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Position Tracking of Ball and Beam System Using Direct Adaptive Controller

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Abstract:-Based on the model approximation, adaptive controller gains are designed to achieve position and tracking performance. The effect of conjecture errors and outsidedisturbances will be attenuate to a particular level using direct Adaptive Controller method gives that all signals involved are bounded in the output ofthe system will asymptotically tracks the desired output path. So, the direct adaptive controller is applied on Ball and Beam system in order to track the position and tracking control of the ball and beam system using MATLAB is demonstrated.

Keywords:-*Ball and Beam, Modeling, Adoptive Controller*.

I. INTRODUCTION

The project deals with MATLAB Design of Adaptive Controller algorithm for Non-Linear system. In this project we will study about the mathematical modeling of Ball and Beam systems. Later Adaptive Controller technique is applied to ball and beam it is used to control the position of the ball without falling down. Adaptive Controller is also used for optimization process and results of Adaptive Controller are plotted and then finally results are compared to prove that Adaptive Controller gives best results of performance Index.

The chapter one deals with the complete introduction of the project, the way the thesis is organized. It also gives a basic idea of the content present in the thesis. The literature review is also given for the further references.

The chapter two deals with the introduction of Adaptive Controller and its design and direct and indirect controllers, Model Reference Adaptive Controller techniques are explained.

The chapter three deals with mathematical modeling of Ball and Beam system. It can be represented in state space equation and transfer function. The principle and physical criteria also have been study in detail.

The chapter four deals with stability of the system and explained about Lyapunov stability and linear matrix inequality.

The chapter five deals with the design of controller and Lyapunov equations are discussed

The chapter six deals with the display of result of the position controlof the given nonlinear system.

The chapter seven deals with overall discussions and conclusion. A few recommendations for the future work

II. ADAPTIVE CONTROLLER

To design and tune a good controller, one need to know:
(i). The required control loop performances
(ii). Dynamic model of the plant to be controlled
(iii). Suitable control for achieving desired performance

Adaptive controllers are two types
1.Direct Adaptive Controller
2. Indirect Adaptive Controller

- *Direct Adaptive Controller:*

The design of the controller is such output of the error between the plant and reference model is zero during initial conditions.

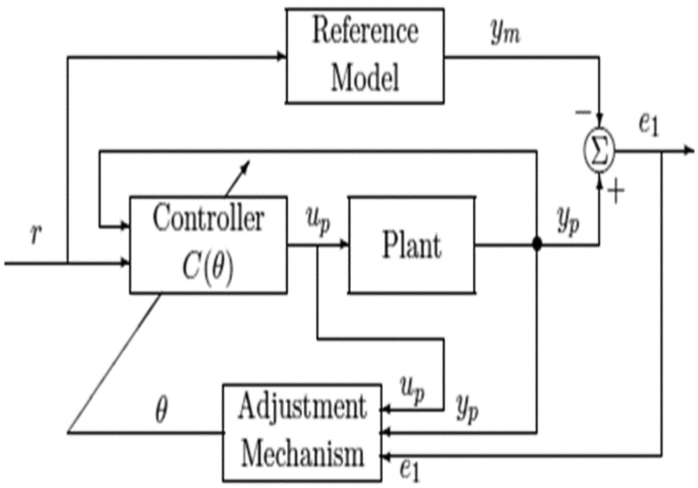

Fig.1 Block Diagram of Adaptive Controller

Here Reference model gives the output y_m and compared with plant output and gives error e1. Based on the error, the control parameters are adjusted such the plant gives required performance. Here error will be Zero.

- *Indirect Adaptive Controller:*

The adaptation mechanism 1 controls theadjustable predictor parameters and these parameters are then used to calculate the controller parameters.

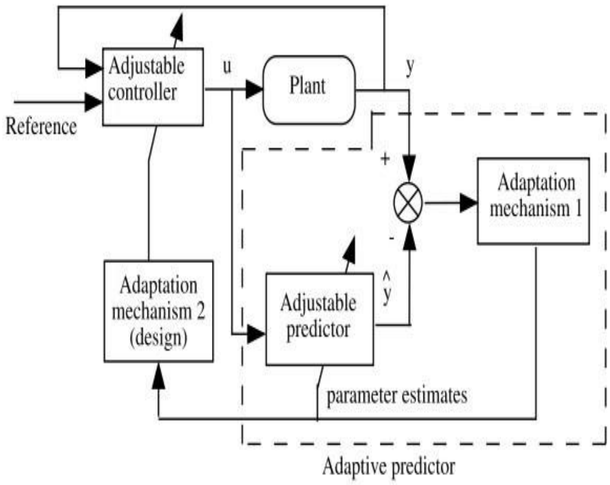

Fig. 2 Indirect Adaptive Controller

The output of the plant y will be given as input to the adjustable controller and also a reference signal will be given as an input to the adjustable controller. The Adaptation mechanism 2 will control the adjustable controller and the output of the adjustable *controller is input to both plant and adjustable predictor. In Indirect Adaptive Controller, error will not be equal to zero but it is a finite small value which can be neglected.*

