Real Time Bidding (RTB) for Dynamic Spectrum Access in 5G Cognitive Radio Networks

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Abstract:- The rapid proliferation of radiocommunication standards and services in recent years is causing the problem of spectrum scarcity. In this context, one of the main objectives of 5G is to facilitate access to the radio spectrum. Our contribution within the framework of this paper is the use of auctions to solve the problem of dynamic spectrum access in the context of 5G using Radio Cognitive (RC) terminals. To achieve this, we have defined a new ecosystem for dynamic spectrum access based on real-time bidding. We assume that the evaluations of the channels made by the secondary operators are dynamic over time. We also formulate the problem of smoothing the budget of the secondary operator in an auction mechanism for access to the primary spectrum. From these considerations, we determine the optimal offer that the secondary operator makes to acquire the spectrum and the income obtained by the primary operator.

Keywords :- *Dynamic Access, Spectrum, Auction, Cognitive Radio, Ecosystem, Channel Valuation.*

I. INTRODUCTION

The new generation of wireless cellular networks (IoT, 5G) are characterized by :

- A mixture of heterogeneous communication links,
- A dense deployment of small base stations.

Several technical challenges must be addressed in terms of interference management and frequency allocation.

Some radio frequency bands which are currently in use are used both as backhaul links (in the case of microwave links) and access links, must be shared by a large number of base stations, which causes congestion of these connections. This will grow exponentially with the deployment of 5G and IoT networks.

Other frequency bands, on the other hand, are underexploited due to the exclusive use of licensees for these bands.

Auctions have recently been proposed as a solution to the problem of spectral congestion.

In the context of 5G cognitive radio, buyers are secondary operators who can use the spectrum for a fee, and sellers (primary operators) provide radio resources to buyers and receive earnings from them by selling unused radio resources or underutilized. The products are radio resources, for example, bandwidth, time-slots and rights to transmit data on the network. Through the auction, primary operators, secondary operators and auctioneers have incentives to participate in order to gain benefits in a cognitive radio system [1] [2].

The components of an auction market are : - the seller (primary operator) wants to sell unused or underutilized portions of spectrum. The seller offers the price and quantity of the product to be auctioned. - the buyer (secondary operator) wants to buy the portion of free spectrum from the primary operator. The buyer submits a bid in terms of price and bid quantity to buy through the auction. - the exchange / compensation price is the price of each product (portion of spectrum) to be traded on an auction market. The negotiation price must satisfy the asking price and the bid price by the seller and the buyer, respectively.

Xiaofei Bu et al. [3] study the problem of spectrum auctions where the primary user wants to share continuous spectrum with secondary users, and each secondary user has a fixed transmission demand. The objective of this work is to design a truthful auction mechanism, which can allocate spectrum with varying bandwidths to secondary users and maximize social efficiency at the same time.

Ayoub Alsarhan et al. [4] offer dynamic auctions where spectrum is periodically auctioned to meet secondary user demands over time. This pattern determines the size of the spectrum to be auctioned for each session. The performance evaluation of the proposed system shows the ability of the system to maximize the reported revenues for the primary operator under different spectrum market conditions.

Ajay Gopinathan et al. [5] address the dynamic spectrum allocation problem from the perspective of the primary user who wishes to maximize auction revenues. It features an online auction framework that dynamically accepts bids and allocates spectrum.

In [6], a dynamic spectrum allocation model based on the theory of auctions in a heterogeneous two-level network is proposed, in which the primary users are the sellers, the auctioneer is the coordinator and the femto base station as a submitting buyer for idle spectrum and acts as a wireless access point that provides communication services for secondary users. Its basic process is as follows: the auctioneer gradually increases the price of spectrum relative to the reserved price; each bidder decides whether or not he participates in the purchase. It features distributed execution and low complexity which can reduce unnecessary information exchange between primary or secondary users. Meanwhile, it can improve the spectrum usage and improve the efficiency of the auction by generating the incentive mechanism.

In [7], the authors studied the problem of access to spectrum in the context of single and multichannel cognitive radio. A framework based on repeated auctioning was adopted. In single-channel spectrum access, secondary users selectively participate in the auction based on their valuation and past auction history. In multichannel networks, a no-regret approach has been proposed. Its performance has been shown to be significantly better than a naïve greedy solution.

In auction-based solutions, each channel is assigned to a single network, that is, there is no notion of secondary user and primary user in the same channel. In the literature, two possibilities are available [8] : - either the regulator allocates the channels to the primary users, the latter independently allocating the unused portions of their channel to the secondary users ; - either the regulator allocates the right to be secondary user or primary user in the channel.

