel porous plates with slip boundary conditions.

II. CONSTRUCTION OF THE PROBLEM

Consider a unidirectional steady flow of the incompressible couple stress fluid in between two infinitely long horizontal parallel porous plates $y = -h$ and $y = h$ as shown in Figure 1. Both the upper and lower plates are porous faced with uniform thickness which are placed at rest.

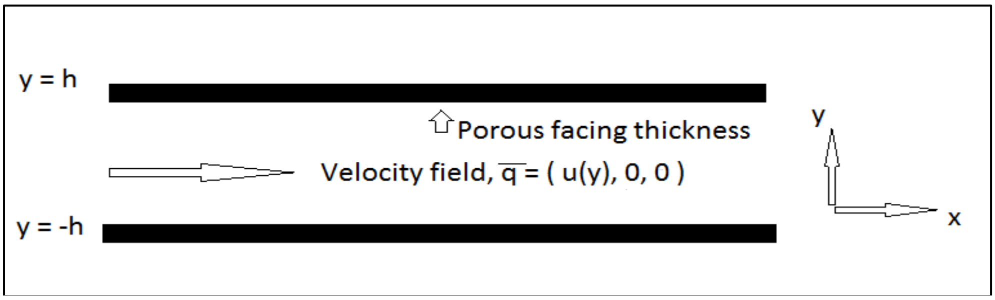

Fig 1:- Flow Configuration of the Plane-Poiseuille flow of couple stress fluid

The flow is due to the constant pressure gradient G in the positive x-direction with velocity field $\bar{q} = (u(y), 0, 0)$. In the porous medium, Darcy law

$$\bar{q}^{*} = (0,0,h) - \frac{\varphi(x,y,z)}{\eta}.\nabla\left(p_1 + p^{*}\right)$$

Where h film thickness, φ permeability, η couple stress viscosity coefficient, p_1 pressure in the porous matrix, p^* is the pressure in the film region and pressure $p = p_1 + p^*$.

The governing equation of an incompressible couple stress fluid in the absence of body forces is given by [10],

$$\eta \frac{d^4u}{dy^4} - \mu \frac{d^2u}{dy^2} + \frac{dp}{dx} = 0$$

where p is the fluid pressure at any point, μ viscosity material constant and η couple stress viscosity coefficient. The slip boundary condition is

$$u(h) = \beta\left[\frac{du}{dy} - \frac{\eta}{\mu}\frac{d^3u}{dy^3}\right] \quad \text{at } y = -h$$

$$u(h) = -\beta\left[\frac{du}{dy} - \frac{\eta}{\mu}\frac{d^3u}{dy^3}\right] \text{ at y = h}$$

where β is slip constant. The couple stresses on the boundary vanishes

$$\frac{d^2u}{dy^2} = 0 \quad \text{at } y = -h \text{ and } y = h$$

Introducing the following non-dimensional parameters,

$$y^* = \frac{y}{h}, \quad u^* = \frac{\rho h}{\mu}u, \quad p^* = \frac{\rho h^2}{\mu^2}p$$

$$a = \sqrt{\frac{\eta}{\mu h^2}}, \quad \alpha = \frac{\beta}{h}, \quad G = -\frac{dp}{dx}$$

the boundary value problem reduce to (asterisks have been dropped for simplicity)

$$a^2\frac{d^4u}{dy^4} - \frac{d^2u}{dy^2} = G$$

with slip boundary condition at y = □1 and y = 1. The precise answers for the proposed problem is

$$u(y) = \frac{G}{2}\left(1 + 2\alpha - y^2\right) - \frac{G}{b^2}\left(1 - \frac{\cosh(by)}{\cosh(b)}\right)$$

where $b = \frac{1}{a} = \sqrt{\frac{\mu h^2}{\eta}}$.

