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Comparative Analysis of a Controller for a Magnetic Ball Levitation System

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Abstract:- Magnetic levitation system is a system that can work on the principle of magnetic attraction and repulsion to levitate an object. It presents a mathematical modelling using a Taylor series approximation method. However, the magnetic ball levitation system is mostly a non-linear and open loop unstable. So, it made the controller strategy more difficult. This thesis work introduces the control techniques of a magnetic ball levitation system depends on linear feedback which is a PID controller. Then the PID controller is more improved by an Adaptive PID with MRAC controller and was obtained a good performance characteristic. Increasing the performance of an Adaptive PID with MRAC Controller was possible with Genetic Algorithms optimization techniques by minimizing the objective function and the error. Lastly, for position control and stabilization of the magnetic ball levitation system an optimal LQR was developed and the weighting matrix Q and R based on the analytical approach was chosen as and are positive semidefinite and positive definite matrix respectively. So, the performance of an Adaptive PID with MRAC controller was more improved by a LQR controller; the performance (i.e., overshoot, Peak amplitude and settling time) obtained by a LQR controller was better when compared to a PID, an Adaptive PID with MRAC Controller and an Adaptive PID controller with GA Optimization techniques.

Keywords:- *Magnetic Levitation System, Proportional-Integral-Differential Controller, Genetic Algorithms, Adaptive with MRAC Controller and Linear Quadratic Regulator Controller.*

I. INTRODUCTION

Engineering systems generally have some basic physical structures. Some of the popular systems are examined for studies about system mechanisms or controller design, like inverted pendulum and magnetic levitation systems [1], [2]. Magnetic levitation is one of the important phenomena to the physical control templates because it rewards in the industrial domain. It is one of the most complexes, nonlinear, and unstable system.

From a control point of view, a magnetic suspension is the simplest way to levitate in the air with no physical contact to the object that means to be levitate under the magnetic material (the one which is needed) and the strength of the magnetic line of force (magnetic field) which is produced by the electromagnetic (having electrical and magnetic characteristics) is regulated to accurately balanced by the gravitational force. Therefore, the system can have only one force, to levitate the object weight so the system is done on the working principle via the attractive force between a magnet and the material.

As an object goes nearer to the electromagnet, the amount of current in the electromagnet will decrease and as the objects goes too far apart, the amount of current in the electromagnet will increase.

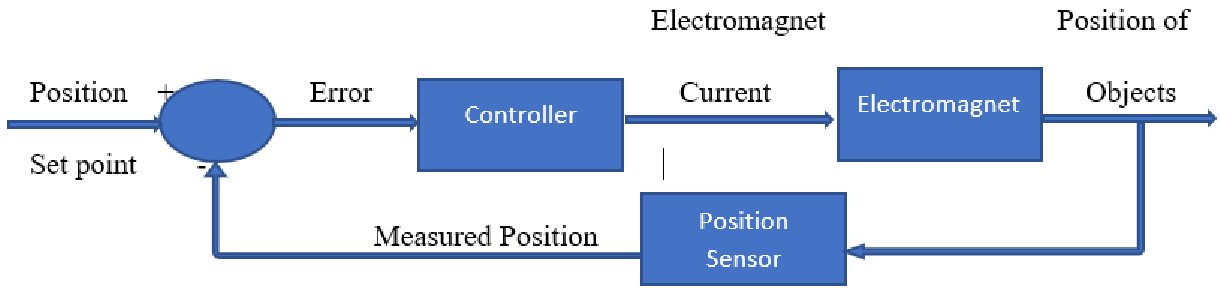

Figure 1: - The control structure of the magnetic ball levitation system.

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II. MATHEMATICAL MODELLING FOR THE SYSTEM

The circuit diagram given below is the overall components of the magnetic ball levitation system. It is comprised of the Mechanical and Electrical sub-systems.

