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The Effect of Different Registers of Semiotic Representations in the Problem Solving Challenge Involving Fractions. Study with Future Primary School Teachers

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Abstract:- This study has as main objective to investigate the efficiency of an alternative methodology for teaching and learning to solve problems involving fractions. It is a methodology that consists of using different registers of representations of a problem composed by 3 variants, structured according to the experimental principles of semiotic variation and concomitant variations. The study was conducted with 29 students, attending teacher training courses for elementary school, through a questionnaire containing 3 variants of a problem. In each variant-problem, 3 to 4 alternative representations of two other types of problem registers were given. The results revealed that deficiencies in the conceptual comprehension of the fraction and its computational rules, difficulties to make the linguistic decoding, are factors that had an unfavorable influence on the performance of the students. However, it has been proved that the use of different registers of semiotic representations induce the student to the comprehension activity of problem, which consists in the identification, articulation and coordination of the units and cognitive reference variables between the registers. Thus, applied in didactic situations, an approach of this nature is an effective method for teaching and learning to solve problems, because it has produced an increase about 10% in the performance of future primary school teachers in problem solving involving fractions.

Keywords:- *Semiotic Representations Registers, Fractions, Future Primary School Teachers, Semiotic Variation, Concomitant Variation, Problem Solving, Proficiency With Fractions.*

I. INTRODUCTION

Fractions are academic knowledge which aretaught at primary and secondary school,as well as the others mathematical contents, the teaching of fractions aims to solve problems. So, the teachers need a solid and deep knowledge of them. According toLamon, 2007 (apud Olanoff et al, 2014), a major goal for mathematics education is proficiency with fractions, because it is fundamental for understanding algebra. However, Olanoff et al (2014, 268) refer about research studies which shows that many teachers possess a limited knowledge of mathematics in key contents areas such as number. It was almost the same problem I observed with many primary teachers in 3 primary schools. I gave them a simple problem involving fractions, and none of them could have a reasonable resolution. This means that these teacher did not acquired proficiency with fractions, during their formations.

To have proficiency with fractions means to have a deep understanding of the different perception and interpretations of fractions. According to Olanoff et al (2014), this is one of the important areas of prospective teachers' knowledge. What makes the students not possess proficiency with fractions?

According to Duval (2011), the deep reason why these teachers do not possess proficiency with fractions, should be sought out in the lack of understanding about the links between the fractional representation of semiotic registries andproblem solving. It is in this perspective that this study was conducted. It's assumed that through this link the students will possess the proficiency with fractions.Ruhama Even (1990) already suggested that teachers needed to understand concepts in different representations and should be able to translate and form links between them, because different representations give different insights that allow a better, deeper, more powerful and more complete understanding of a concept (p. 525).

So, this study aims to investigate the efficiency of using different registers of representations,as methodologies of teaching and learning to solveproblems which involve fractions.

II. THEORETICAL BACKGROUND

In order to gain a deeper understanding of rational numbers in general, researchers have tended to agree that one must be familiar with many different interpretations of fractions (Olanoff et al (2014, p. 271). Pinilla (2007) list different interpretations that an apparently intuitive definition of fractions can give rise: as part of one-whole, at

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times continuous and at times discrete; as a quotient, a division not carried out, $\frac{a}{b}$, which should be interpreted as *a*:*b*; as indicating ratio; as an operator; as an important part of work on probability; as score; as a measure; as a quantity of choice in a set; as a percentage; as rational number; as a point positioned on a directed straight line; and in terms of everyday language, for telling the time (a quarter to ten) or describing a slop (a 10% rise) often far from a scholastic idea of fractions (p. 97).

There are, therefore, so many different meanings of fraction, and each of them is associated with several situations. Thus, for the learning of a mathematical concept, Vergnaud proposed a unifying principle. He defined a concept *C* by a tendency of sets(*S*, *I*, *L*), where *S* is the *referent* (set of situations that give the sense of the concept fraction), *I* is the *meaning* (set of variants on which is based the operability of the scheme), and *L* is *the signifier* (a set of linguistic forms that allow the symbolic representation of the fraction, the treatment situations and procedures). In parallel, Vergnaud also proposed the theory of "conceptual fields. A “conceptual field” is a set of problems and situations that its handling requires different concepts, procedures and representations which are strictly interconnected.

