Sliding Mode Controller Design For Interacting and Non Interacting Two Tank System

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Abstract:- Sliding mode control approach is recognized as one of the efficient tools to design robust controller for complex higher-order dynamic plant operating under uncertainty conditions. The main advantages of sliding mode are low sensitivity to plant parameter variations and disturbances can be completely eliminated. Due to this virtue, sliding mode control has been widely applied and research on sliding mode control theory is greatly motivated as challenges and demand grow. It is noticed that a variety of improved sliding mode control schemes have been invented which possess additional desired characteristics. In this thesis, Sliding mode control strategy is applied to non interacting and interacting two tank systems with uncertainty being present. Here these systems are dynamically modeled. SMC theory and design for first order and second order sliding mode strategy is briefly presented in this work. The second order sliding mode scheme covered in this thesis are: Twisting algorithm and Super-Twisting algorithm. Since sliding mode control provides an elegant way of designing controllers feasibly for nonlinear systems. In this thesis, first order and second order sliding mode controller is applied to non interacting and interacting two tank systems.

Keywords:- Siding Mode Control (SMC), Second Order Sliding Mode Control (SOSMC), First Order Sliding Mode Control (SOSMC).

I. INTRODUCTION

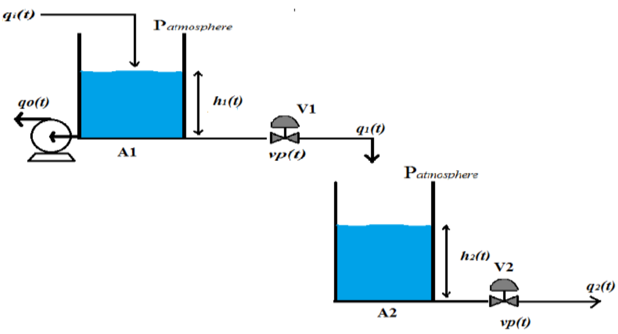
Sliding mode control has its roots in (continous-time) relay control. It was originated in the Soviet Union somewhere in late 1950's, but it was not published outside the Soviet Union until the publication of [2]. After this publication, there was rapid growth of list of publications and SMC had aquired a strong position in the field of both linear and nonlinear control theory. we refer to the standard work of V.I Utkin for obtaining a good overview of continuous-time sliding mode, [3],many other good publications are for example [4],[1],[5] and [6].Sliding mode control for nonlinear systems are reported in [7] by J. J. E. Slotine, and in [8] by Lu and Spurgeon, etc.. More application of sliding mode control technique are reported in [9, [10], [11], [12] and [13]. In [14], [15], [16] and [17], sliding mode control for uncertain systems are proposed. Output feedback is recognized one of the effective approach for tracking error elimination. It has been combined with sliding mode control as Lu and Spurgeon mentioned in [22],[23] and also [24].

In more recent years, a so-called "Higher Order Sliding Mode Control (HOSMC)" has been proposed [37],[21],[38] for nonlinear sliding mode design. And so-called "Twisting Controller" and "Super-Twisting Controller" are the most popular types of higher order sliding mode controller .The Structure of the controller and parameter selection rules of such controller are provided in [37]. The benefit of this HOSM controller, the control signal is being made continuous for comparing with first order sliding mode control.

The control of two tank liquid level system has gained attention of many researchers all over the world. Due to its nonlinear characteristics it is being one of the most challenging benchmarks controlling problem due to its nonlinear characteristics. The control objective in a two tank system is that a required liquid level of the liquid in tank is to be maintained when there will be an inflow and outflow of water out of the tank respectively. The coupled tank system is a single-input single output system (SISO) where input is control voltage and output is water level.

II. MODELLING OF TWO TANK SYSTEMS

Non interacting system:



The above figure shows shows the state coupled two tank system.This system has of two tanks with orifices and liquid sensors at the bottom each tank, liquid basin and a pump .The diameter of the two tanks is same and can be with different output diameters of orifices.The pump feeds the first tank. The output from the first tank is the input supply for the second tank.the outflow of the tank 2 is emptied into basin.

The dynamic mathematical equations for the liquid level in the two tanks are derived as given below. The rate of change of liquid level w.r.t to time in each tank is given by

$$L_i(t)=\frac{1}{A_i}(F_i^{in}(t)-F_i^{out}(t))\,cm/s,\ \ i=1,2 \quad ---(1)$$

where $L_i(t)$, A_i, $F_i^{in}(t)$, $F_i^{out}(t)$ are liquid level, area of cross sectional, rate of inflow, rate of outflow respectively for the ith tank.

Inflow rate of tank one is given as
$F_i^{in}(t) = K_p V_p(t)$ cm^3/s. ---(2)

Where Kp is the pump constant and Vp(t) is the applied voltage to the pump.

Using bernoulis theorm the velocityof outflow at the bottom of the each tank is given as

$$v_i^{out} = \sqrt{2gL_i(t)} \text{ cm/s.} \quad i=1,2 \qquad \text{---(3)}$$

Where ‘ g ‘ is the gravitational acceleration.
The outflow from each tank is given as

$$F_i^{out}(t) = a_i\sqrt{2gL_i(t)} \text{ cm}^3\text{/s} \quad i=1,2 \qquad (4)$$

Where a_i is area of cross sectional of output orifice at the bottom of the i^{th} tank.