In [9], the authors adopt second-price auction mechanisms to solve the spectrum allocation problem as well as the band detection problem. They introduce the notion of fictitious money by realizing the payment system in real time. In another approach, the authors propose a new auction based on waiting time for dynamic spectrum allocation. With this approach, user submissions are their wait time instead of money [10]. In [11], the authors propose an auction framework for cognitive radio networks to allow unauthorized secondary users to share available spectrum with authorized primary users in a fair and efficient manner, subject to the constraint of interference for each primary user. They then extend the scope of the proposed auction for the most difficult scenario with free frequency bands where the secondary user will have the choice between paying and having good QoS or accessing the channels for free and risking interference with users. In some scenarios, spectrum is used more efficiently by granting a band to multiple secondary users simultaneously, which sets it apart from traditional auctions where only one user can be the winner. However, the multi-winner auction is a new concept that poses new challenges to traditional auction mechanisms. Therefore, in [12], the authors propose an efficient collusion-resistant multi-win spectrum auction mechanism where multiple secondary users can gain access to a single channel.

Not all of these dynamic channel allocation approaches take channel quality into account in their ratings. While the quality of the primary channel can influence the offers of secondary operators. Most of them do not consider the dynamics of channel ratings over time and against the performance indicators of those channels.

This article overcomes these limitations by considering on the one hand that the evaluations of secondary operators on the primary channels depend on the performance indicators of these channels, and on the other hand that these evaluations vary over time according to the variation of the channels. We then appreciate the impact of these variations on the strategies of the bidders and the revenues of the primary operator.

II. RTB ECOSYSTEM FOR DYNAMIC ACCESS TO SPECTRUM

Auctions are based on the concept of selling and buying goods or services. The main aim of the use of these auctions in 5G networks is to provide motivation to secondary users in order to fully optimize the use of the spectrum.

Starting from real-time auctions for the acquisition of advertising space on the internet, we define a similar ecosystem for the acquisition of free frequency bands left by primary operators.

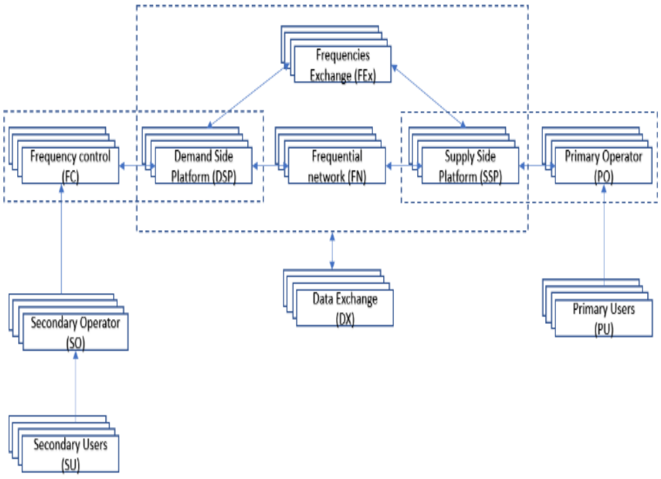

Figure 1 :*RTB ecosystem for dynamic spectrum access*

The POs (Primary Operators) hold licenses for the use of frequency bands.

The PU (Primary User) are the subscribers of the POs who use the portions of the spectrum made available to them by the latter.

The SO (Secondary Operator) wishes to use the white spaces left free by the POs.

The FC (Frequency Control) manages the placement of frequency portions similar to advertising agencies.

FN (Frequency Network) offers real-time auction systems in secondary spectrum markets.

Demand Side Platforms (DSP) allow FC to bid on multiple FNs in parallel.

The SSP (Supply Side Platform) allows PO to market its portions of spectrum on several FN simultaneously.

FEx combines multiple FNs, playing both the role of DSP and SSP, creating a platform to match supply and demand in the market way.

Data Exchange (DX), also known as Data Management Platform (DMP), provides primary channel information (in real time) to DSPs, SSPs, and ADXs.

The stages of the auction mechanism for the sale of portions of primary spectra are :

- The PO sends a portion of free spectrum to its SSP
- The SSP sends a BID REQUEST to all the DSPs
- Within each DSP, all TDs are eligible to acquire this portion of spectrum
- Within each TD, several requests may be eligible for the acquisition of this portion of spectrum
- Each eligible request returns a maximum auction price
- Each DSP receives BID ANSWERS or best bids from TD
- ANSWERS BIDs are sent to SSP and auction takes place. The top bidder wins the spectrum portion
- The highest SO bidder is awarded the portion of spectrum for exploitation

III. AUCTIONS FOR DYNAMIC SPECTRUM ACCESS

We consider a PO facing N OS neutral in terms of risk. Each SO$_i$ attributes to the desired portion of spectrum a value v$_i$, unknown to the others ;

All the SOs, including the PO, consider the evaluations of the others as independent random variables, resulting from a probability distribution of a strictly increasing distribution function over an interval$[\underline{v}, \overline{v}]$ with$F\ (\underline{v}) = 0\ and\ F\ (\overline{v}) = 1$, and of probability density*f(v)*. The probability distribution is common knowledge.