The non-dimensional volume flow rate of the channel is given by

$$q = \int_{-1}^{1} u(y)dy = \frac{2G}{3} + 2G\alpha - \frac{2G}{b^2}\left(1 - \frac{\sinh(b)}{b\cosh(b)}\right)$$

III. RESULTS AND DISCUSSIONS

With numerous flow parameters, the velocity profiles U and volume flow rate q are plotted in the graph. The problem is described by the effect of pressure gradient G on velocity, couple stress parameter a, slip parameter α and volume flow rate q. The solutions of this problem are acquired by employing the classical viscous Newtonian model. When $a \rightarrow 0$ or $b \rightarrow \infty$ for Newtonian flow in the absence of couple stresses, the solutions are also considered and compared with viscous Newtonian fluid.

- *Influences of Slip Factor along with Pressure Gradient in Velocity profiles*

Figure 2 illustrates the dissimilarity of velocity profiles of plane poiseuille flow for various values of couple stress factor a when the other factors held fixed in absence of slip parameter α. Figure 3 to Figure 8 represents the dissimilarity of velocity profiles of plane poiseuille flow for numerous values of couple stress factor a. It is observed from the graphs that, as the couple stress factor a escalations there is a decrease in the velocity which shows that growing of couple stresses has declining outcome on the velocity.

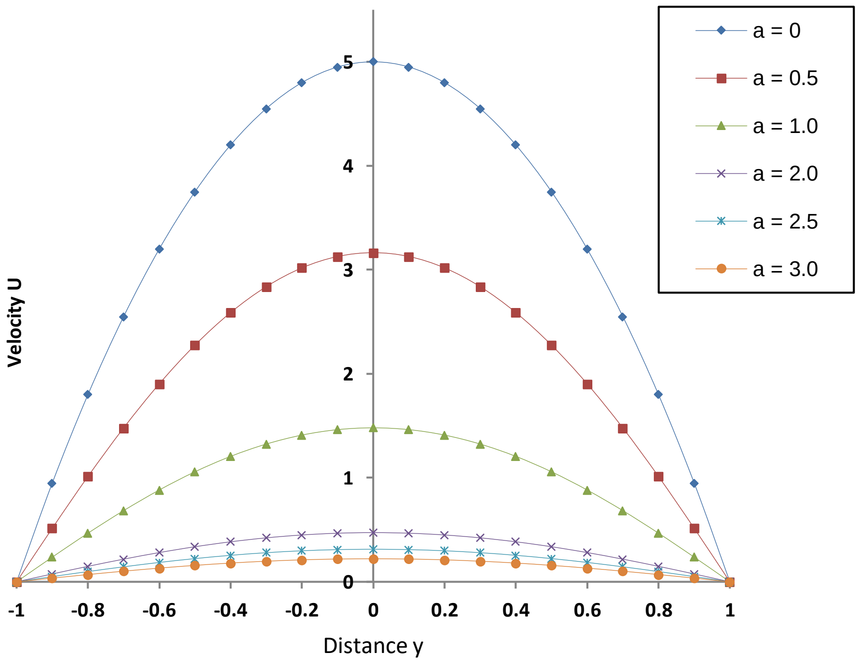

Fig 2:- Velocity profiles with various couple stress parameter a for fixed values of slip parameter $\alpha = 0.0$ and *pressure gradient G=10*

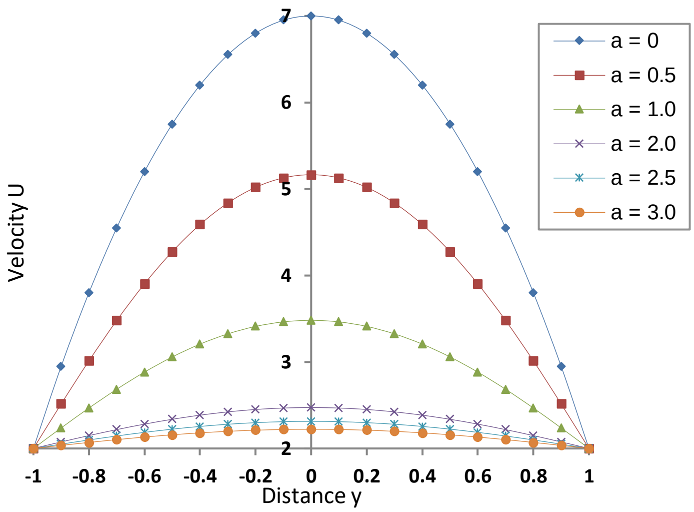

Fig 3:- Velocity profiles with various couple stress parameter *a* for fixed values of slip parameter $\alpha = 0.2$ and *pressure gradient* $G = 10$