But the output from the first tank is the input supply for the second tank

$$\text{So } F_2^{in}(t) = F_1^{out}(t) \qquad \text{---(5)}$$

Using the equations from 1-5 dynamic equations of two tank system is obtained.

$$L_1(\dot{t}) = \frac{a_1}{A_1}\sqrt{2gL_1(t)} + \frac{K_P}{A_1}V_P \qquad \text{---(.6)}$$

$$L_2(\dot{t}) = \frac{a_1}{A_2}\sqrt{2gL_1(t)} - \frac{a_2}{A_2}\sqrt{2gL_2(t)} \qquad \text{---(7)}$$

Modeling of interacting two tank system

The simple nonlinear model of the two tank system [1] is obtained by considering the mass balance principle, which is relating the water level H_1, H_2 and applied voltage ‘q’ to the pump.

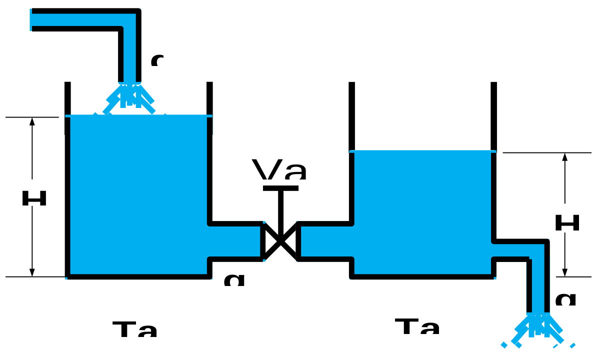


Fig.2 Interacting two tank systems

Interacting system, which are mostly of order higher than 1 are frequently seen in a process industry. A 2nd level order process is considered here. In this process liquid flows from one tank to other tank through a valve and then flow out of the second tank through another valve. A pump which has ability to pump-in: qi(t) and pumpout:qo(t) liquid in the first tank is being controlled to maintain the level of liquid h2(t) in second tank. Both valves are not controlled and left open at a particular value everytime when the flow between both tanks depends on the levels in each tanks h1(t), h2(t).The cause-and-effect relationship is a two-way path.

The coupled tank is of two tanks which function as a storage element. Using a pipe we will pump the liquid to the tank 1. Through another pipe this liquid will be sent to another tank. ‘A_1’ and ‘A_2 ’ are the areas of cross-section of the tank 1 and tank 2 respectively. ‘q’ is the rate of flow of the liquid for the tank 1. ‘q_1’ is the rate of flow of between tank 1 and tank 2.

‘q_2 ‘ is the rate of out flow of liquid from tank 2. H_1 and H_2 are the height of the level of liquid in tank 1 and tank 2 respectively.

The parameter B_1, B_2, B_{12} are proportionality constants and dependends on the discharge coefficients, gravitational constant and the area of cross-sectional of each outlet. Pump feeds water to the tank 1 and these two tanks were interconnected using a small pipe with a valve. And there will be an outlet connected to tank 2. The liquid being used in the plant is assumed to be steady, non-viscous and incompressible leading to the use of Bernoulli’s equation for obtaining a set of non-linear state equations:

Applying the flow balance equation for tank 1 and tank 2 are:

$$\frac{dH_1}{dt} = \frac{1}{A_1}(q - q_1)$$

$$\frac{dH_2}{dt} = \frac{1}{A_2}(q_1 - q_2) \qquad (8)$$

In eq(3.1) ‘q1’ and ‘q2 ’can be expressed as
$q_1 = a_{12}\sqrt{2g(H_1 - H_2)}$ for $H_1 > H_2$

$$q_2 = a_2\sqrt{2gH_2}$$

for $H_2 > 0$ (9)

Where a_{12} and a_1 are the area of cross-section of pipes interconnecting the two tanks and outlet respectively.

The coupled tank system is being modelled by the two differential equations given below

$$A_1\frac{dH_1}{dt} = q - B_{12}\sqrt{|H_1 - H_2|}sign(H_1 - H_2) \qquad (10)$$

$$A_2\frac{dH_2}{dt} = B_2\sqrt{H_2} - B_{12}\sqrt{|H_1 - H_2|}sign(H_1 - H_2) \qquad (11)$$

As there is no leakage provided in tank 1, hence, where is B_1 =0. Here B_2 is discharge proportionalityconstant of output and is given by equation (11).
$B_2 = a_2\sqrt{2g}$ (12)

And B_{12} is discharge proportionality constant between the two tanks.

