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Derivations of BF- Algebras

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Abstract:- The concept of derivation in a BF- algebra has been introduced . In addition a left –right anda right- left- derivation of BF_2 – algebras, left and right - derivation of ideal in BF– algebras are investigated. Different characterization of right-left-derivations,left-right - derivation,self map and fixed subalgebras have been discussed. We have also discussed derivation of BF-algebra if left and right –derivations are equal. In general different new theorems, Lemmas,Propositions and Corollaries have been proved.

Keywords:- *BCI-algebra,B-Algebra, BG- algebra, Derivation of B-algebra,and BF-algebra.*
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I. INTRODUCTION

Neggers and Hee Sik Kim in [5] introduced B-algebras which is related to a class of algebras several classes of algebras such as BCH/BCK-agebras . Tao Sum, Junjie Zhao and Xiquan Liang in [7] investigated BCI-algebras with conditions and their properties. The notion of BF-algebras as ageneraization of B-algebras initaited by Andrzes in [1]. He also introduce ideals and normal ideal in BF-algebras. Nora.O. and Al-shehrie in[6] introduced the notion of left-right(right-left) derivation of B-algebra and some related properties. Jianming Zhan and Yong Lin Liu in [3] discussed on f- derivation of BCI- algebras.

Abujabaland etal in [2] introduced left-right-derivation of BCI –algebras and Mostofa and etal in [4] discussed about properties of derivation of Ku-algebras.

In this paper the derivation of BF-algebra with different properties and left and right -derivatives of ideals in BF-algebras havebeen introduced.

II. MATERIALS AND METHODS

In [7]a non-empty set A with abinary operation *, and a constant 0 is calleda BCI-algebra, if it satisfies the following axioms:

1. $((a*b)*(a*c)*(c*b)=0$
2. $(a*(a*b))*b=0$
3. $a*a=0$
4. $a*b=0$ and $b*a=0$ *implies* $a=b$. *a,b,c in A .*

Again In[5]a non-empty set A with abinary operation $*$*, and a constant 0 satisfying the following axioms:*

1. $a*a=0$.
2. $a*0=a$.
3. $(a*b)*c=a*(c*(0*b))$ *,for all a,b,c in A is said to a B-algebra.*

In [5] an algebra which satisfies conditions:

1. $(a*b)*(0*b)=a$.
2. $a*(b*c)=(a*(0*c)*b$.
3. $a*b=0$ *Implies* $a=b$.
4. $0*(0*a)=a$.

is also called a B-algebra.

An algebra $(A, *, 0)$ *is said to be aBH- algebra if it satisfies the following holds:*

1. $a*a=0.$
2. $a*0=a.$
3. $a*b=0$ *and* $b*a=0 \Rightarrow a=b.$

*In addition a non- empty set A with a binary operation *, and a constant 0 is called a BG-algebra if* $a,b\in A$ *satisfies the following axioms:*

1. $a*a=0.$
2. $a*0=a.$
3. $a=(a*b)*(0*b).$

Theorem 2.1. [5] If $(A, *, 0)$ *is a B-algebra, then* $(A, *, 0)$ *is a BG-algebra.*

In [1] an algebra $(A, *, 0)$ *of type(2,0) is called BF-algebra if it satisfies the following axioms for all* $a,b\in A$:

1. $a*a=0.$
2. $a*0=a.$
3. $0*(a*b)=b*a$.

In [5] Ifa non-empty set A with a binary operation, and a constant zero is calleda B-algebra and* $a,b\in A$*, then the following holds:*

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1. $0*(a*b)=b*a.$
2. $a=(a*b)*(0*b).$
3. $a*b=0 \quad and \quad b*a=0 \Rightarrow a=b.$

Example 2.2. Let $(R, *, 0)$ *be the algebra with the operation * defined by*

$$a*b=\begin{cases} a & if\ b=0 \\ b & if\ a=0 \\ 0 & otherwise \end{cases}$$

. Then $(R, *, 0)$ *is a B-algebra.Where R is a real number.*

*In [1]a non-empty set A with a binary operation * ,and a constant 0 is said to be a BF- algebra and for all* $a,b \in A$*, then the following holds:*