➢ *Direct Adaptive Controller:*

IN DAC adaptation law searches for the parameters such that the response of the plant under adaptive control is same as that of reference model to minimize the error. DAC means system which gives desired performance is expressed in terms of a reference model, which gives desired performance to the reference signal.

The system output is compared with desired response from a reference model and the control parameters are update based on this error. Using DAC, choose a reference model such that it will respond quickly to a step input with a short settling time.

The DACconsists of two loops
(i). inner loop
(ii). Outer loop.

Inner loop consisting of plant and the regulator. Outer loop consists of regulator parameters to drive the error between model output and plant output to zero.

Reference model: It is used to give an ideal response of the adaptive control system to the reference input.

Controller: It consists of different parameters that are controllableto achieve the error to zero.

Plant: Plant is assumed to have known structure but the parameters are unknown.

Adjustment mechanism: It is used to control the parameters of the controller in order to obtain the desired performance of the plant.

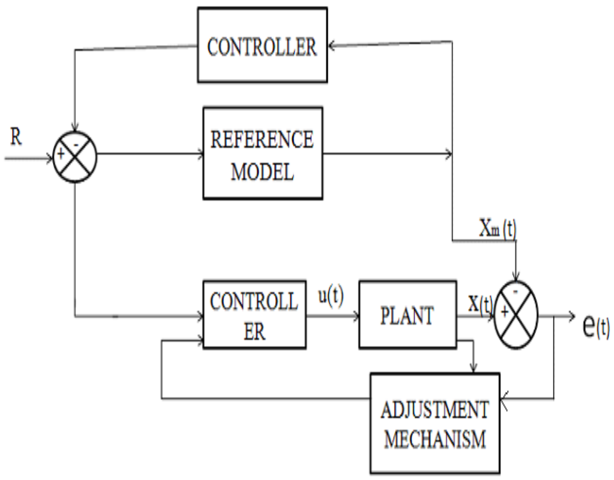

Fig. 3Block diagram of Direct adaptive controller

III. MODELLING OF BALL AND BEAM SYSTEM

The practical model of the ball and beam system is as shown in below fig 4.

Fig 4. Practical model of Ball and Beam system

Working: A sensor is placed on the beam in order to track the position of the ball. In order to meet the required beam angle, torque will be applied to the beam by using actuator. The Controller controls the position of the ball by moving the beam using motors.

The angular velocity of the beam can be neglected during slow motion of the ball. The Ball position can be changed without the limit for a fixed input of beam angle. This property has made the system a suitable device to test different control strategies such as Fuzzy logic and PID.

➢ *Modelling of ball and beam system:*

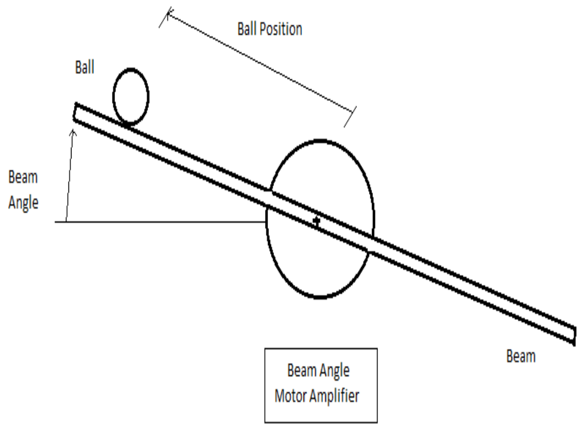

Fig 5.Modelling of ball and beam system.