3.1 Static considerations

➢ Determination of the SO optimal bid (strategy)

SO$_i$makes the lowest possible$b_i$offer, its objective being to try to keep an annuity$z(b_i, v_i)$.

Let$b_i = b(v_i)$be the strategy (i.e. its bid for the acquisition of the spectral portion up for auction) of player i, determined from the function b (.) Common to all players. b (.) being an increasing function of the reservation value (evaluation of the primary channel).

Each bid$b_i$corresponds to a probability of winning the bid for player$i$.

$$Prob(b_i\ winner) = Prob\big(b_i > b_j,\ \forall j \neq i\big) \quad (1)$$

Let's pose $q(b_i) = b^{-1}(b_i) = v_i$

$$Prob(b_i\ winner) = F^{n-1}\big(q(b_i)\big) \quad (2)$$

The utility of monetary profit is :$\mathrm{U}\big(z(b_i, v_i)\big) = z(b_i, v_i)$. The mathematical expectation of utility of profit under a Bayesian equilibrium is given by:

$$E_i = \mathrm{U}\big(z(b_i, v_i)\big).F^{n-1}\big(q(b_i)\big) \qquad (3)$$

The objective ofSO$_i$is the maximization ofE_i, i.e. to find the bidb_i which maximizesE_i :

$$\frac{\partial E_i}{\partial b_i} = 0 \Leftrightarrow \frac{\partial \mathrm{U}\big(z(b_i, v_i)\big).F^{n-1}\big(q(b_i)\big)}{\partial b_i} = 0$$

$$\frac{\partial E_i}{\partial b_i} = \frac{\partial U}{\partial b_i} F^{n-1}\big(q(b_i)\big) + \frac{\partial F^{n-1}\big(q(b_i)\big)}{\partial b_i} \mathrm{U}\big(z(b_i, v_i)\big)$$

$$\frac{\partial E_i}{\partial b_i} = \frac{\partial U}{\partial z}\frac{\partial z}{\partial b_i} F^{n-1}\big(q(b_i)\big) + \mathrm{U}\big(z(b_i, v_i)\big)(n - 1)F^{n-2}\big(q(b_i)\big)\frac{\partial F}{\partial q}\frac{\partial q}{\partial b_i}$$

$$\mathrm{U}\big(z(b_i, v_i)\big) = \frac{\mathrm{U}\Big(z\big(b(\underline{v}), \underline{v}\big)\Big).F^{n-1}(\underline{v}) + \int_{\underline{v}}^{v_i} \frac{\partial U}{\partial z}\frac{\partial z}{\partial s} F^{n-1}(s)ds}{F^{n-1}(v_i)}$$

By explaining$z(b_i, v_i) = v_i - b(v_i)$, we have :

$$v_i - b(v_i) = \frac{\big(\underline{v} - b(\underline{v})\big).F^{n-1}(\underline{v}) + \int_{\underline{v}}^{v_i} F^{n-1}(s)ds}{F^{n-1}(v_i)}$$

$$b(v_i) = v_i - \frac{\int_{\underline{v}}^{v_i} F^{n-1}(s)ds}{F^{n-1}(v_i)} \qquad (4)$$

$\underline{v} = v_0$being the reserve price of the seller (primary operator), that is to say the value below which the item cannot be attributed.

Any bid$v_i < v_0$is rejected by the PO.

➢ Determination of the income of the PO

The income expectation of the PO is given by [13]:

$$R = \int_{v_0}^{\bar{v}} b(v) g_n(v) dv \qquad (5)$$

$g_n(v)$represents the density function of the random variable "rank n evaluation", that is to say the density function of the maximum of the n independent evaluations of the SO. From the properties of the ordered variables [14], we have:

$$g_n(v) = nF^{n-1}(v)\mathrm{f(v)}$$

$$R = \int_{v_0}^{\bar{v}} \left(v - \frac{\int_{v_0}^{v_i} F^{n-1}(s)ds}{F^{n-1}(v)} \right) nF^{n-1}(v)\mathrm{f(v)}\, dv$$

$$R = n \int_{v_0}^{\bar{v}} \left(vF^{n-1}(v) - \int_{v_0}^{v} F^{n-1}(s)ds \right) f(v)\, dv \qquad (6)$$

➢ **Determination of the SO expenditure**

For a first-price auction (the price paid corresponds to the bid made when the auction was won), the expenditure expected by the *SO*

$$S_i = prix(b(v_i)) \times Prob(b_i\ winner)$$

$$S_i = b(v_i)F^{n-1}(v_i)$$

$$S_i = \left(v_i - \frac{\int_{v_0}^{v_i} F^{n-1}(s)ds}{F^{n-1}(v_i)} \right) F^{n-1}(v_i)$$

$$S_i = v_iF^{n-1}(v_i) - \int_{v_0}^{v_i} F^{n-1}(s)ds \qquad (7)$$

The expressions (4), (6) and (7) obtained are a function of the distribution of probabilities *F* of the evaluations of the primary channels made by the SOs.