$B_{12} = a_{12}\sqrt{2g}$ (13)

The dynamic model can be represented as

$$\frac{dH_1}{dt} = \frac{U}{A_1} - \frac{B_{12}}{A_1}\sqrt{|H_1 - H_2|}sign(H_1 - H_2) \quad (14)$$

$$\frac{dH_2}{dt} = -\frac{B_2}{A_2}\sqrt{H_2} + \frac{B_{12}}{A_2}\sqrt{|H_1 - H_2|}sign(H_1 - H_2) \quad (15)$$

III. SLIDING MODE CONTROL

FIRST ORDER SLIDING MODE CONTROL:

Consider the nonlinear model state equation of the two tank sytems as mentioned above.Designing of FOSMC involves two most important steps: (a) properly defining the sliding surface, (b) Designing the control law so that system trajectories reaches the sliding surface.

(i)Defining Sliding Surface:

The sliding surface is defined by using the equation (3.5).

$$S = \{x \in R^n : s = 0\} \quad (3.5)$$

where $x \in R^n$ is state vector and on sliding surface trajectories obeys "$s = 0$".

(ii) Designing Control Law:

The control for FOSMC is expressed as follows:

$$u = u_{eq} + u_d \quad (3.6)$$

with u_{eq} is the equivalent control and u_d is the discontinuous control. The discontinuous control signal is selected so as to bring the state trajectories towards the switching surface or sliding surface. The equivalent control is a control action necessary for maintaining an ideal sliding motion in other words it ensures the state trajectory to remain on the switching surface $s = 0$ during the sliding mode.

The equivalent control u_{eq} is obtained by setting $\dot{s}(x,u,t)|_{u=u_{eq}} = 0$ where $\dot{s} = f(x,u,t)$. The discontinuous control can be defined as

$$u_d = -k.sign(s) \quad (3.7)$$

where, $k = \frac{a|s|}{\sqrt{2}}$ and 'a" is obtained from $t_r £ \frac{2.v^{1/2}(0)}{a}$, $v = \frac{1}{2}s^2$.

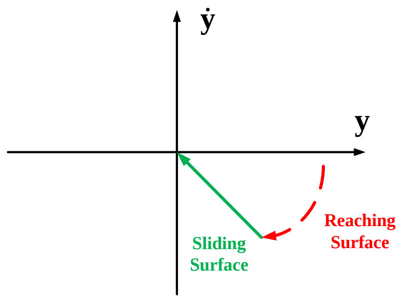
SECOND ORDER SLIDING MODE CONTROL:

Second-order sliding mode controller is design involves design of (i) switching surface and also (ii) design of controller. Switching surface design involves designing a surface/plane 's' which is a the linear combination of the state variables. The role of the SOSMC controller is to keep the variables on this surface and also to make the system insensitive to the uncertainties. If the state variables are deviated from the pre defined switching surface then the controller has to bring these variables onto the surface. The switching surface is defined by equation $s = \dot{s} = 0$.

(a). Defining a sliding surface

In SOSM control approach the history of all state variables will converge to an equilibrium point when they obey $s = \dot{s} = 0$ of the state plane. The switching surface of the SOSMC strategy is stated using equation (5).

$$s = \{X \in R^K ; s = \dot{s} = 0\} \quad (5)$$



Fig(1) Sliding mode control

The mathematical representation of switching surface, 's' is stated as s(x) = Sx, where x is a state vector and $s \in R^n$ is a switching surface vector. The switching function, 's' and it's first time derivative with respect to time is continuous and its second time derivative w.r.t time is discontinuous.

$$\dot{s} = \frac{\partial s}{\partial x}\dot{x} = \frac{\partial s}{\partial x}(f(x) + b(x)u)$$

Differentiating twice the switching variable "s" produces a relation as given in eqn.().

$$\ddot{s} = \frac{\partial \dot{s}}{\partial x}(f(x) + b(x)u) + \frac{\partial \dot{s}}{\partial u}\dot{u}$$

$$\ddot{s} = \alpha(x) + \beta(x)\dot{u}$$

If $|s(x)| < s_0$, the following inequalities are assumed.

$$|\alpha(x)| < \phi \ , \ 0 < \Gamma_m \le \beta(x) \le \Gamma_M \ , \ \beta(x) = \frac{\partial \dot{s}}{\partial u}$$

(b). Control Law

The control laws using twisting ,super-twistinG are described in the following paragraphs.

Twisting Algorithm

Using the twisting algorithm the trajectories are plotted using s and sdot variables are twisting around the origin and its characteristic is as shown in the fig.(2). In the twisting algorithm of the second order SMC assumes that after a finite time interval the point **s**=ṡ=0 will be reached. The control algorithm is defined by the control law as given in eqn.(10).