1. $0*(0*a)=a$
2. $0*a=0*b \Rightarrow a=b$
3. $a*b=0 \Rightarrow b*a=0$

*In [1]a non-empty set A with a binary operation *,and a constant 0 iscalled a* BF_1- *algebra if and only if for all* $a,b \in A$ *the following holds:*

1. $a*a=0.$
2. $0*(a*b)=b*a.$
3. $a=(a*b)=b*a.$

Lemma 2.3. [1] Let $(A,*, 0)$ *be a* $BG-$*algebra. Then the following holds for all* $a,b \in A$*:*

1. *The right cancellation law holds in A. That is* $a*b=c*b \Rightarrow a=c.$
2. $0*(0*a)=a.$
3. *If* $a*b=0$*, then a = b.*
4. *If* $0*a=o*b$*, then a = b .*
5. $(a*(0*a))*a=a.$

Definition 2.4. [5]a non-empty set A with a binary operation *, and a constant 0 is said to be 0- Commutative B- algebra if $a*(0*b)=b*(0*a)$,for all a,b in A.

In [1] a BF-algebra $(A,*, 0)$ *is 0- Commutative if* $a*(0*b)=b*(0*a)$.

Remark 2.5. If a BF-algebra is 0- Commutative , then for all $a,b \in A$*.*

1. $a*(a*b)=b.$
2. $a \cap b=b*(b*a).$

III. RESULTS

3.1. Derivation ofBF- Algebras

Definition 3.1.1. If $(A,*, 0)$ *be a BF-algebra,then we have the following:*

1. *By a left-right-derivation of A is a self-map* $d:A \to A$ *satisfying the identity* $d(a*b)=(d(a)*b)\cap(a*d(b))$ *for all a,b in A.*
2. *A right-left-derivation of A satisfying the identity* $d(a*b)=(a*d(b))\cap(d(a)*b)$ *for all a, b in A.*
3. *If d satisfy both a left-rightand a right-left-derivation,then d is called a derivation of A.*

Remark 3.1.2. If $(A,*, 0)$ *be a* BF- *algebra, then* $a \cap b=b*(b*a)$ *for all a, b in A.*

Example 3.1.3. Let $A=\{0, a, b, c\}$ *be a set defined by the table below:*

*	0	a	b	c
0	0	b	a	c
a	a	0	c	b
b	b	c	0	a
c	c	a	b	0

Then $(A,*, 0)$ *is a BF- algebra.*

Define $d:A \to A$ *by* $d(e)=\begin{cases} c & if\ e=0 \\ 0 & if\ e=a \\ a & if\ e=b \\ 0 & if\ e=c \end{cases}$

Now,

$$d(a*b)=(d(a)*b)\cap(a*d(b))=(0*b)\cap(a*a)=0\cap0$$

Hence d is a left-right- derivation.

Again

$$d(a*b)=(a*d(b))\cap(d(a)*b)=(a*a)\cap(0*b)=0\cap0$$

Hence d is a $(R,L)-$*derivation of A. Therefor d is a derivation of A.*

Definition 3.1.4. A self-map of a BF-algebra A is called regular if $d(0)=0.$

Proposition 3.1.5. Let d be a $(L,R)-$*derivation of*BF-algebra *A. Then*

1. $d(0)=d(a)*a$*, for all a in A.*
2. *d is one -to- one.*
3. *If d is regular, then it is the identity map.*
4. *If there is an element a in A such that* $d(a)=a$*, then d is the identity map.*

5. *If there is an element ain A such that* $d(b)*a=0$ *or* $a*d(b)=0$, *forall b in A, that is d is constant.*

Proposition 3.1.6. Let d be $(R,L)-$ *derivation of BF-algebra A. Then*

1. $d(0)=a*d(a)$ *,forall a in A.*
2. $d(a)=d(a)\cap a$ *,for all a in A.*
3. d *is one -to -one.*
4. *If d is regular, then it is the identity map.*
5. *If there is an element a in A such that* $d(a)=a$, *then d is the identity map.*
6. *If there is an element a in Asuch that* $d(b)*a=0$ *or* $a*d(b)=0$, *for all b in A,then* $d(b)=a$ *,for all b in A. That is d is constant.*

Proof.