- **Case of uniform distribution over**$[v_0, \overline{v}]$
-

$$f(v) = \begin{cases} 1 & if\ \ v \in [v_0, \overline{v}] \\ 0 & else \end{cases}$$

$$F(v) = \begin{cases} 0 & si\ \ v < 0 \\ v\ \ if & v \in [v_0, \overline{v}] \\ 1 & if\ \ v > \overline{v} \end{cases}$$

thus,

$$b(v_i) = v_i - \frac{\int_{v_0}^{v_i} s^{n-1}ds}{v^{n-1}} = v_i - \frac{{v_i}^n - {v_0}^n}{n{v_i}^{n-1}} \qquad (8)$$

$$\lim_{n \to \infty} b(v_i) = v_i$$

This result shows that the more secondary operators there are for the portion of spectrum, the secondary operator must tell the truth about his true value of the channel.

$$R = n \int_{v_0}^{\bar{v}} \left(vF^{n-1}(v) - \int_{v_0}^{v} F^{n-1}(s)ds \right) f(v)\, dv$$

$$= n \int_{v_0}^{\bar{v}} \left(v^n - \int_{v_0}^{v} s^{n-1}(s)ds \right) f(v)\, dv$$

$$= n \int_{v_0}^{\bar{v}} \left(v^n - \frac{v^n - v_0^n}{n} \right) dv$$

$$R = \frac{n-1}{n+1}\bar{v}^{n+1} + v_0^n\bar{v} \qquad (9)$$

$$S_i = v_iF^{n-1}(v_i) - \int_{v_0}^{v_i} F^{n-1}(s)ds = {v_i}^n - \int_{v_0}^{v_i} s^{n-1}ds$$

$$S_i = {v_i}^n - \frac{{v_i}^n - v_0^n}{n} \qquad (10)$$

- **Estimation of SO evaluations by the PO**

The PO that receives the offers from the SOs can use the history of these offers to estimate the probability density function of the primary channel evaluations made by the SOs. And thereby estimate the expectation of his income.

By applying probability density estimation techniques, the PO can then predict the probability law of assessments of the portion of the spectrum.

- **Modeling of the evaluation of the primary channel by SOs**

Each SO has a type of flow to transmit. Depending on this type of flow, the SO will favor certain performance indicators of the transmission channel (throughput, delay, reliability, jitter).

Cognitive 5G radio transmitters / receivers operate on the cycle of cognition [15]. They observe their environment, detect the white spaces left by the POs, evaluate these spaces by estimating their performance indicators. Thanks to their learning engine, cognitive radios can predict these performance indicators of PO radio channels.

For a given type of flow to be transmitted, the SO will set value thresholds for the performance indicators :SNR_{th}, $Delay_{th}$, $Reliability_{th}$, $Jitter_{th}$.

After estimating the real indicators (SNR, Delay, Reliability, Jitter) from the environmental data collected (thanks to the learning engine), the SO compares these real values with the set threshold values and thus gives an evaluation of the primary channel.

$$v = f(SNR, Delay, Reliability, Jitter)$$

$$v = \left(\frac{SNR}{SNR_{th}} + \frac{Delay}{Delay_{th}} + \frac{Reliability}{Reliability_{th}} + \frac{Jitter}{Jitter_{th}} \right) \times perception\ that\ SO\ has\ for\ the\ primary\ canal$$

This perception depends on its economic weight, the potential income it may have due to the transmission of its flow, ...: this is a value set by the SO at a given time.

Example :

SNR	SNR_t	$Dela$	$Delay_t$	$Reliabili$	$Reliabilit$	$Jitte$	$Jitter_t$
10^6	10^5	100	10	0.7	0.8	-0.4	-0.5

$$perception = 50$$

$$v = \left(\frac{10^6}{10^5} + \frac{100}{10} + \frac{0.7}{0.8} + \frac{-0.4}{-0.5}\right) \times 50 = 1048.75$$

If the actual indicators vary, then v also varies. But v can also vary according to the thresholds since they can vary according to the type of traffic.

3.2 Dynamic considerations

We also consider that the offer made by an OS is a function of the evaluation it makes on the portion of spectrum at a given time.

Which allows to pose: $b_i = b(v_i(t)) = b_i(t)$.Thus,

the SO auction profit is:

$$z(b(v(t)), v(t)) = v(t) - b\big(v(t)\big) \quad (11)$$

The profit utility of the SO is:

$$\mathrm{U}\big(z(b(v(t)), v(t))\big) = z(b(v(t)), v(t)) = v(t) - b\big(v(t)\big) \ (12)$$

The profit expectation of the SO is:

$$E(t) = \mathrm{U}\left(z\left(b\big(v(t)\big), v(t)\right)\right).F^{n-1}\big(v(t)\big) \quad (13)$$

The SO seeks to maximize this expectation.