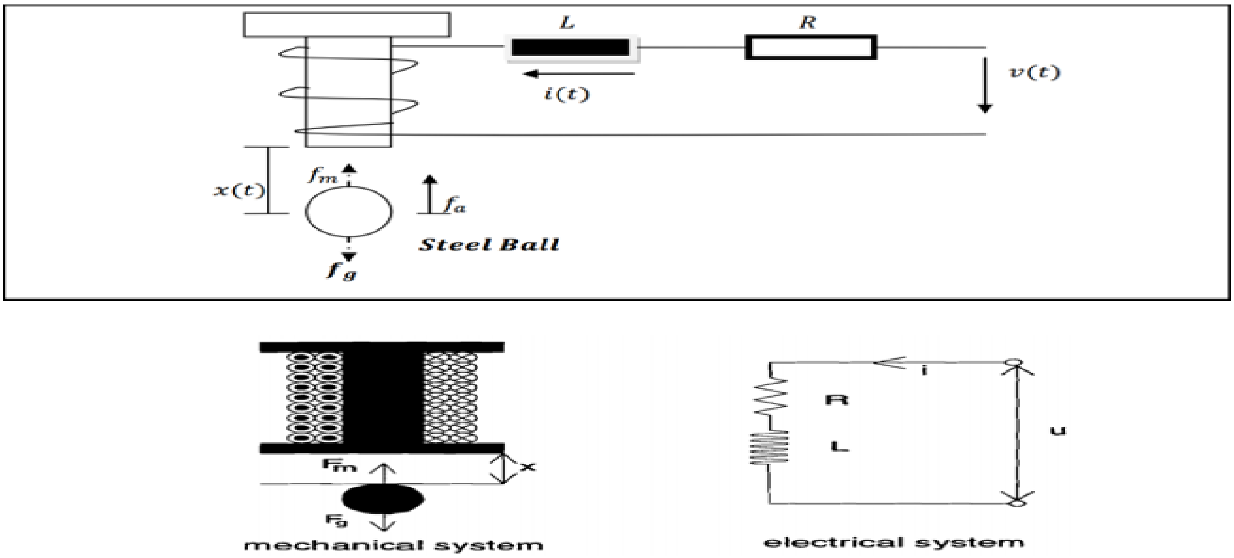

Figure 2: - Main components of Magnetic Ball Levitation System [7]

The following assumptions were put for modelling the system.

i. The system has zero frictional force
ii. The losses due to the mechanical and electrical system were negligible.

Modelling for mechanical sub-systems

As we know the energy stored in the inductor can be expressed by the formula as: -

$W_e = {}^1\!/_2\, Li^2$ ………………………………………2

$P_e = \frac{dW_e}{dt}$ and $P_m = -f_m \frac{dx}{dt}$, therefore $-f_m \frac{dx}{dt} = \frac{dW_e}{dt}$

$f_m = -\frac{dW_e}{dt}\frac{dt}{dx} = -\frac{dW_e}{dx}$ ……………….............3

Where f_m is electromagnetic force now substituting equation (2) in equation (3) and got: -

$f_m = -{}^1\!/_2\, i^2 \frac{d}{dx}\left(\frac{K_c}{X}\right)$ ……………………………………………………

$= -{}^1\!/_2\, i^2 \left(-\frac{K_c}{x^2}\right)$ ………………….. 4

$= {}^1\!/_2\, K_c \frac{i^2}{x^2}$ ………………………….

If f_m magnetic force produced by input current, f_g the gravitational force and f_{net} is the net force acting on the ball. The equation of force can be written as: -

$m\frac{d^2x}{dt^2} = \text{mg} - {}^1\!/_2\, K_c \frac{i^2}{x^2} \Rightarrow \frac{d^2x}{dt^2} = g - {}^1\!/_2\, K_c \frac{i^2}{mx^2} = \boldsymbol{f_1}\ (\boldsymbol{\ddot{x}}, \boldsymbol{\dot{x}}, \mathbf{x}, \mathbf{i})$ ...………………..5

By applying Kirchhoff's voltage law, the mathematical equation for the electrical part given in figure (2) is expressed by the formula as: -

$v = v_R + v_L \Rightarrow v(t) = Ri(t) + L\frac{di(t)}{dt}$

$L\frac{di(t)}{dt} = -Ri(t) + v(t) \Rightarrow \frac{di}{dt} = -\frac{R}{L}i + \frac{1}{L}v = \boldsymbol{f_2}\ (\mathbf{i}, \mathbf{v})$ ……………………………6

Where *V, I, R* and *L* is source voltage, input current in the electromagnetic coil, coils resistance and coils inductance respectively.

To linearize a nonlinear system, it is always advisable to use some equilibrium point. Therefore, to obtain the equilibrium point use the condition.