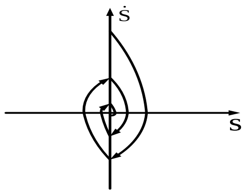


Fig.(2). Trajectory of s and sdot (Twisting)

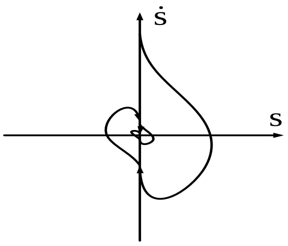
$$\dot{u}_{TW} = \begin{cases} -u_{TW} & \text{for } |u| > u_{max} \\ -k_m \operatorname{sign}(s) & \text{for } s\dot{s} \le 0 \text{ and } |u| \le u_{max} \\ -k_M \operatorname{sign}(s) & \text{for } s\dot{s} > 0 \text{ and } |u| \le u_{max} \end{cases} \qquad (10)$$

$$k_M > k_m > 0,\ k_m > \frac{4\Gamma_M}{s_0},\ \ k_m > \frac{\phi}{\Gamma_m},\ \ \Gamma_m k_M - \phi > \Gamma_M k_m + \phi$$

where

Super Twisting

Using super twisting algorithm, the trajectories plotted using s and sdot variables are twisted around the origin along with travel on sdot axis (super-twisting) and its characteristic is as shown in fig.(3). The super twisting SMC algorithm depends on inserting an integrator into the control loop so that control becomes continuous time function (u_{st}). The control law u_{ST} is defined by two things, the first is defined in terms of an integral of a discontinuous function of sliding variable, while the second is a continous function of the sliding variable. The control law is defined mathematically using eqn.(11).



Fig(3)Trajectory of s and sdot(Super twisting)

$$u_{ST} = u_1 + u_2$$

$$\dot{u}_1 = \begin{cases} -u_{ST}, & \text{for } |u_{ST}| > u_{max} \\ -w \operatorname{sign}(s), & \text{for } |u_{ST}| \le u_{max} \end{cases}$$

$$u_2 = \begin{cases} -\lambda |s_o|^{\rho} \operatorname{sign}(s), \text{for } |s| > s_o \\ -\lambda |s|^{\rho} \operatorname{sign}(s), \ \text{for } |s| \le s_o \end{cases} \qquad (11)$$

where

$$w > \frac{\phi}{\Gamma_m}, \qquad 0 < \rho \le 0.5$$

$$w > \frac{4\Gamma_M}{s_o}, \quad \rho(\lambda\Gamma_m)^{\frac{1}{\rho}} > (\Gamma_M w + \phi)(2\Gamma_M)^{\frac{1}{\rho}-2}$$

IV. SIMULATION RESULTS

For Non Interacting two tank system:

The proposed control algorithm to regulate the liquid level in tank 2 with step signal. The objective is to maintain the liquid level in tank 2 for 8cm. To achieve his liquid level the SM controller is made to regulate the flow rate 'q' at the tank 1, so that, desired level of tank 2 is achieved. The values of tuning parameters are considered as K=10

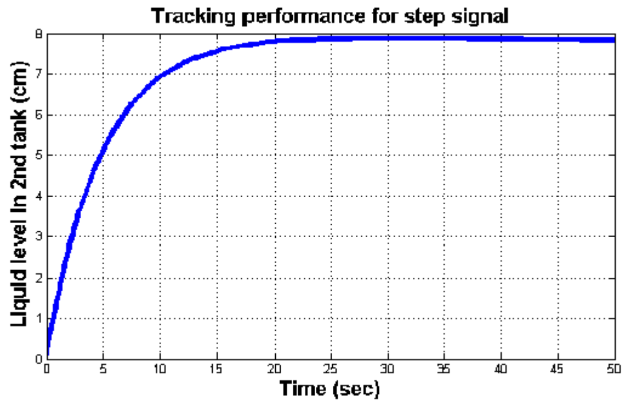


Fig 1 tracking performance for non interacting system with FOSMC

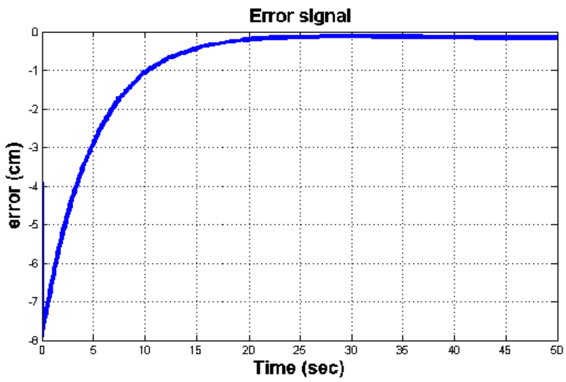


Fig 2 Error signal for non interacting system with FOSMC

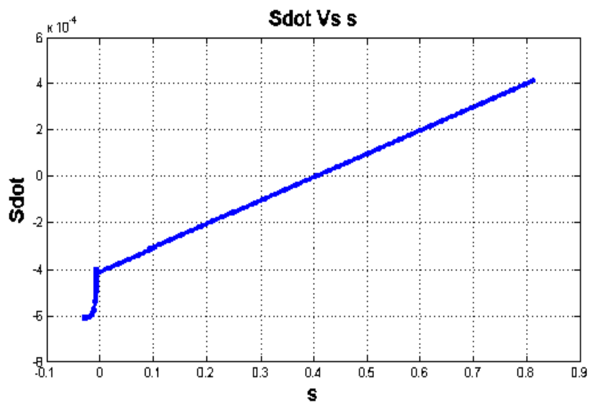


Fig.3 sdot vs s for non interacting system with FOSMC

In the twisting algorithm, the constants k,α,λ are chosensuch that, they ensures the system trajectories converge in afinite time of the sliding surface. From the avialables conditions,the constants obtained are k=300,α=0.05,$\lambda = 1$