1.Let ain A. Then $a*a=0$ *and*

$$d(0)=d(a*a)=(a*d(a))\cap(d(a)*a)$$
$$=(d(a)*a)*[(d(a)*a)*(a*d(a))]$$
$$=[(d(a)*a)\circ(0*(a*d(a))]*(d(a)*a)$$
$$=[(d(a)*a)*(d(a)*a)]*a]*(d(a)*a)$$
$$=0*(d(a)*a)=a*d(a).$$

Hence $d(0)=a*d(a)$.

2. *Let* $(A,*,\ 0)$ *be a BF- algebra. Then* $a*0=a$ *by definition of BF- algebra.*

So that $d(a)=d(a*0)=(a*d(0))\cap(d(a)*0)$

$$=(a*d(0))\cap(d(a)*0)$$
$$=d(a)*[(d(a)*a)*d(0)]$$
$$=d(a)*[d(a)*(a*a)*d(a)] \text{ by (1)}$$
$$\Rightarrow d(a)*0=d(a)*[d(a)*(a*(a*d(a))].$$

We have $d(a)*[a*(a*d(a))]=0$ implies $d(a)=[a*(a*d(a))]=d(a)\cap a$.

Hence $d(a)=d(a)\cap a$.

3. *Let a, b in A such that* $d(a)=d(b)$, *then by (1)* $d(0)=a*d(a)$.

Also by (1) $d(0)=b*d(b)$. *Thus* $a*d(a)=b*d(b)$,

But $d(a)=d(b)$.

We get $a*d(a)=b*d(a)$.

Hence $a=b$.

Therefore d is one- to -one.

4. *Let d be regular, and ain A. Then* $d(0)=0$, *so, we have* $0=a*d(a)$ *by (1).*

Hence $d(a)=a$ *,for all ain A. That is d is the identity map.*

5. *Assume* $d(a)=a$, *for some a in A . Then* $a*d(a)=0$ *and d(0) = 0.*

Thus d is the identity map.

6. *Assume* $d(a)*a=0$ *or* $a*d(b)=0$ *for all b in A. Then* $d(0)=0$ *and* $d(b)*a=0$.

Hence $d(b)=a$.

On the similar manner, $a*d(b)=0$ *and* $d(0)=0$, *implies* $d(b)=a$.

Thus d is the identity map.□

Example 3.1.7. Let Q be a rational numbers and "-" theoperations on Q. Then (Q,- ,0) is a BF- algebra. Since

1. $a-a=o$.
2. $a-0=a$.
3. $0-(a-b)=-(a-b)=b-a$.

Let $d:Q\to Q$ *defined by* $d(a)=a-1$, *for all ain Q. Then*

$$d(a-b)=(d(a)-b)\cap(a-d(b)).$$
$$=((a-1)-b)\cap(a-(b-1)).$$
$$=(a-b-1)\cap(a-b+1).$$
$$=(a-b+1)-((a-b+1)-(a-b-1).$$
$$=(a-b+1)-(a-b+1-a+b+1).$$
$$=(a-b+1)-2=a-b+1-2=a-b-1.$$
$$=d(a-b) \text{ ,for alla,b in } Q.$$

So d is a left-right- derivation of A. But

$$d(1-0)=(1-d(0))\cap(d(1)-0)=(1-(0-1))\cap((1-1)-0$$

$$=2\cap 0=0-(0-2)=2.$$
$$0=1-1=d(1)=d(1-0).$$

Hence $2\le 0$. *Therefore d is not a right-left- derivation of Q.*

In addition

$$d(1-0)=(d(1)-0)\cap(1-d(0)).$$
$$=((1-1)-0)\cap(1-(0-1)).$$
$$=(0-0)\cap(1-(0-1))=0\cap 2.$$
$$==2-(2-0)=2-2=0.$$

Hence $2\le 0$. *Thus d is not derivation of Q.*□

Definition 3.1.8. Let $(A,*,\ 0)$ *be a* BF_2- *algebra. Then*

1. *A left- right-derivation of* BF_2- *algebra is a self-map* $d:A\to A$ *satisfy* $d(a*b)=(d(a)*b)\cap(a*d(b))$ *for all a,b in A.*
2. *A right-left- derivation of A satisfying the identity* $d(a*b)=(a*d(b))\cap(d(a)*b)$ *forall a, b in A.*

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3. *If dsatisfies both a left-rightand a right-left- derivation, then d is called a derivation of A.*

Lemma 3.1.9. Let $(A,*,\ 0)$ *be a* BF_2 *−algebra and let a in A. Then* $a*a=0$.