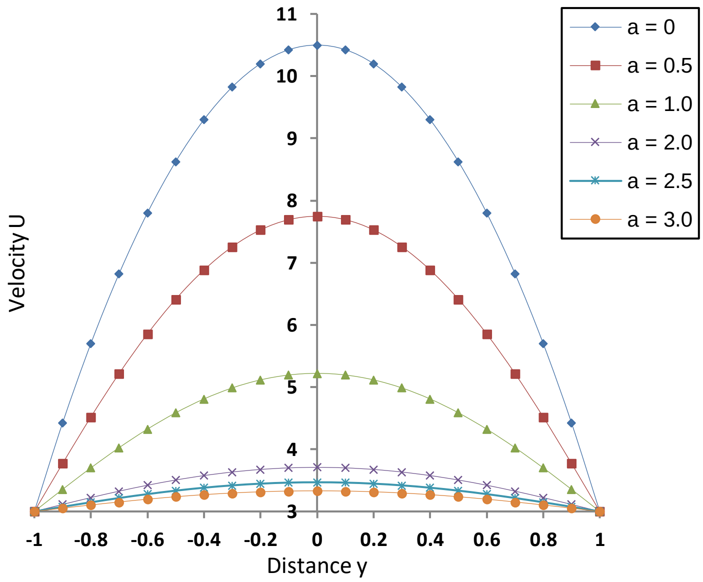

Fig 4:- Velocity profiles with various couple stress parameter *a* for fixed values of slip parameter $\alpha = 0.2$ and *pressure gradient* $G = 15$

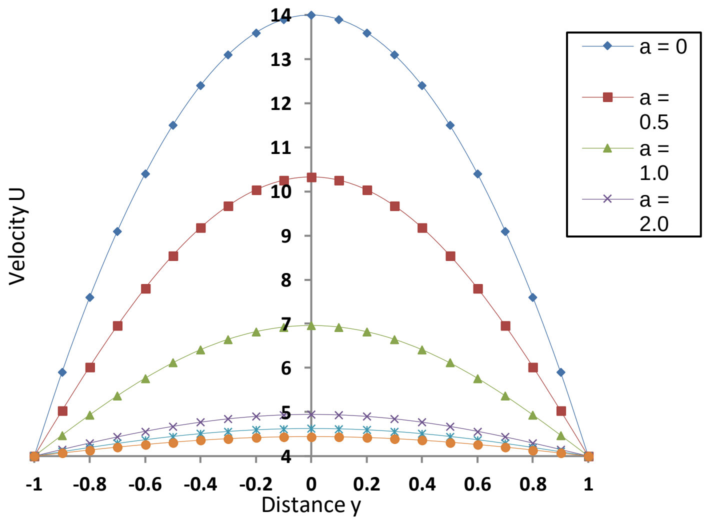

Fig 5:- Velocity profiles with various couple stress parameter *a* for fixed values of slip parameter $\alpha = 0.2$ and *pressure gradient* $G = 20$