$\ddot{x} = \dot{x} = 0, \frac{di}{dt} = 0,$ ……………………7

From the condition given in (7) the acceleration $\boldsymbol{\frac{d^2x}{dt^2}} = \mathbf{0}$ and $\boldsymbol{\frac{di}{dt}} = \mathbf{0}$. This means that the two forces are equal.

From the linearized state equations and the output equations the corresponding coefficient matrices

$A = \begin{pmatrix} 0 & 1 & 0 \\ 1962 & 0 & -56.7011 \\ 0 & 0 & -350 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 100 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$ and $D = 0$ ……………8

Hence, from equation (8) the state space to transfer function **G(s)** of the system was found using the analytical formula was given as: -

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$$G(S) = -\frac{5670.11}{(S+350)(S^2-1962)} = -\frac{5670.11}{S^3+350S^2-1962S-686700} \quad \ldots\ldots\ldots\ldots\ldots 9$$

III. CONTROLLER DESIGN

PID Controller

PID controller is the most widely control strategy used in the areas of large industry. They do, however, in control and instrumentation engineering it faces some challenges for tuning the parameters of the gains needed for stability and for better performance characteristics.

A **Proportional-Integral-Derivative controller** (**PID controller**) is a control loop feedback mechanism (controller) widely used in industrial control systems. A PID controller evaluates the ***error*** which is the difference between the actual output and a reference set-point. The controller needs to reduce the *error* by adjusting the system through manipulated variable.

Controller designer arrange the Proportional, Integral and Derivative modes into three different controller algorithms or controller structures. These are called Interactive, Non-interactive, and Parallel algorithms.

$$u(t) = K_p e(t) + K_i \int_o^t e(\tau)d\tau + K_d \frac{d}{dt} e(t$$

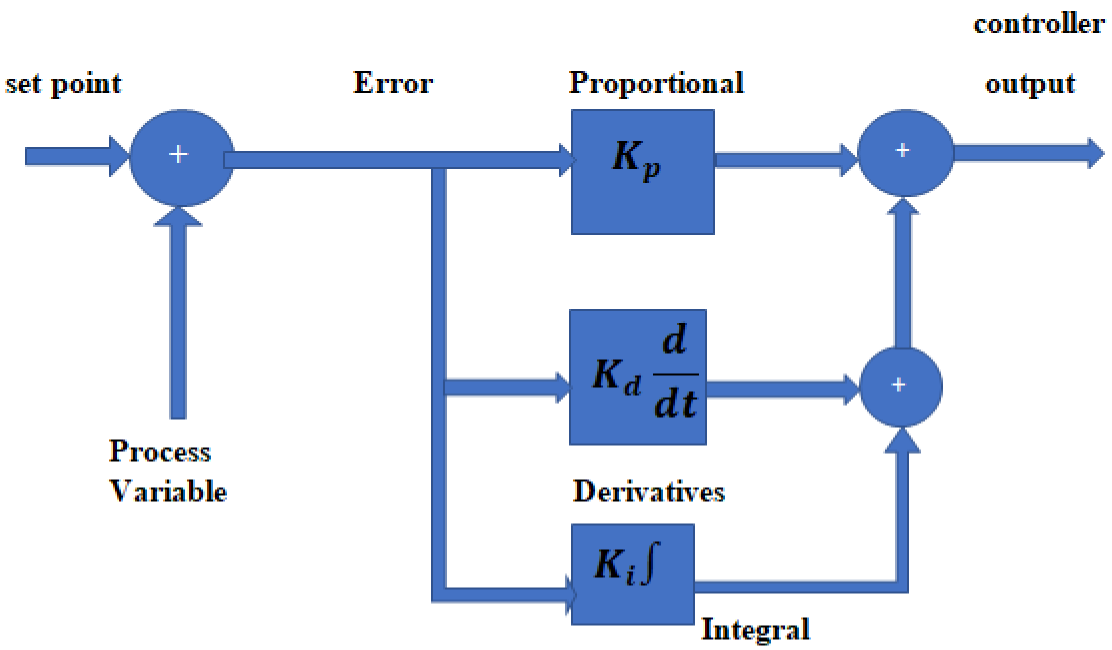

Figure 3: - Parallel Algorithms

Software Tools (PID Tuning Toolbox in MATLAB)

The software toolbox method was used to tune the PID controller parameters online.