$$\frac{\partial E(t)}{\partial t} = \frac{\partial \mathrm{U}.F^{n-1}(v(t))}{\partial t} = \frac{\partial U}{\partial t}F^{n-1} + U\frac{\partial F^{n-1}}{\partial t}$$
$$= \left(v'(t) - v'(t)\frac{\partial b}{\partial v}\big(v(t)\big)\right)F^{n-1} + \Big(v(t) - b\big(v(t)\big)\Big)(n-1)v'(t)\frac{\partial F}{\partial v}\big(v(t)\big)F^{n-2}$$

$$\frac{\partial E(t)}{\partial t} = 0 \Leftrightarrow \left(v'(t) - v'(t)\frac{\partial b}{\partial v}\big(v(t)\big)\right)F^{n-1}\big(v(t)\big) + \Big(v(t) - b\big(v(t)\big)\Big)(n-1)v'(t)\frac{\partial F}{\partial v}\big(v(t)\big)F^{n-2}\big(v(t)\big) = 0$$

Consider a uniform distribution, that is to say$F\big(v(t)\big) = v(t)$, we have:

$$v(t)\frac{\partial b}{\partial v}\big(v(t)\big) + (n-1)b\big(v(t)\big) - nv(t) = 0$$

Let pose$x = v(t)\ et\ y = b(x)$

$$x\dot{y} + (n-1)y = nx$$

Solving this equation gives the variation of the optimal supply of the SO :

$$b\big(v(t)\big) = v(t) + \frac{c}{v^{n-1}(t)} \qquad (14)$$

$c \leq 0$is a parameter adjustable by the bidder to make an offer.

The expected income of the PO is:

$$R = \int_0^T b(v(t))g_n(v(t))dt$$

$$= \int_0^T \left(v(t) + \frac{c}{v(t)^{n-1}}\right)nF^{n-1}(v(t))\mathrm{f}(v(t))dt$$

$$R = n\left(\frac{v^{n+1}(T) - v^{n+1}(0)}{n+1} + c(v(T) - v(0))\right) \quad (15)$$

The expected expenditure of the SO is:

$$S = \left(v(t) + \frac{c}{v^{n-1}(t)}\right)v^{n-1}(t)$$

$$S = v^n(t) + c \qquad (16)$$

Let suppose that$v(t)$follows a Rayleigh law. Indeed, during propagation between a transmitter and a receiver, the transmitted signal is often subject to several phenomena linked to the propagation environment (Reflection, Diffraction, etc.). On reception, this results in a signal consisting of multiple elementary signals arriving with a given angular distribution which differs depending on the channel crossed. These elementary signals take different paths and therefore have different amplitudes and propagation times. They thus exhibit phase shifts which can lead to recombination in a constructive and destructive manner, thus causing a total disappearance of the signal. This latter phenomenon, more commonly referred to as "fading" or "fading", can affect the performance of mobile communication systems. The fading amplitude of the received signal can follow several statistical distributions such as: Rayleigh, Nakagami [16]. We assume here that the statistical distribution of the fading amplitude follows a Rayleigh law. Thus, we can have :

$$v(t) = \left(\frac{t}{\sigma^2}\exp\left(\frac{-t^2}{2\sigma^2}\right)\right) \times perception\ that\ SO\ has\ for\ the\ primary\ canal$$

3.3 Problem of budget smoothing

The secondary operator has $S \in \mathbb{R}_+$ to spend on acquiring the portions of spectrum left by the primary operator.

It is assumed that the number of auction requests varies continuously over time.

Let $\mu : t \mapsto \mu(t)$ be the function representing the number of auction requests per unit of time.

Thus, the total number of auction requests over a time interval [0, T] is:

$$\int_0^T \mu(t)dt$$

The auction algorithm for portions of spectrum is implemented in a geographic area by the primary operator.

At each instant $t \in [0, T]$ the algorithm controls the bid level *b(t)*(offer) offered by secondary users in order to maximize the number of spectral portions to be acquired.
We then define the following quantities:

- The auction rate won *(winrate):*
$$\omega : b \mapsto \omega(b)$$
- The cost of a portion of spectrum:
$$p : b \mapsto p(b)$$

The set $(b(t))_{t\in[0,T]}$ is the bidding strategy.

For a bidding strategy $(b(t))_{t\in[0,T]}$, we define the evolution of the portions of spectrum purchased over time and the expenses incurred.