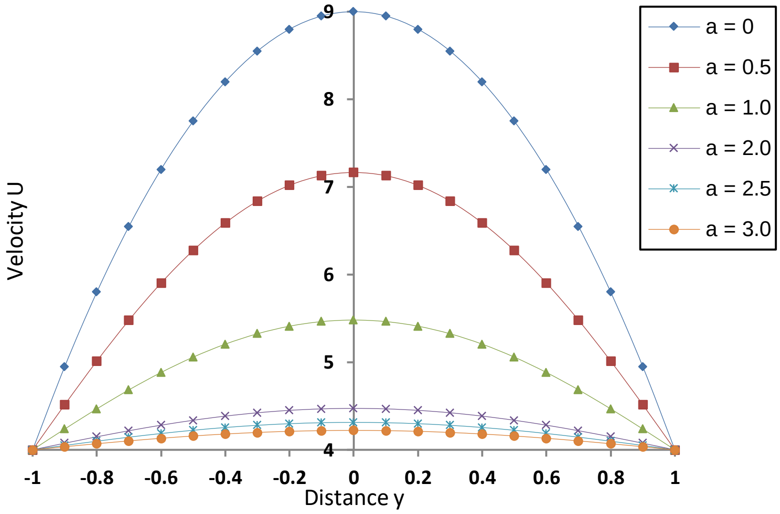

Fig 6:- Velocity profiles with various couple stress parameter *a* for fixed values of slip parameter $\alpha = 0.4$ and *pressure gradient G=10*

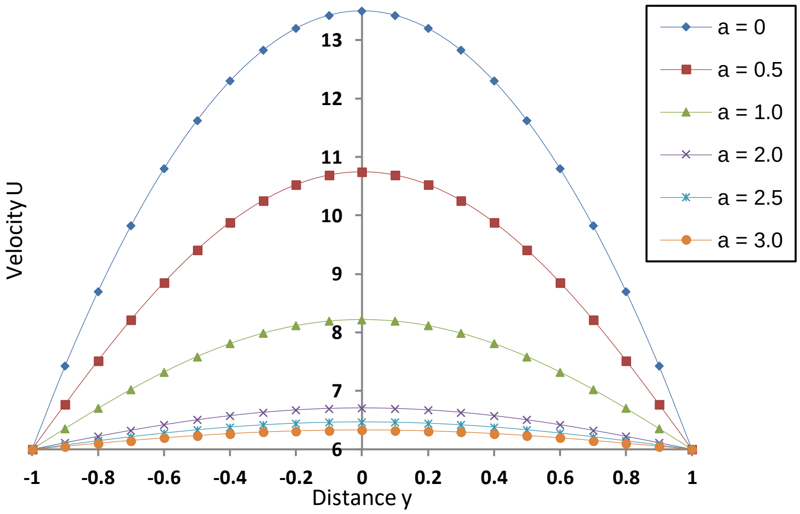

Fig 7:- Velocity profiles with various couple stress parameter *a* for fixed values of slip parameter $\alpha = 0.4$ and *pressure gradient G=15*

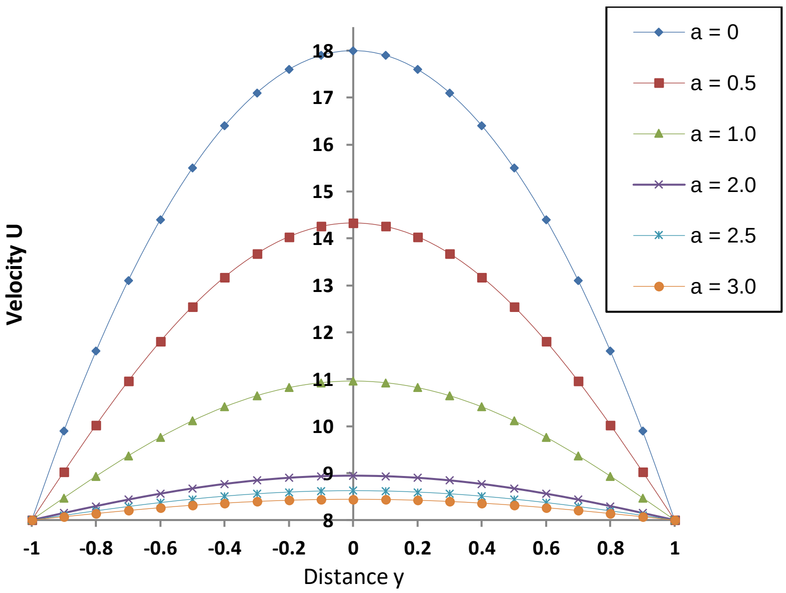

Fig 8:- Velocity profiles with various couple stress parameter *a* for fixed values of slip parameter $\alpha = 0.4$ and *pressure gradient G=20*

➢ *Influence of Slip Parameter in Volume Flow Rate*

Figure 9 to Figure 14 exhibits the dissimilarity of volume flow rate q with growing values of pressure gradient G for numerous flow parameter of viscous Newtonian fluid a and slip parameter α. From the graph, It is observed that the volume flow rate q of the channel increases with the increasing values of pressure gradient G, while it decreases with the increasing of couple stress parameter a with slip parameter α. However, for larger values of slip parameter α, no flow parameter affects the volume flow rate q, for various values of couple stress fluid.