Ball and beam system is a non-linear system. The dynamic equations of ball and beam system are explained below

Gravitational force in the x-direction:

F_{tx} = MgsinΘ

The torque produced by the ball rotational motion: $T_r = F_{rx}R = \frac{J_b}{R}\ddot{r}$

Corresponding linear force:

$$F_r = \frac{T_r}{R} = \frac{J_b}{R^2}\ddot{r}$$

Centrifugal force on ball about point 'o': $F_c = Mr\left(\frac{d\theta}{dt}\right)^2 = Mr\dot{\theta}^2$

Applying Newton's second law of motion:

$$F = F_c - F_{rx} - F_{tx}$$

$$M\ddot{r} = Mr\dot{\theta}^2 - \frac{J_b}{R^2}\ddot{r} - Mg\sin\theta$$

$$M\ddot{r} + \frac{J_b}{R^2}\ddot{r} = Mr\dot{\theta}^2 - Mg\sin\theta$$

$$(M + \frac{J_b}{R^2})\ddot{r} = Mr\dot{\theta}^2 - Mg\sin\theta$$

$$\ddot{r} = \frac{M}{(M + \frac{J_b}{R^2})}(r\dot{\theta}^2 - g\sin\theta)$$

$$\ddot{r} = B(r\dot{\theta}^2 - g\sin\theta)$$

Where $B = \frac{M}{(M + \frac{J_b}{R^2})}$

$$\ddot{r} = B(r\dot{\theta}^2 - g\sin\theta)$$

$$\ddot{\theta} = u(t)$$

The state space representation is

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = B(x_1 x_4{}^2 - g\sin x_3)$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = u$$

Where,

x_1 = Ball position; x_2 = Ball velocity

x_3 = Beam angle; x_4 = Beam velocity

Table 1 The physical parameters of the Ball and Beam system

Parameter	Value
Mass of the Ball, M	0.05 Kg
Radius of the Ball, R	0.01m
Gravity, g	$9.81\, m/\sec^2$
moment of inertia of the Ball, J_b	2×10^{-6} Kg-m^2

The nonlinear functions of the above equation it can be chosen as:

$$f_1(x) = \frac{-Bg\sin(x_3)}{x_3}$$

$$f_2(x) = Bx_1x_4$$

Depend on the states of the system and these functions can be taken as premise variables for the Adaptive model. Hence, the ball and beam dynamics Which can be represented as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & f_1 & f_2\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ 1\end{bmatrix}u$$

Linearization of Non-linear System:

Consider a system of fourth order i.e having four state variables. Then the matrices "A,B" are formulated by using Jacobian Matrix.

$\dot{x}$. = f(x,u)

$$\mathbf{A}= \begin{bmatrix}\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4}\\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4}\\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4}\\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4}\end{bmatrix} \quad \mathbf{B} = \begin{bmatrix}\frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \frac{\partial f_1}{\partial u_3} & \frac{\partial f_1}{\partial u_4}\\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \frac{\partial f_2}{\partial u_3} & \frac{\partial f_2}{\partial u_4}\\ \frac{\partial f_3}{\partial u_1} & \frac{\partial f_3}{\partial u_2} & \frac{\partial f_3}{\partial u_3} & \frac{\partial f}{\partial u_4}\\ \frac{\partial f_4}{\partial u_1} & \frac{\partial f_4}{\partial u_2} & \frac{\partial f_4}{\partial u_3} & \frac{\partial f_4}{\partial u_4}\end{bmatrix}$$

IV. STABILITY OF THE SYSTEM

➢ ***LYAPNOV STABILITY ANALYSIS:***

To design any controller first we have to consider stability. It means of checking the stability is necessary for any system. The Lyapunov direct method involves finding a Lyapunov function for a system. If such a function exists, then the system is stable.

A Lyapunov function is defined as a scalar function $V(X)$ that satisfies the two conditions:
i. $V(X)$ is a positive definite function
ii. $\dot{V}(X)$is a negative definite function

Lyapunov function is a function in time, whose scalar value is always positive, with a negative derivative. It must converge to the point where $V(X) = \dot{V}(X) = 0$as $t \to \infty$. In general, $V(X)$ is a quadratic function of $X(t)$, $V(0) = 0$when $X = 0$. This means that system states must converge to origin, in the absence of input. This is basically same as asymptotic stability.

By using Lyapunov stability analysis, we can measure the Gains of the Nonlinear systems. Lyapunov theory is used to make conclusions about Trajectories of the system. It concerns the stability of the solutions near to the equilibrium point.