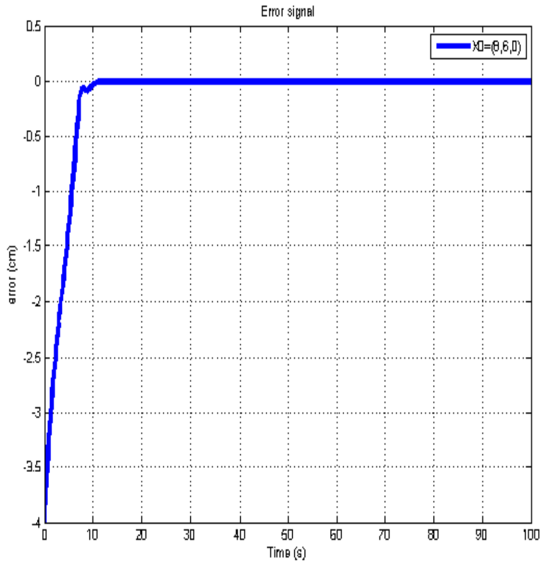


Fig 4 Error signal for non interacting system with Twisting SMC

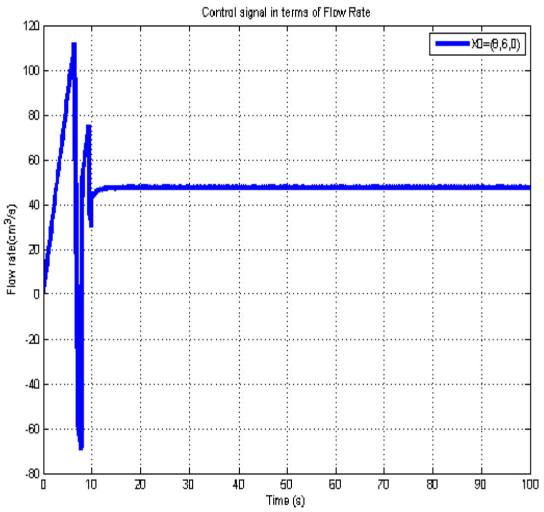


Fig 5 control signal in terms of flow rate for non interacting system with Twisting SMC

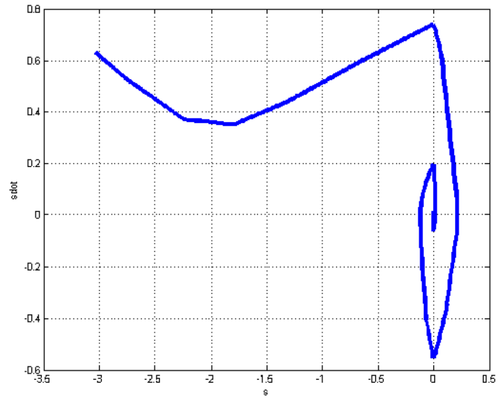


Fig 6 sdot vs s for non interacting system with Twisting SMC

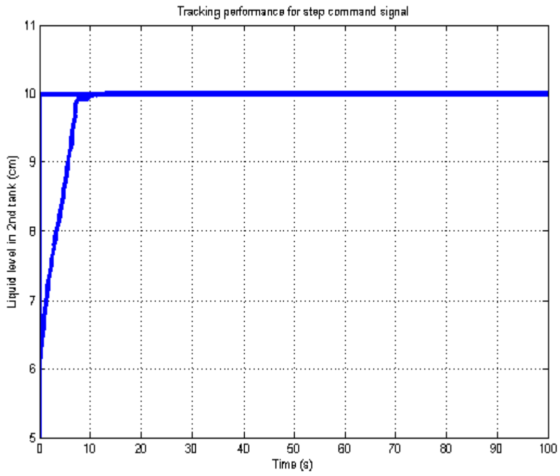


Fig7 Tracking performance for step signal for non interacting system with Twisting SMC

In the super twisting algorithm, the constants k,w, λ arechosen so that, they ensures the finite time convergence ofstate trajectories into the switching surface.From the availableconditions, the constants obtained are k=10,w=20,$\lambda = 0.75$.