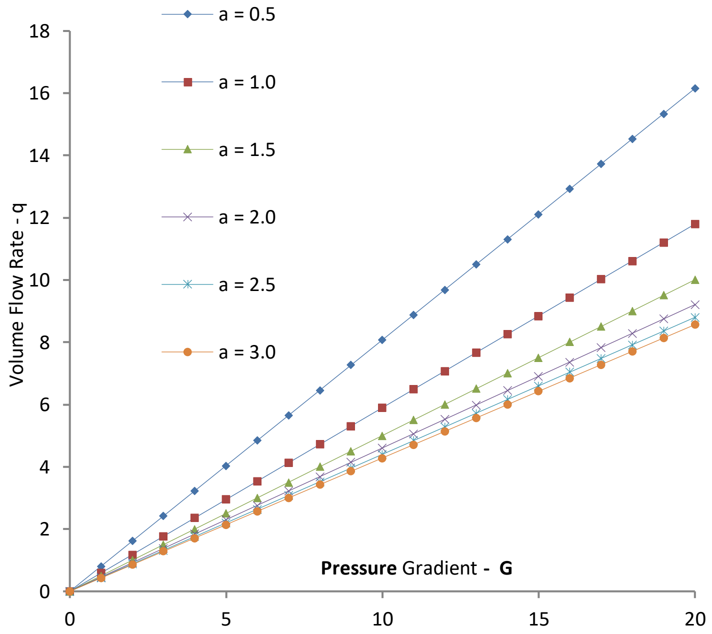

Fig 9:- Volume Flow Rate with various couple stress parameter a for fixed values of slip parameter $\alpha = 0.2$

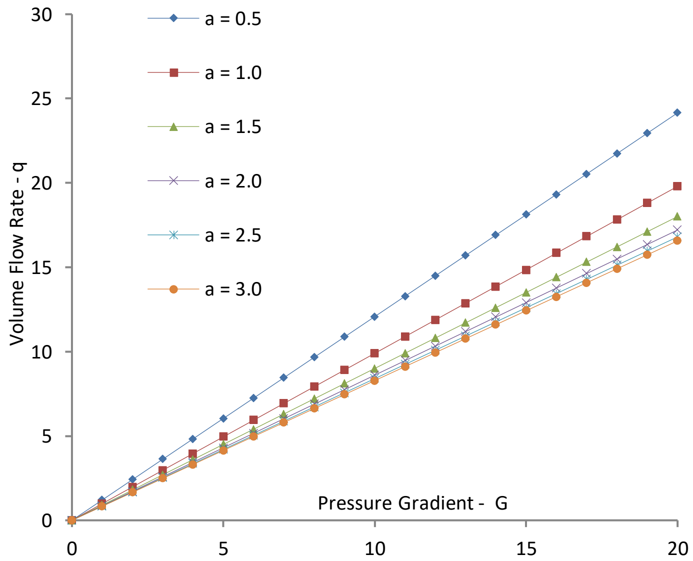

Fig 10:- Volume Flow Rate with various couple stress parameter a for fixed values of slip parameter $\alpha = 0.4$