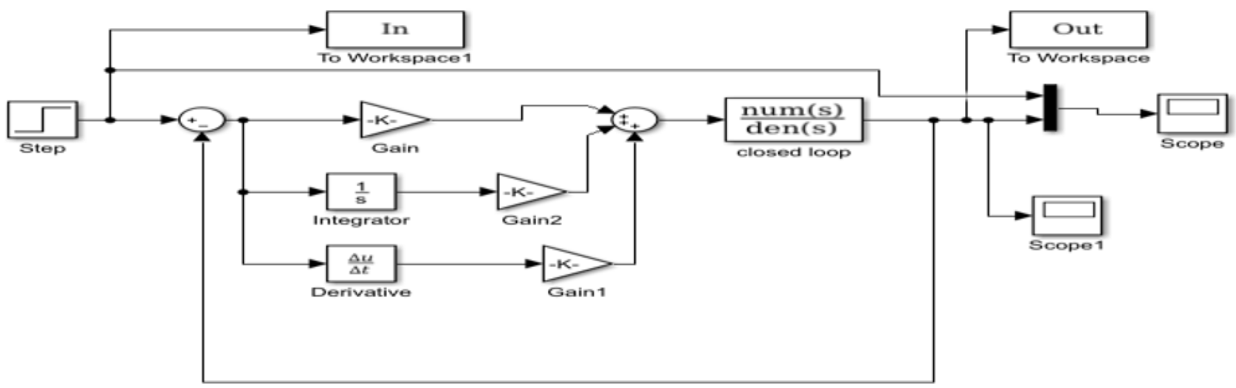

Figure 4: - Simulink block diagram for the PID Controller.

Figure 5: - Simulation result for the closed loop system after placing a PID controller into the system

The simulation result shows that the PID controller can stabilize the unstable part of the closed loop system or plant.

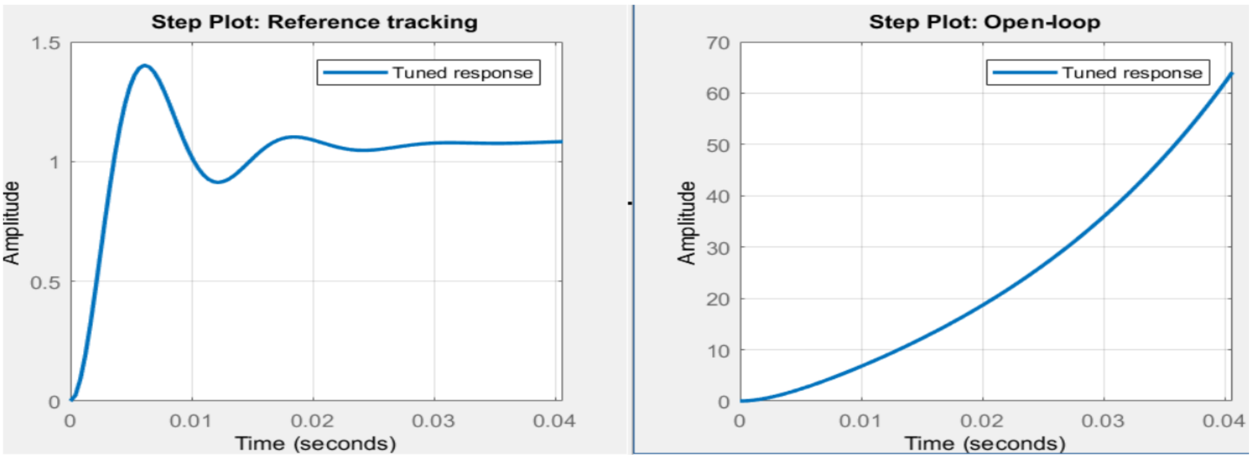

Figure 6: - Simulation result of the closed loop system from PIDTuner.