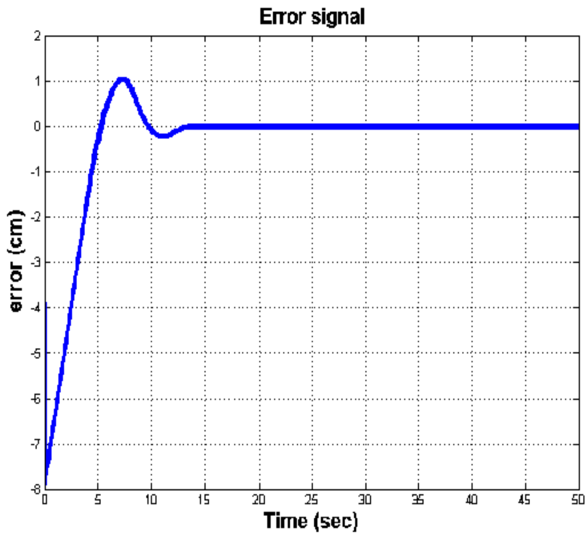


Fig 8 Error signal for non interacting system with super twisting SMC

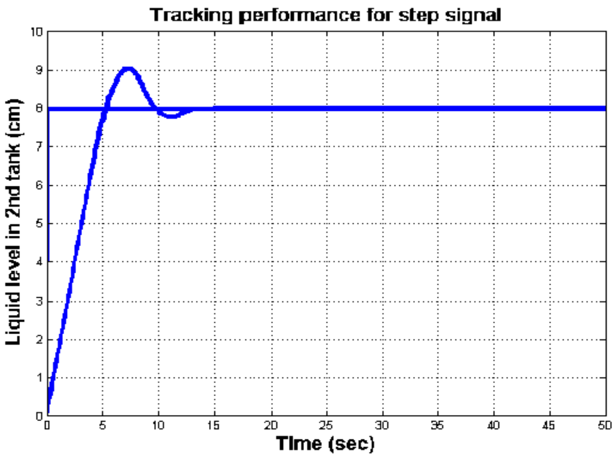


Fig 9 Tracking performance for step signal for non interacting system with Twisting SMC

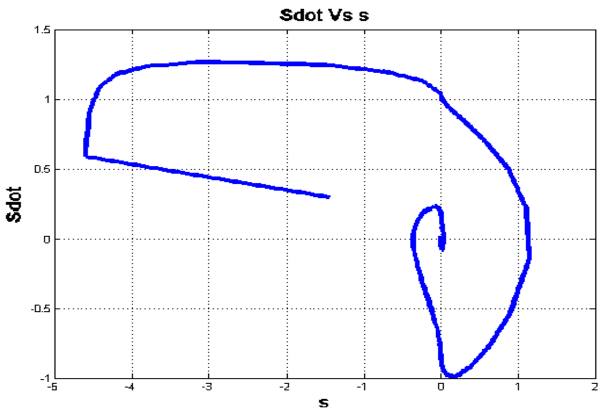


Fig 10 sdot vs s for non interacting system with super twisting SMC

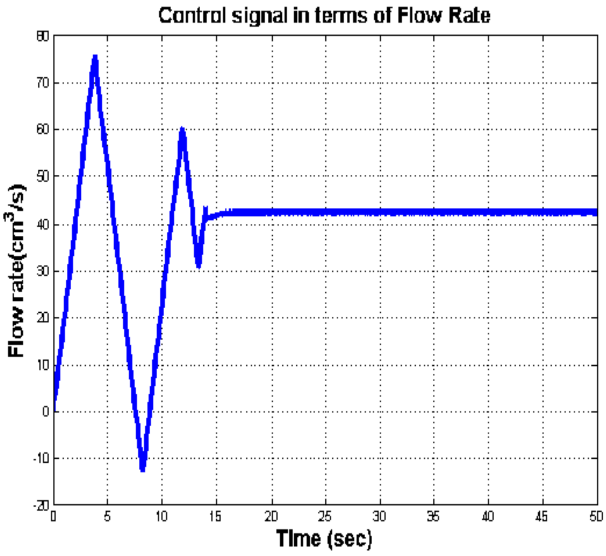


Fig 11 control signal in terms of flow rate for non interacting system with super twisting SMC

Interacting Two Tank System:

The proposed control algorithm to regulate the liquid level in tank 2 with step signal. The objective is to maintain liquid level in tank2. To achieve his liquid level the SM controller is made to regulate the flow rate ‘q’ at the tank 1, so that, desired level of tank 2 is achieved. The values of tuning parameters are considered as K=10.