The unique solution is

$$-Q = A^T P + PA$$

The Lyapunov inequality: $A^T X + XA < 0$.......(1)

Where X is unknown matrix

V. DESIGN OF CONTROLLER

Adaptive controllers are designed to control real physical systems ;
1.structural vibrations
2. coulomb friction
3. Measurement noise
4. Errors and sampling delays

Theorem: The closed adaptive system is globally is asymptotically stable if there exists a common P for all the subsystems which satisfies the following inequalities

$$G_{ii}^T P + PG_{ii} < 0$$

$$\left(\frac{G_{ij}+G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij}+G_{ji}}{2}\right) \le 0 \quad i<j$$

Where,$G_{ii} = A_i - B_i K_i$,
$G_{ij} = A_i - B_i K_j i, j$ =1,2,…..,r

Pre-multiplying and post-multiplying both sides of inequalities in standard equation by P^{-1} and take $X = P^{-1}, M_i = K_i X$

Finally, the following LMIs of equation (1.8) is

$$-XA_i^T - A_i X + M_i^T B_i^T + B_i M_i > 0$$

$$-XA_i^T - A_i X - XA_j^T - A_j X + M_j^T B_i^T + B_i M_j + M_i^T B_j^T + B_j M_i \ge 0, i < j$$

Finally, the stability analysis of the adaptive control system is reduced to a problem of finding a common *P*.

By using Lyapunov stability and LMI (Linear Inequality Matrix) technique we get the gains of controller (K), Positive definite matrix (P).

Hence the Lyapunov stability model for the ball and beam system can be given as equation

$$A^T P + PA = -Q$$

$$A^T P + PA < 0$$

$$Q = A^T P + PA$$

$$B^T Q + QB < 0$$

We know the structure of the plant but we don't know the parameters of the plant. By using model reference adaptive controller, we are getting the parameters of the plant. Here the parameters of the plant are:

$$A = \begin{bmatrix} 0 & 1.0000 & 0 & 0 \\ 0 & 0 & -0.4122 & 0 \\ 0 & 0 & 0 & 1.000 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 71.2069 \end{bmatrix}$$

$$P=\begin{bmatrix} 27.1850 & -9.9943 & -2.4662 & -0.0000 \\ -9.9943 & 10.7268 & -6.8475 & 0.0000 \\ -2.4662 & 6.8475 & 28.6258 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 53.2754 \end{bmatrix}$$

$$Q=\begin{bmatrix} 0.0197 & 0.0401 & -0.0236 & -0.0100 \\ 0.0401 & 0.1398 & -0.0718 & -0.0375 \\ -0.0236 & -0.0718 & -0.0509 & 0.0223 \\ -0.0100 & -0.0375 & -0.0223 & -0.0200 \end{bmatrix}$$

$$K=[-0.0032 \quad -0.0105 \quad 0.0077 \quad 0.0041]$$

The gains of the controller are obtained. This value of gain K makes the system stable.

VI. MATLAB RESULTS

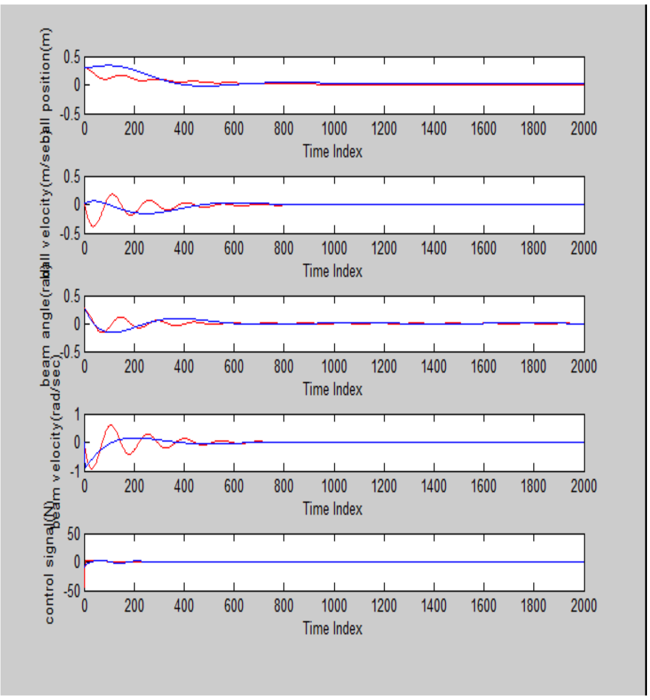

Fig. 6:-MATLAB results for Ball and Beam system for initial position 0.3m

Here Blue line indicates reference signal and Red line indicates actual control signal**.**

VII. CONCLUSION AND FUTURE WORK

We have studied the problem of designing various controllers for nonlinear systems. The adaptive model is able to describe the system dynamics accurately with any operating range. For stabilizing the system, a state feedback controller is designed based on adaptation mechanism.

The control problem has been investigated for Ball and Beam system with the subsystem that exchanges information through networks. The parameters of the ball and beam system are known by using Lyapunov stability analysis.

➢ ***Future work:***

Direct adaptive controller for ball and beam can be implemented in a hardware to observe actual stability of a system. This technique can be extended for indirect adaptive controller.

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