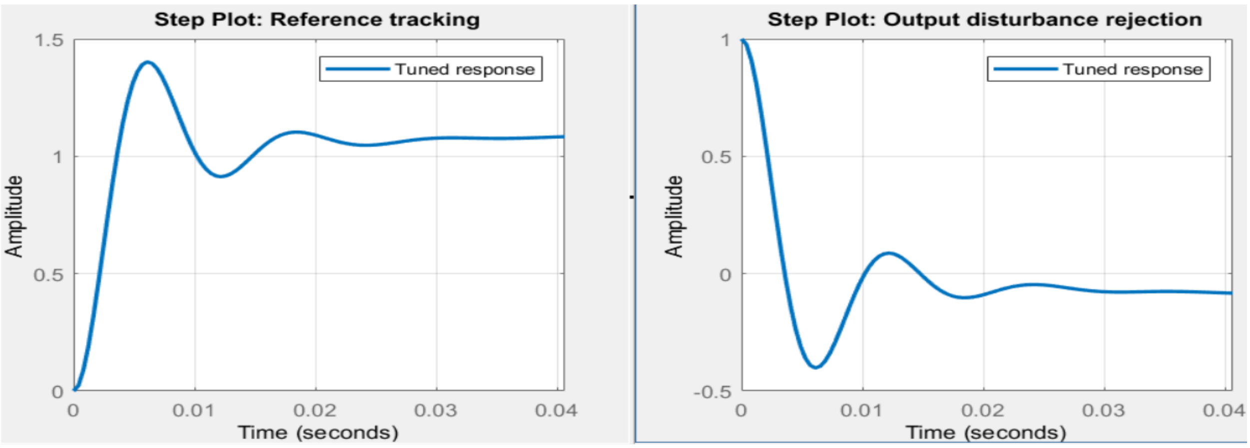

Figure 7: - Simulation result of the output disturbance

From the simulation result stated in figure (6) and figure (7) the PID Tuner as shown in above and from its characteristics performance in the table given above you can conclude that the PID Tuner could find a stabilizing proportional-integral-derivatives controller for the system and it verifies that the output disturbance or the steady state error becomes almost zero in time domain.

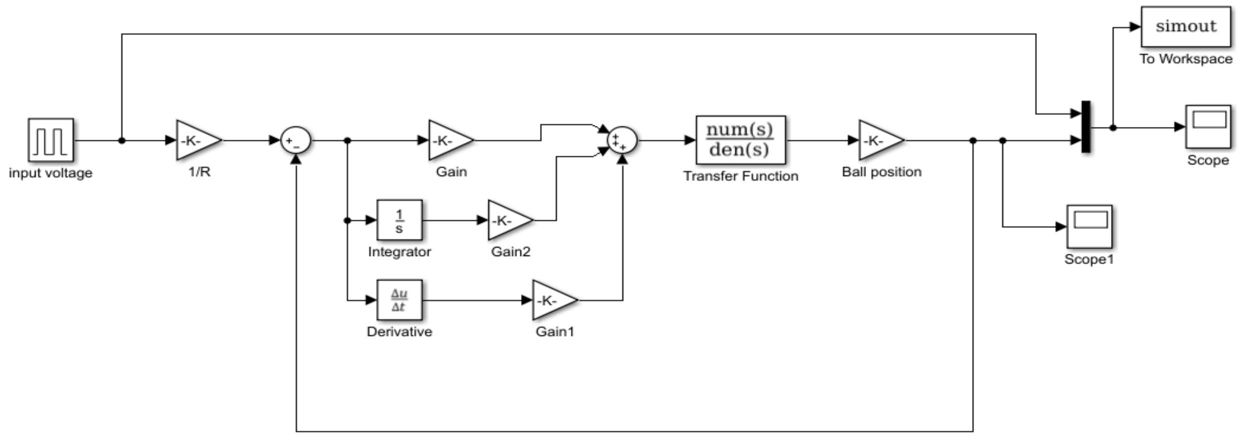

Figure 8: - SIMULINK model of the linearized state space for one dimensional ball system