❑ Portions of spectrum purchased in progress (until time t):

$$I : t \mapsto \mathrm{I(t)}\ ,\quad \mathrm{I}(0) = 0$$

At time t, the number of purchased spectrum portions is:

$$I'(t) = \underbrace{\mu(t)}_{nomber\ of\ requests} \times \underbrace{\omega(b)}_{auction\ win\ rate}$$

❑ Current expenditure (up to time t):

$$S : t \mapsto \mathrm{S(t)}\ ,\quad S(0) = 0$$

At time t, the expenditure made is:

$$S'(t) = \underbrace{I'(t)}_{number\ of\ portions\ of\ spectrum\ purchased} \times \underbrace{p(b)}_{price\ of\ spectrum\ portion}$$

By posing (change of variable) :

$$\tau(t) = \int_0^t \mu(s)ds$$

$\tau(t)$ is the number of bidding requests up to time t.
We obtain :

$$I'(t) = \tilde{I}'(\tau)\mu(t)$$
$$S'(t) = \tilde{S}'(\tau)\mu(t)$$

Which leads us to the following dynamic equations:

$$\tilde{I}'(\tau) = \omega(b(\tau))$$
$$\tilde{S}'(\tau) = \tilde{I}'(\tau)p(b(\tau))$$

(where the ~ symbol indicates that the underlying variable is τ*, ie.* $I(t) = \tilde{I}(\tau(t))$ *and* $S(t) = \tilde{S}(\tau(t))$*)*

Suppose the functions ω and p are strictly increasing and differentiable.
So,

$$\tilde{S}'(\tau) = \tilde{I}'(\tau)(p\circ\omega^{-1})\left(\tilde{I}'(\tau)\right)$$

For the sake of simplification, we will work without the ~ symbol.

The goal of the auction algorithm here is to minimize the budget S by having fixed the number of spectrum portions, that is to say:

$$\min_{\tau(T)} S = \int_0^{\tau(T)} \tilde{S}'(\tau)d\tau = \int_0^{\tau(T)} \tilde{I}'(\tau)(p\circ\omega^{-1})\left(\tilde{I}'(\tau)\right)d\tau$$

by fixing the total of the purchased spectra portions.

We will then use the techniques of Euler-Lagrange variational calculus to solve this problem.

Considering our functional :

$$\int_0^{\tau(T)} \tilde{I}'(\tau)(p\circ\omega^{-1})\left(\tilde{I}'(\tau)\right)d\tau$$

We have :

$$\left(\frac{\partial}{\partial I} - \frac{d}{d\tau}\frac{\partial}{\partial \dot{I}}\right)\left[\dot{I}(p\circ\omega^{-1})(\dot{I})\right] = 0$$

Since $\dot{I}(p\circ\omega^{-1})(\dot{I})$ does not depend on I, it remains:

$$\frac{d}{d\tau}\left(\frac{\partial}{\partial \dot{I}}\left[\dot{I}(p\circ\omega^{-1})(\dot{I})\right]\right) = 0$$

Thus, there exists a constant C with respect to τ such that

$$\frac{\partial}{\partial \dot{I}}\left[\dot{I}(p\circ\omega^{-1})(\dot{I})\right] = C$$

Let pose: $\mathrm{x} = \dot{I}\ and\ y = p\circ\omega^{-1}(\dot{I})$

We then have $y = f(x)\ \ with\ f(x) = p\circ\omega^{-1}(x)$

The equation then becomes

$$\frac{\partial}{\partial x}[xy] = C \Longrightarrow y\frac{\partial}{\partial x}x + x\frac{\partial}{\partial x}y = C$$

ie.

$$y + \mathrm{x}\dot{y} = C$$

which is a 1st degree equation with variable coefficients and second member.

The general solution of this equation is given by:

$$y = C + \frac{k}{x}\ ,\ \ k = cte$$

From where

$$p \circ \omega^{-1}(\dot{I}) = C + \frac{k}{\dot{I}}$$

Knowing that

$$S'(\tau) = I'(\tau)(p \circ \omega^{-1})(I'(\tau))$$

We have

$$S'(\tau) = I'(\tau)\left(C + \frac{k}{I'(\tau)}\right) = CI'(\tau) + k$$

$$S = \int_0^{\tau(T)} S'(\tau)d\tau = \int_0^{\tau(T)} CI'(\tau) + k\ d\tau$$

$$S = C\int_0^{\tau(T)} I'(\tau)\ d\tau + \int_0^{\tau(T)} kd\tau$$

$$S = CI(\tau(T)) + k\tau(T)$$

$$I(\tau(T)) = \frac{S - k\tau(T)}{C}$$

$\tau(T)$is the number of auction requests during the time interval [0, T]

If we know the variation in the number of purchased spectrum portions$I$and the variation in the number of auction requests, we determine the variation in the budget$S$.