The Vergnaud approach is important and useful, because it aims at the development of a conceptual image, fundamental for the learning of mathematics. Therefore, according to Pinilla (2007), the occurrences of the mathematical object “fraction” are multiple and refer back to a variety of registers of representations, each one belonging to an appropriate system of registers of semiotic representations (p. 98). So, for Duval (2003) the approach such as of Vergnaudis not enough to characterize the specificity of mathematical thinking, about fractions for example. Duval thinks that the conceptualization passes through the “register of representation” that expresses its own object.

According to Duval (2003), the difference between the cognitive activity required by mathematics and that required in other domains should not be sought in the concepts, but in the importance of the *semiotic representations* and in the great *variety of semiotic representations* used in mathematics (p. 13).For him, the reason for the blocks of understanding that many students experience in mathematics should be sought not in mathematical concepts and their epistemological complexities, but in the production system of semiotic representation register.

Indeed, Gagatsis& Elia (2004, p.447), based on the studies of Sierpinska (1992) and Lesh, Behr & Post (1987), point out that in the educational community, it is strongly believed that students can learn the meaning of mathematical concepts by experimenting with multiple mathematical representations. It is because the central cognitive process in mathematical activity is to change a representation register by another that it is equivalent referentially. So, the progressive development of the use of different representations undoubtedly enriches the meaning, the knowledge and the understanding of the object (D’Amore, 2007, p. 82).

The change of representations register can be done in two ways, known as “treatment” or “conversion”. Treatment refers to the transformation of representations made within the same system; whereas “conversion” is the transformation of representations from one register to another target register, while retaining same reference objects (Duval, 2003, p. 14-15).

A conversion is made through an activity of articulation and coordination of the different registers. Consequently, according to Duval (2003, p. 16), the conversion’s activity is the fundamental activity of transformation in mathematics, because it leads to the mechanisms underlying the understanding of mathematical knowledge area.

Therefore, the diversity of the registers of semiotic representations plays a central role in the comprehension of mathematics. So, a pertinent model of teaching and learning a mathematical concept is that one which presents or requires two or more registers of semiotic representations of that object.

In general, a mathematical problem, such as that which involves fractions, appear represented by natural language register (word problem). To solve a problem, Polya (1978) suggests four steps: understanding the problem; establishing a plan; executing the plan, and finally, the retrospective; and also described the procedures necessary to realize each step. The first two steps are most important, because, to solve a problem before the person needs to reconstruct the meaning of the text in a mathematical approach. To do this, it is necessary the understanding of the statement of the problem and of the information that it brings, as well as the conceptual relations that give the meaning to this information (Lorensatti, 2009, p. 95). Therefore, to solve a problem, the student will depend on their knowledge of the linguistic and mathematical codes that are in the statement of the problem.

However, even with all the hints given by Polya (1978), experiences show that students have a lot of difficulties in developing their skill,solving mathematic problems. So, Duval (2003) suggests to use the conversion as an instrument for cognitive analysis of the problem solving,and establishes the model which is called *experimental principles of semiotic variation* and *concomitant variations*, which consists of the following steps:

- Give the most elementary representation possible, R_1 of an object in an output register A and its converted representation $R_1^{'}$ in an arrival register B;
- Carry out all the possible variations of $R_1, \ldots, R_n$ which retain in the different representations a value of representation of something in the output register A, and

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observe the concomitant variations of $R_1^{'}$ in the arrival register B (Duval, 2003, p. 25)

The cognitive variations are only those by which a modification in the starting register A cause a modification in the arrival register B (Maranhão&Iglori, 2003, p. 66).

This Duval's model seeks to articulate the use of different forms of representations with the semantics of verbal problems. The study made by Gagatsis& Elia (2004, 449) showed that with this model, students' ability to solve a step change problem with additive structure is highly associated with verbal problem-solving skills.

III. RESEARCH METHODOLOGY

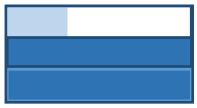
This study was conducted with 29 students attending teacher training courses for elementary school (future primary teachers), at a college called Inhamízua Primary Teacher Training Center, located in Beira town, in Mozambique.

The research was done through two questionnaires, which occurred in two different sessions.The first sessionwas only for diagnostic knowledge of the students about representation of fractions and their problem-solving skills, and it contained only 1 exercise and 1 problem to solve.