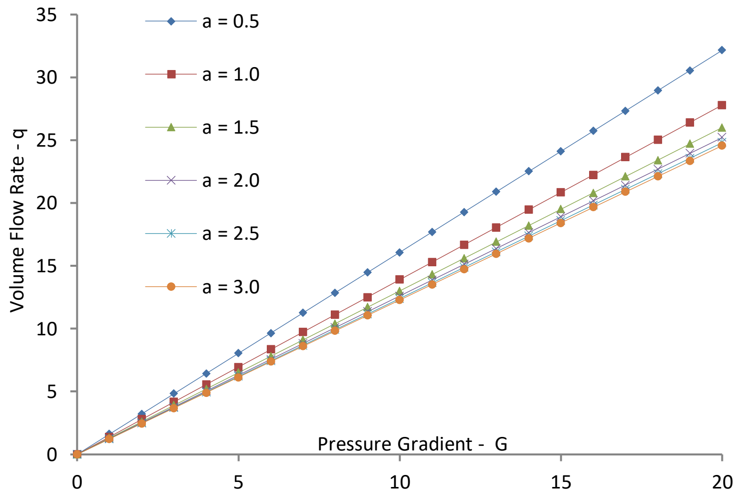

Fig 11:- Volume Flow Rate with various couple stress parameter *a* for fixed values of slip parameter $\alpha = 0.6$

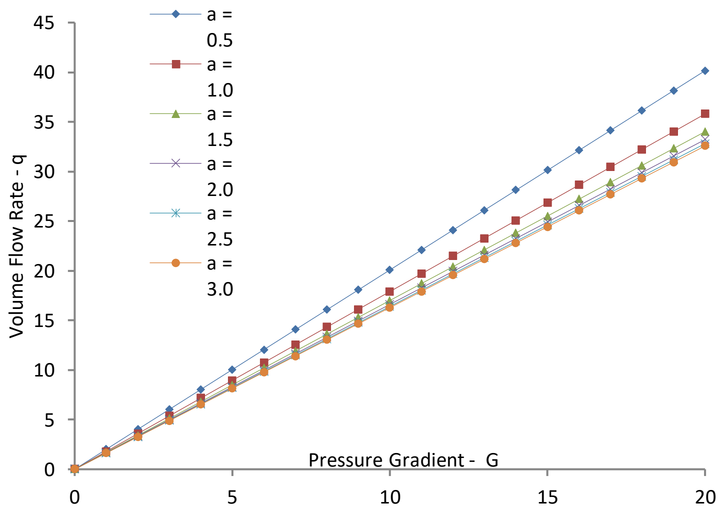

Fig 12:- Volume Flow Rate with various couple stress parameter *a* for fixed values of slip parameter $\alpha = 0.8$

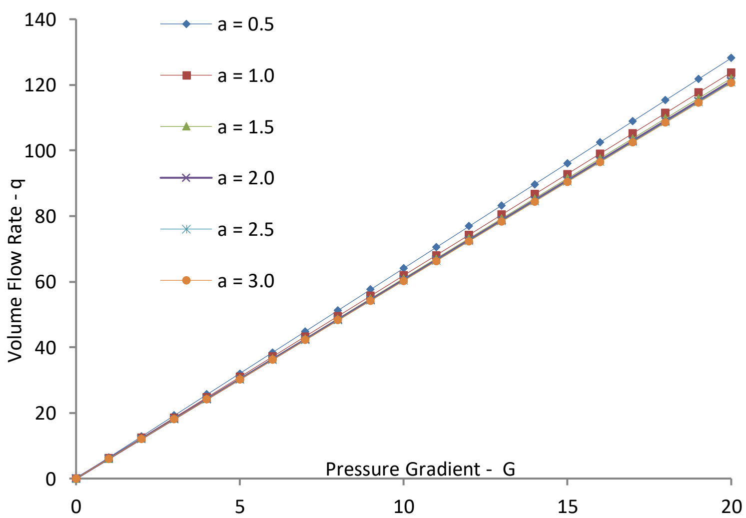

Fig 13:- Volume Flow Rate with various couple stress parameter *a* for fixed values of slip parameter $\alpha = 3.0$