IV. SIMULATIONS AND COMMENTS

4.1. Effects of the number of SOs and their evaluations

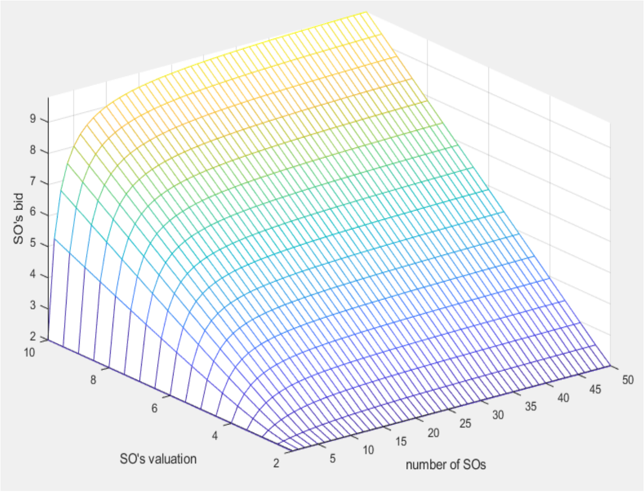

Figure 2 : evolution of the bid of an SO according to the number and evaluations of SOs involved in the auction

The higher the valuation, the higher the offer made by the SO. This supply also grows with the number of SO, but reaches a maximum at a certain number of SO. This maximum is the true estimate of SO.

$$\lim_{n\to\infty} b(v_i) = v_i$$

SO has no chance of winning the auction unless it declares its true assessment of the channel being sought.

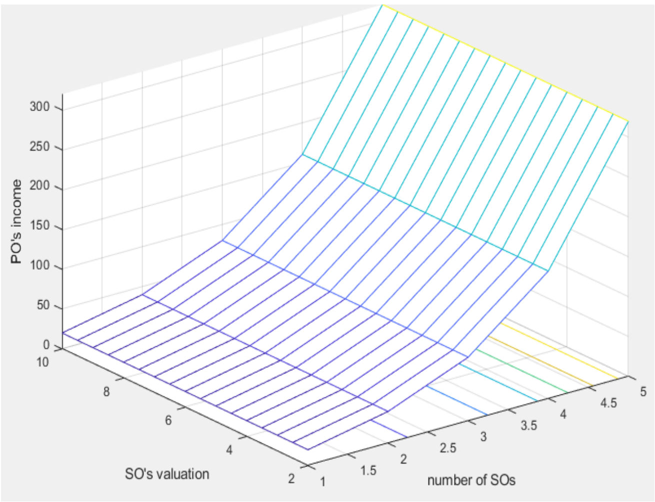

Figure 3: evolution of the expected income of the PO

The more bidders there are, the more the PO's expected income grows. This growth is independent of the evaluations made by the SOs on its channel. The PO will then have an interest in inviting as many bidders as possible to maximize its income.

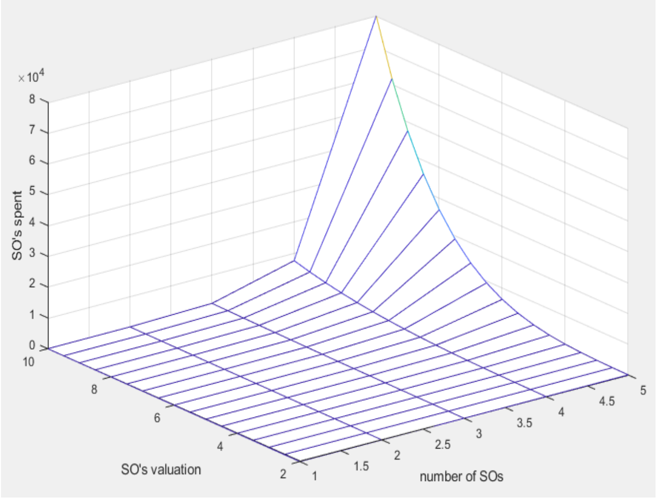

Figure 4 : evolution of the expected expenditure of the SO

The expected expenditure of the SO increases with the evaluation made of the primary channel, but does not depend on the number of SOs engaged in the auction.