Questionnaire of the diagnostic:

1. Consider the figure, and:

(a) writes a sum of simple fractions corresponding to the shaded part; **(b)** Determine, showing the calculations, the fraction corresponding to the unshaded part of the figure



2. Solve the problem:

Maria has 1/3 of the money to buy a motorcycle. The husband gave her 1/2 of the remaining amount to buy thatmotorbike.Which fraction of the money does she still need to buy the motorbike?

The second session was specifically to attending the objective of this study, because previous experiences and the results of the diagnostic proved that the students had many difficulties to solve problems which involve fractions.

The second questionnaire contained 3 variants of a problem. In each variant-problem, 3 to 4 alternative representations of two other types of problem registers were given (figures and fractions).

The problems of this questionnaire were structured according to the Duval model, which is called by *experimental principles of semiotic variation* and *concomitant variations*.

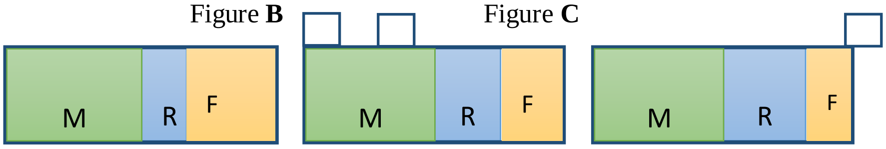
Thus, this questionnaire worked as an alternative methodology of teaching and learning of problem solving involving fractions. It was used to test its efficiency.

Questionnaire of the study (Duval model):

Problem 1

Mary had 1/2 of the money to buy a motorbike. She received a certain amount of money from her husband for the purchase of that motorbike, and she still lacked one-third of the motorbike price. Which fraction of the motorbike price Mary received from her husband?

(a) Figure **A** Figure **B** Figure **C**

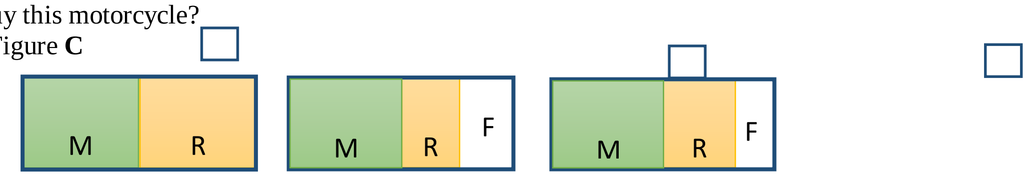


(b) $\frac{1}{2}$ $\frac{5}{6}$ $\frac{1}{4}$ $\frac{1}{6}$

Problem 2

Mary had 1/2 of the money to buy a motorbike. She received from her husband 1/2 of the part of the money that lacked for the purchase of that motorbike. Which fraction of themotorbike price is still lacked for Mary to buy this motorcycle?

(a) Figure **A**Figure **B**Figure **C**



M = fraction corresponding to the money Mary had

R = fraction corresponding to the

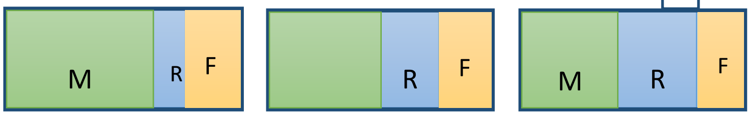
F = a fraction corresponding to the money that, after receiving from her husband, money Mary received from her husband still lacked for Mary to buy the motorbike

(b) $\frac{1}{2}\ \frac{0}{2} = 0\ \frac{1}{4}\ \frac{1}{6}$ ☐ ☐ ☐ ☐

Problem 3

Mary had a certain fraction of the money to buy a motorbike. She received from her husband 1/3 of the money that is lacked for the purchase of that motorbike, and still it was lacked 1/4 of the price. Which fraction of the motorbike price did Mary have before receiving from her husband?

(a) Figure **A** ☐ Figure **B** Figure **C** ☐ ☐



(b) $\frac{1}{3}\ \frac{1}{4}\ \frac{5}{8}\ \frac{7}{12}$ ☐ ☐ ☐ ☐

In each problem the students had to answer the following questions:

(a) Each of the following figures represent the price of a motorcycle. Analyze each figure according to the data, and indicate, by placing an X, the figure that correctly represents the fraction of the problem situations.

(b) Place an X in the square corresponding to the fraction of the price that still is missing for, Mary buy the motorcycle, after having already received the fraction of the husband. And, based on the figure, show here the calculations you made to determine the chosen fraction.