Proposition 3.1.10.Let $(A,*,\ 0)$ *be a* BF_2 *− algebra and let d be a* (L,R)*− derivation of* BF_2 *− algebra A. Then*

1. $d(a)=d(a)*0$.
2. $d(a*b)=d(0)$ *if and only if a = b ,for all a,b in A.*

Lemma 1.1.11. Let $(A,*,\ 0)$ *be a* BF_2 *−algebra and let d be a derivation of* BF_2 *− algebra. Then* $d(a*b)=d(0)$ *if and only if* $d(a)=a,\ \ d(b)=b$ *and* $a*b=0$.

Proof. *Let* $(A,*,\ 0)$ *be a* BF_2 *−algebra and* $d:A\to A$ *be a derivation of* BF_2 *− algebra. Then assume* $d(a*b)=d(0)$*,a,b,0 in A.*

Now, $d(a*b)=(d(a)*b)\cap(a*d(b))$.

$=(a*d(b))*(a*d(b))*(d(a)*b)$

$=[a*d(b))*(0*d(b))*a]*(b*d(a))$.

$=0*(b*d(a))=d(a)*b$ *by* (L,R)*− derivation.*

The rest of the proof followstrivially.□

Proposition 3.1.12. Let A be a BF_2 *− algebra and let d be derivation of A. Then the following holds.*

1. *If d is regular, then* $d(a*b)=0$ *for some a,b in A.*
2. *d is one-to one*
3. *If d is regular, then d is the identity map.*
4. *If there is an element a in Asuch that* $d(a)=a$*, then d is the identity map.*

Definition 3.1.13. Let A be a BF_2 *− algebra and let d be a derivation of A. Then the fixed derivation of A is defined by* $Fix_d(a)=\{a\in A: d(a)=a\}$.

Proposition 3.1.14. Let A be a BF_2 *− algebra and let d be a derivation of A. Then* $Fix_d(a)$ *is a subalgebra of A.*

Definition 3.1.15. Let A be a BF *−algebra and* $d:A\to A$ *be a self-map. Then d is called a left derivation of* BF *−ideal of A if it satisfies the following conditions:*

1. $0\in P$.
2. $d(a*b)\in P$ *and* $d(a)\in P$ *implies* $d(b)\in P$*, for any a,b in A . Similarly d is called a right derivation of* BF *− algebra if it satisfies the following conditions:*

1. $0\in P$.
2. $d(a*b)\in P$ *and* $d(b)\in P$ *implies* $d(a)\in P$*, for any a, b in A .*

Example 3.1.16. Let $A=\{0,a,\ b,\ c\}$ *and * be defined by the table below:*

*	0	a	b	c
0	0	a	b	c
a	a	0	c	0
b	b	c	0	c
c	c	0	b	0

Hence $(A,*,0)$ *is a* BF *−algebra.*

Let $P=\{0,a,c\}$ *be ideals of A. Define* $d:A\to A$ *by*

$$d(a)=\begin{cases}0, if & a=0,a,b\\ a, if & a=c\end{cases}$$ *, we have*

1. $0\in P$.
2. $d(a*c)=d(0)=0\in P$ *and* $d(a)=0\in P$*, implies* $d(c)=a\in P$.

Hence d is a left derivation of ideal of A.

Again d is also a right derivation of ideals of A. Since

1. $0\in P$.
2. $d(a*c)=d(0)=0\in P$ *and* $d(c)=a\in P$, *implies* $d(a)=0\in P$.

IV. DISCUSSION

In this paper we introduced derivation of BF- algebra which is important for the growth of the theory to wards applications in algebraic coding theory which become new area of research.

V. CONCLUSIONS

In this paper we introduced the concepts of derivations in BF-algebra,the left -right and right-left derivation of BF_2 − algebra has been introduced. In addition,left- right-derivation of ideals of BF_2 −algebra has been investigated. Finally,different characterization Theorems,Lemmas and corollaries have been proved.

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