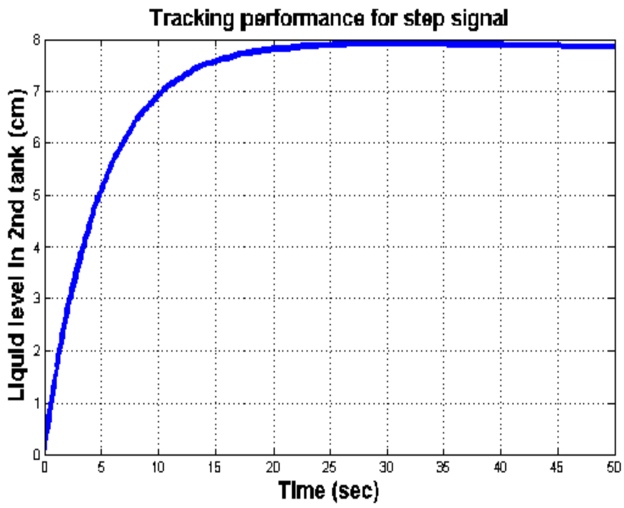


Fig 12 tracking performance for interacting system with FOSMC

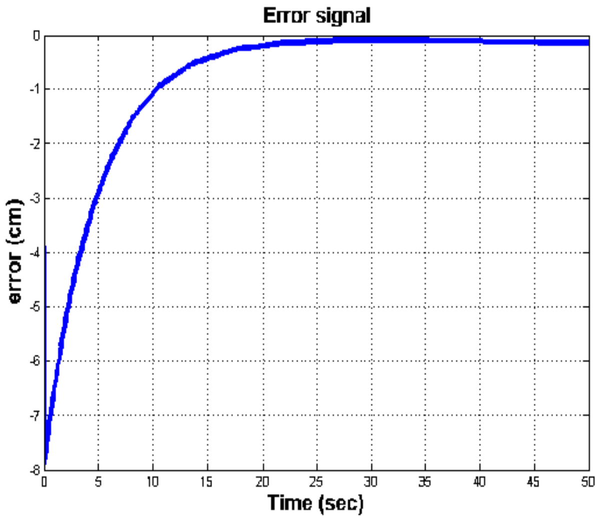


Fig 13 Error signal for interacting system with FOSMC

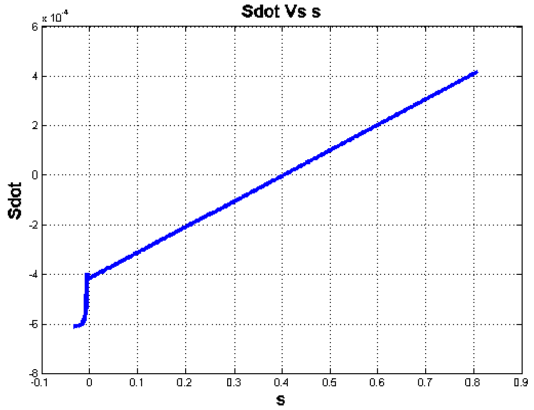


Fig 14 sdot vs s for interacting system with FOSMC

Simulation results of second order SMC With Twisting algorithm:

In the twisting algorithm, the constants k,α,λ are chosen so that, they ensures the system trajectories converge in a finite time to the sliding surface. From the conditions, the constants obtained are k=320,α=0.05,λ = 1

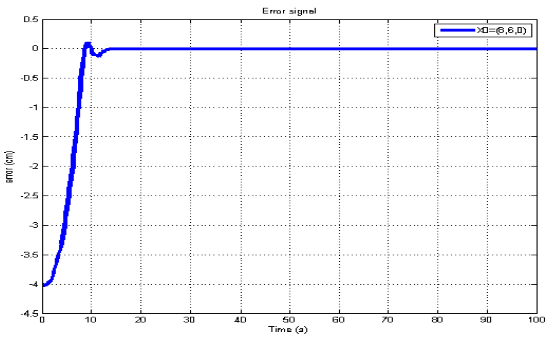


Fig 15 Error signal for interacting system with Twisting SMC

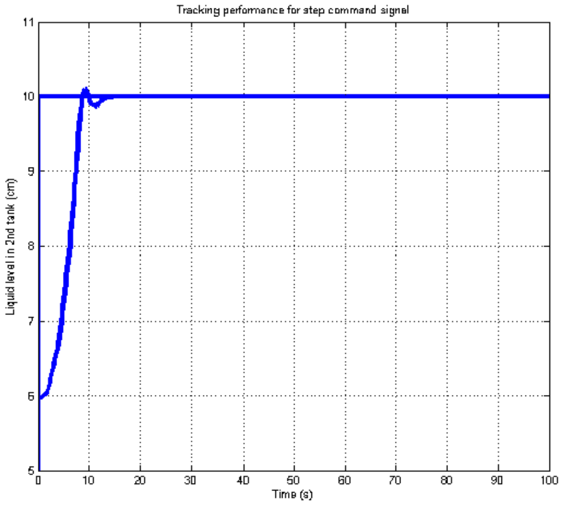


Fig16 Tracking performance for step signal for interacting system with Twisting SMC

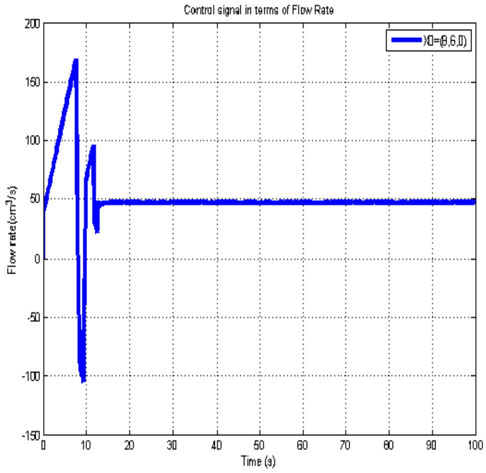


Fig 17 control signal in terms of flow rate for interacting system with Twisting SMC

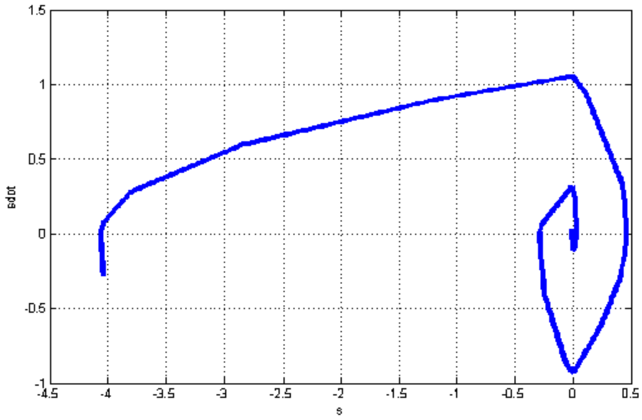


Fig 18 sdot vs s for interacting system with Twisting SMC

In the super twisting algorithm, the constant k,w,λare chosen so that, they ensures the finite time convergence of thestate trajectories to the switching surface.From the conditions, the constants obtained are k=10,w=20,$\lambda = 0.75$.