Note that this questionnaire follows the Duval model, because:

1. The position of the unknown in the statements of the problem has undergone the structural variations ≡ principle of semiotic variation.

Mathematical models of the structure of eachproblem

problem 1	problem 2	problem 3
a ⊕**X**⊕b = p	a ⊕c⊕**X** = p	**X**⊕ c⊕ b = p

X is the unknown. It changes positions from one problem to another

2. There is a structural variation of the statements of the problem, and also in some of the registers in the alternatives≡ principle ofconcomitant variations.

IV. RESULTS AND DISCUSSION

(A)Results of the questionnaire of diagnostic

Only 17.2% of the students were able to solve correctly (a) and (b).

13.7% of the students presented correct sums for (a), but wrong solutions for (b); and 44.8% presented the correct answer for (b), but wrong solution for (a).

It can be said that the students still have difficulties to recognize the fractions and its operations within different semiotic representationregisters of the mathematical object “fraction”. The difference of percentages between (a) and (b) may indicate that students had little experience with type (a) exercises, but some experience with type (b) exercises.

No students (0.0%) were able to solve the given problem. However, 3.4% presented the following attempt that reveals, at least, a good reasoning, but with a small problem at the end of the equation (it is not understood how the term *ax* appeared at the end of the equation):

$\frac{1}{3}x+\frac{1}{2}(x-\frac{1}{3}x)+ax=x$; $\frac{1}{3}+\frac{1}{2}-\frac{1}{6}+a=1$; $\frac{2+3-1}{6}+a=1$; $\frac{4}{6}+a=1 \Rightarrow a=\frac{3-2}{3}; a=\frac{1}{3}$

The diagnostic questionnaire results show that the students have a very limited knowledge about fractions. Although the students studied fractions in primary and secondary school, most of them know little about the meaning of the several registers of semiotic representations offractions, as well as the significance of operations with fractions. So, we can conclude they did not have proficiency with fractions.

(B)Results of thequestionnaireof this study (test of the Duval model)

Results of problem 1:

There was only 24.1% of the students who indicated the appropriate figure (figure A) and the appropriate fraction (fraction 1/6). 27.6% chose the correct figure and a wrong fraction; and 6.8% chose the correct fraction and a wrong figure. Some of the reasoning followed by choosing Figure A and fraction 1/6 are as follows:

“Mary had one half, the other half was missing,= 1/2. The husband gave her a part, and it was still missing 1/3, or 0.3. Therefore, it means that the husband gave her 0.2” (It is a good reasoning!!)

“First, we find the part of Mary's value, which is half; then we compare the squares according to the values that Mary had and what was missing to add”

“Figure A indicatesthe value that does not exceed half”

The calculations made to determine the chosen fraction (1/6) were as follows:

- $\frac{1}{2}-\frac{1}{3}=\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$ (17.2% of the students. It's a good reasoning!)
- $x+\frac{1}{2}+\frac{1}{3}=1$; $x=1-\frac{1}{2}-\frac{1}{3}$; $x=\frac{6-3-2}{6}=\frac{1}{6}$ (only 6.9% of the students, but it's an excellentreasoning!)
- $\frac{1}{3}$ of $\frac{1}{2}=\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$ (Only 6.9% of the students, but it's also a good reasoning!)

Results of problem 2:

Only 34.5% of students indicated the appropriate figure (figure B) and the appropriate fraction (fraction 1/4). 10.3% chose the appropriate figure, but not the appropriate fraction. And 3.4% chose the appropriate fraction but not the appropriate figure.

Some of the reasoning followed by choosing Figure B and fraction 1/4 are as follows:

"Half of the money Mary had was half of the rectangle, M, and she received half of the missing money, which is half of the half of the rectangle, R, which is 1/4; then the other half of the half of the rectangle is ¼". (This is the best answer).

"The husband gave half the value that was missing, not giving complete; left half of the half to complete the purchase value of motorcycle"

"Because half of the value was still missing. So, is this which the husband gave her, 0.25"

"Half of the missing part corresponds to R, (1/4), even so, it was missing half of the value she received from her husband, which corresponds to 1/4 (Fr) of the figure".