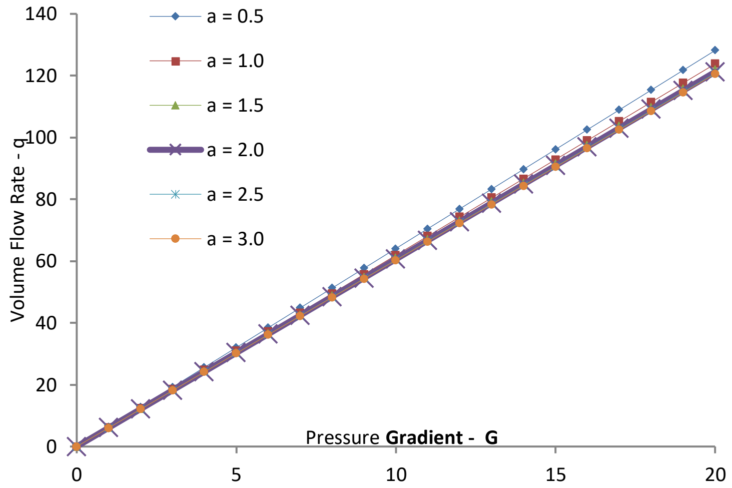

Fig 14:- Volume Flow Rate with various couple stress parameter *a* for fixed values of slip parameter *α= 4.0*

- *Influence of Slip Parameter in Velocity profile*

Figure 15. demonstrations that the dissimilarity of velocity for different values of the slip factor α while the other parameters couple stress *a* and pressure gradient *G* are fixed. We observed that the velocity has increasing impact due to increasing of slip factor. This shows that the velocity affected by the motion of the boundary is less due to more the fluid slips at the boundary.

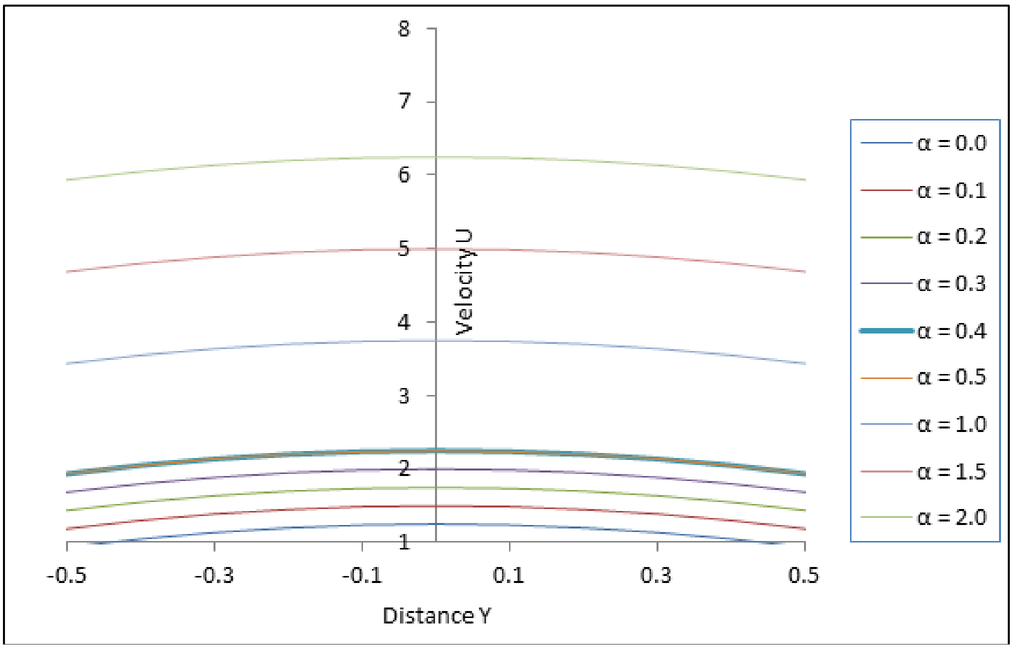

Fig 15:- Variation of velocity for various values of the slip parameter *α* for *a* and G are fixed

IV. CONCLUSION

The classical flow problem of plane-poiseuille flow has been considered for an incompressible couple stress fluid between parallel plates with slip boundary conditions. In the absence of couple stress, the results for the liming case as $a \to 0$ or $b \to \infty$ for Newtonian flow are achieved. It is quite interesting to see that these limiting solutions are well in agreement with the solutions of Newtonian fluid. Due to presence of couple stresses has a declining impact on the fluid velocity and the volume flow rate.

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