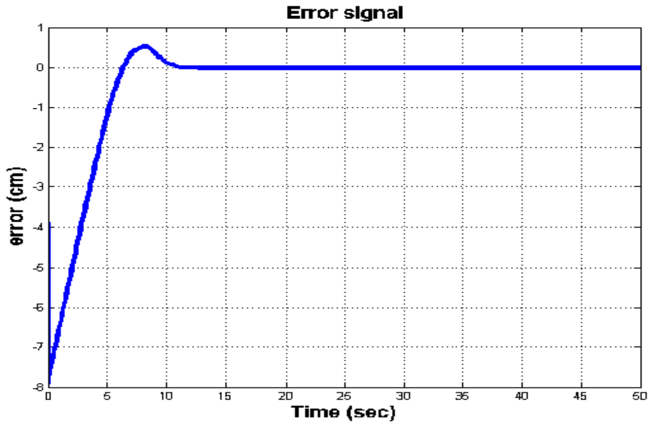


Fig 19 Error signal for interacting system with super twisting SMC

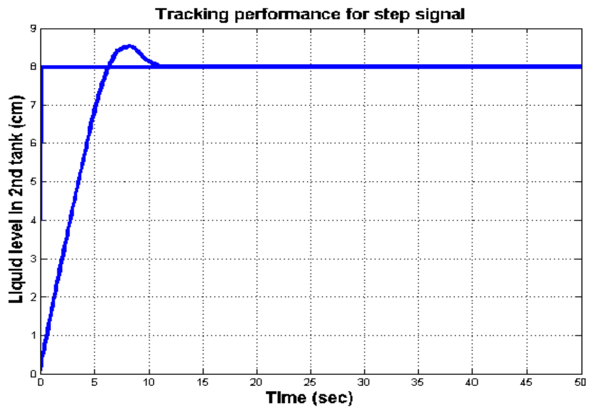


Fig 20 Tracking performance for step signal for interacting system with super Twisting SMC

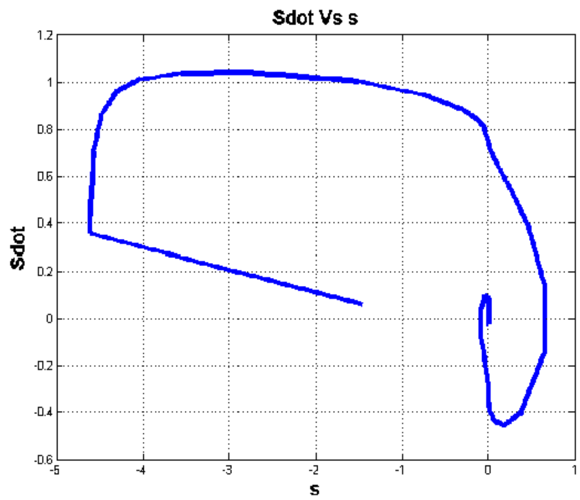


Fig 21 sdot vs s for interacting system with super twisting SMC

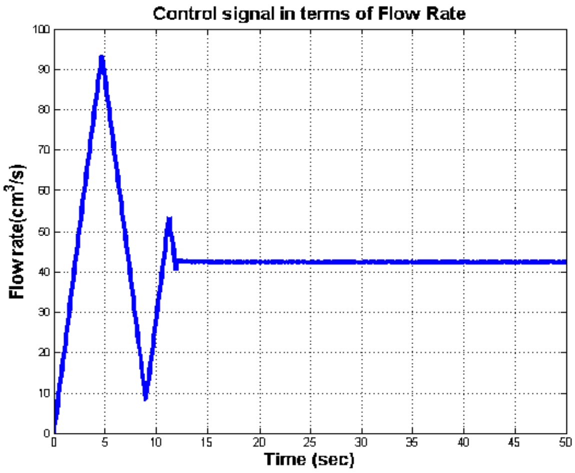


Fig 22 control signal in terms of flow rate for interacting system with super twisting SMC

V. CONCLUSIONS

The mathematical model for interacting and non interacting two tank systems. The conventional sliding mode or first order sliding mode is applied for interacting and non interacting two tank systems , since conventional sliding controller exhibits discontinuous control action which is one of the demerits of this controller. So, to overcome this problem second order sliding mode is proposed in this work. In second order sliding mode control, two algorithms are considered in this thesis namely twisting and super-twisting algorithms. These first order and second order sliding mode controller are successfully implemented to for interacting and non interacting two tank systems in MATLAB environment.

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