The calculations made to determine the chosen fraction (1/4) were as follows:

- $\frac{1}{2}+\frac{1}{4}+x=1$(Only 10.3% of the students presented thisequation, and its correct resolution. it's a calculation of a good reasoning);
- $\frac{1}{2}=0.5$ and $0.5:2=0.25=\frac{1}{4}$ (6.9% of the students)
- $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$ for to correspond half of ½(6.9% of the students)

Within the 34.5% of the students who indicated the appropriate figure (figure B) and the appropriate fraction (fraction 1/4), in this problem 2, there are those who also indicated, in the problem 1, the appropriate figure (figure A) and the appropriate fraction (fraction 1/4). There are also 17.9% of the students who resolved the two problems correctly.

Results of problem 3:

Only 6.9% of the students indicated appropriate figure (figure A) and appropriate fraction (fraction 5/8). Although the students indicated appropriate figure or appropriate fraction, they did not give theirs reasoning.

34.5% chose the correct figure and wrong fraction; and 0% chose the correct fraction and wrong figure. Some of the reasoning and calculations that they presented were as follows:

"Because it corresponds"

"I added 1/3 with 1/4, and it resulted in this very small number 7/12"

"$\frac{1}{3}$ has to match half of the remaining value"

- $\frac{1}{4}:\frac{2}{5}=\frac{1}{4}\times\frac{5}{2}=\frac{5}{8}$ (Presented by only 3.4% of the students. It's difficult to understand this thought)
- $x-\frac{1}{3}=\frac{1}{4}$, $x=\frac{1}{4}+\frac{1}{3}=\frac{3+4}{12}=\frac{7}{12}$ (presented by 24.1% of the students. It was the Mode).

Also here, there are only 6.9% of the students who gave appropriate answers in both problems 1 and 3. Or better, only 6.9% of the students correctly answered problems 1 and 3. However, no student was able to present correct answers to all problems.

Analyzing the results, problem by problem, we can say all are negative. We can note that problem 1 seems to be the simplest of all. So, it was to be expected that the students would perform best there, because one may think that it is easy to articulate the data of the statement of the problem (she still lacked one-third of the motorbike price) with the proportions of the parts of each figure, and to distinguish the correct figure. Identified the correct figure,then the corresponding fraction can easily be identified.

The failure of the majority of thestudents can be explained from the following results (already presented above): "27.6% chose the correct figure and a wrong fraction; and 6.8% chose the correct fraction and a wrong figure".

These results may mean at least one of the following two ideas: these students had a great deal of difficulty in understandingand interpreting the problem, and then,they made a random choice,because they did not understand the meaning of the different semiotic representations registers of the "fraction" object. This conclusion is valid to other problems.

Therefore, we can conclude that the majority of students (34.4% plus those who did not respond) cannot yet make an interpretation of a problem involving fractions, because they did not understand the meaning of the different semiotic representations' registers of the "fraction" object.

However, despite problem 2, it is structurally more difficult (She received from her husband 1/2 of the part of the money that lacked for the purchase of that motorbike),with respect to problem 1.Let us note that from problem 1 to problem 2, the percentage of students who answered correctly increased in 10.4% (from 24.1% of problem 1, to 34.5% of problem 2); and, the percentage of aleatory choices decreased in 20.7% (from 34.4% of problem 1, to 13.7% of problem 2). In addition, 17% of the students were able to correctly solve both problems 1 and 2 (note that 17% means to be very good if compared with 0.0% of result of the diagnostic).

So, if we combine these results with the reasoning of the calculations presented by the students, we can believe that the students improved their comprehension of the problems, because the presence of the different registers of semiotic representations of the fractions, it induced them to identify, to articulate and to coordinate the units and cognitive reference variables between the registers, and consequently increased their performance in solving the problems given.

Problem 3 was the one that had the most critical results, because it is relatively the most difficult of all. However, given its complexity and compared with the results of the diagnosis (6.9% is better than 0%), we can say that there was significant improvement.

V. CONCLUSION

Deficiencies in the conceptual comprehension of the fraction and its computational rules, difficulties to make the linguistic decoding, are factors that had an unfavorable influence on the performance of future primary school teachers to solve problems. This study proved that these factors can be overcome when we use the different registers of semiotic representations of problem, because it induces the student to the comprehension activity of problem, which consists in the identification, articulation and coordination of the units and cognitive reference variables between the registers. These results can mean that the *experimental principles of semiotic variation* and *concomitant variations*, applied in didactic situations, are an effective method for teaching and learning to solve problems involving fractions, because it has produced an increase in the performance of the students in problem solving.

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