4.2. Effects of the number of SOs over time

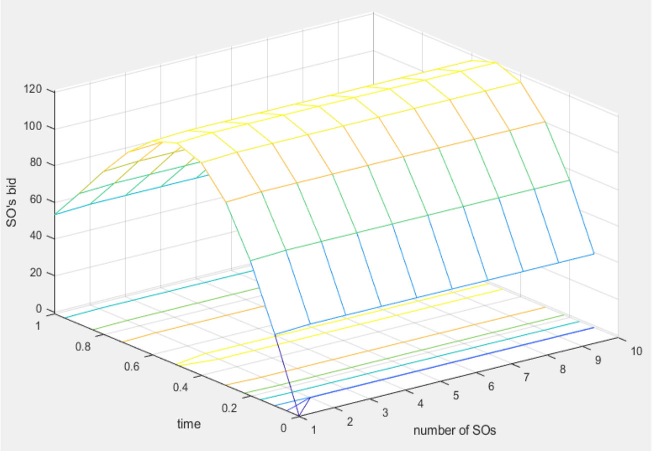

Figure 5: evolution of SO bid over time forc $= -1$

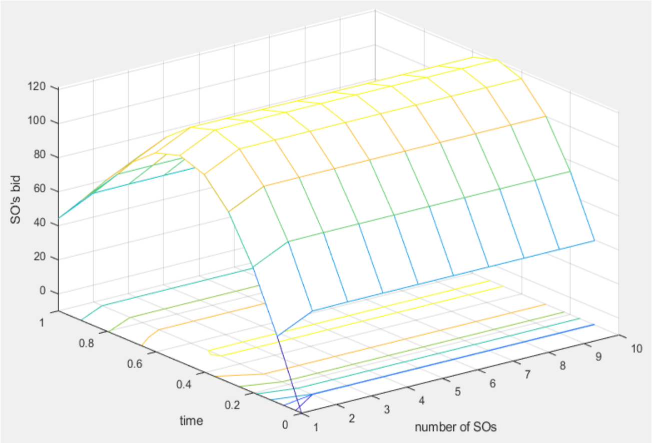

Figure 6: evolution of SO bid over time forc $= -10$

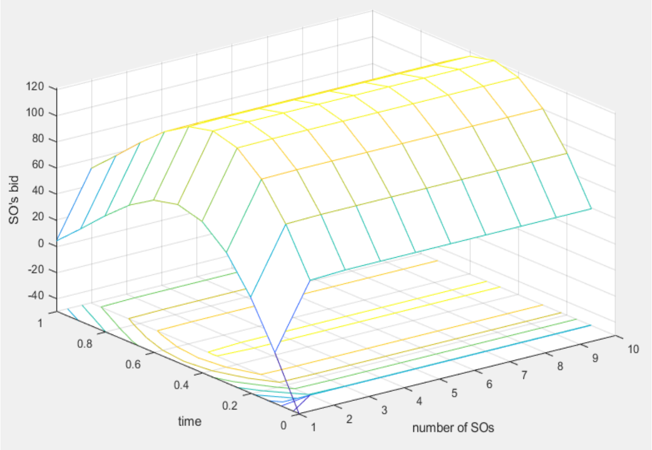

Figure 7: evolution of SO bid over time forc $= -50$

The supply of SO grows over time until it reaches a maximum and begins to decrease. This is due to the evolution of the Rayleigh channel considered in this study. This offer depends only on the channel, but not on the number of SOs engaged in the auction. The adjustable parameter c influences the value of the offer: the smaller it is, the more the supply decreases without impacting its growth.

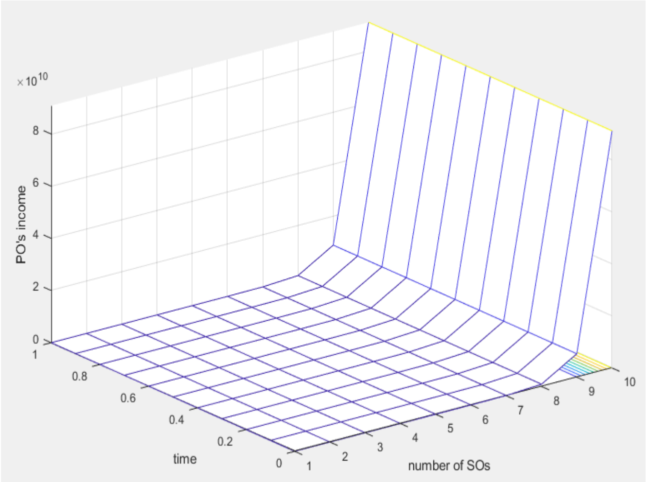

Figure 8: evolution of PO expected income over time

The expected income of the PO depends only on the number of SOs in the auction mechanism.

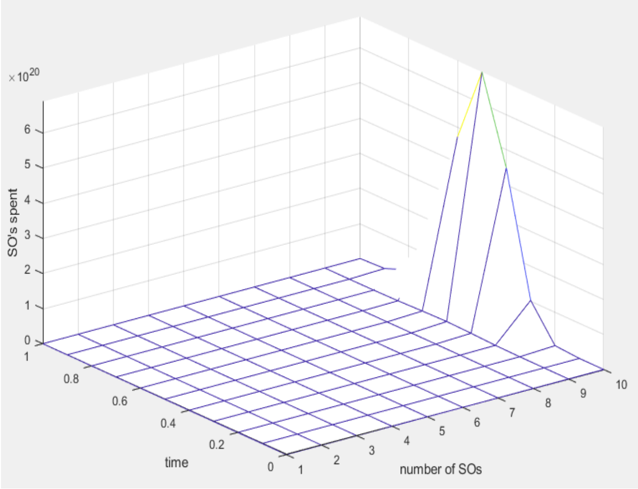

Figure 9: evolution of the expected SO expenditure

The expected expenditure of SO depends on the channel, but increases exponentially from a number of SOs.

V. CONCLUSION

This study focuses on the economic interaction between secondary operators and primary operators in cognitive radiocommunication networks. Operators have frequency channels which they auction in secondary markets. They then offer auction mechanisms that maximize their income. The secondary operators who are the bidders develop bidding strategies in order to maximize their chance of winning the auction. Primary channel evaluations made by secondary operators depend on performance indicators and may vary over time depending on channel variation and the type of flow that the secondary operator wishes to transmit.

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