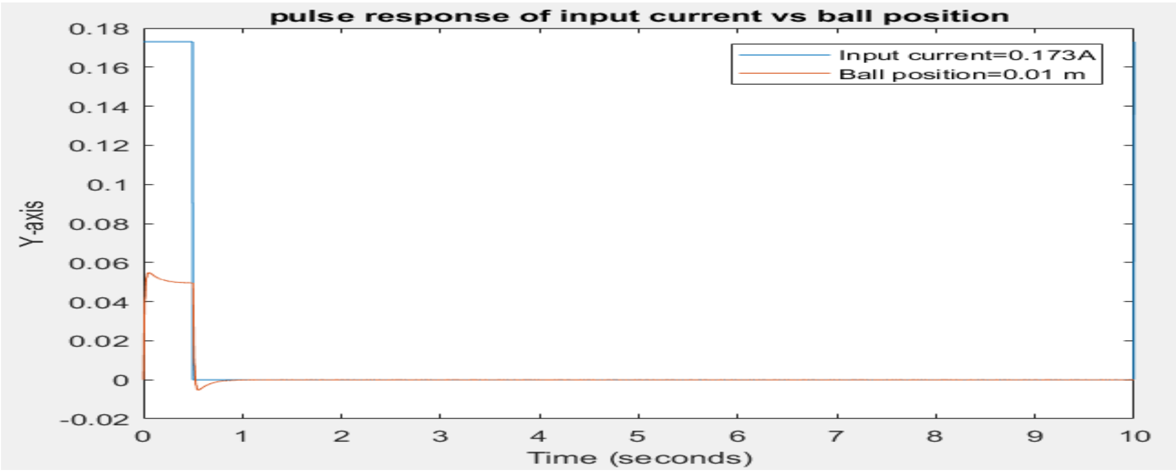

Figure 9: - The ball levitates in the vertical component with the equilibrium current of 0.173 A.

The simulation result shows that the one-dimensional ball levitation system is levitated at the ball position of 0.01m with equilibrium current of 0.173 A. But the ball position is gradually leading to the equilibrium point.

Adaptive PID with MRAC Controller based on GA optimization techniques.

The word adaptive means to adapt which deals with the variation of behavior of the process to the best dynamic system. An adaptive controller is a type of modern control system which is inherently nonlinear. And also, the basic idea behind an adaptive control is to estimate uncertain plant/controller parameters on-line, while using measured system signals. So, it can modify its state in response to variation in the dynamics of the system and the behavior of the unwanted noise [3]. Adaptive Control is also dealing with complicated process in which the system has unpredictable parameter variation/deviation and uncertainties.

A PID controller can be expressed in a generalized time domain form as: -

$$U(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de}{dt}$$ ……………………………………….....10

Where K_p is proportional gain, K_i is the integral gain, and K_d is the derivative gain and its Laplace transformation was given as: -

$$G_c(s) = K_p + K_d s + \frac{Ki}{s}$$ ……………………………………….…………….…..11

Based on the MIT rule it can get K_p as: -

$$\frac{d\varepsilon}{dK_p} = \frac{G_p E}{(1+G_p K_p+\frac{G_p K_i}{s}+G_p K_d s)}$$ …………………………….……………….…............12

Based on the MIT rule it can get K_i as: -

$$\frac{d\varepsilon}{dK_i} = \frac{\frac{G_p}{s} E}{(1+G_p K_p+\frac{G_p K_i}{s}+G_p K_d s)}$$ ………………………………………………………………13

Based on the MIT rule get K_d as: -

$$\frac{d\varepsilon}{dK_d} = \frac{G_p s y_p}{(1+G_p K_p+\frac{G_p K_i}{s}+G_p K_d s)}$$ ………………………………………………………….14

$$G_m(s) = \frac{{\omega_n}^2}{s^2+2\zeta\omega_n s+{\omega_n}^2} \ldots\ldots\ldots\ldots 15$$

$$G_m(s) = \frac{5.92e5}{s^2+800s+5.92e5} \ldots\ldots\ldots\ldots 16$$

Therefore, the tuning terms are given as follows: -

- $\frac{d\theta_1}{dt} = \frac{dK_p}{dt} = \gamma\varepsilon\frac{d\varepsilon}{dK_p} = (-\frac{\gamma_p}{s})\varepsilon(\frac{5.92e5}{s^2+800s+5.92e5})e$ ………………17
- $\frac{d\theta_2}{dt} = \frac{dK_i}{dt} = \gamma\varepsilon\frac{d\varepsilon}{dK_i} = (-\frac{\gamma_i}{s})\varepsilon(\frac{5.92e5}{s^2+800s+5.92e5})e$ ………………..18
- $\frac{d\theta_3}{dt} = \frac{dK_d}{dt} = \gamma\varepsilon\frac{d\varepsilon}{dK_d} = (-\frac{\gamma_p}{s})\varepsilon(\frac{5.92e5}{s^2+800s+5.92e5})y_p$ ………………19

GA Optimization techniques.

Genetic algorithms are an approach to optimization and learning based loosely on principles of biological evolution, these are simple to construct, and its implementation does not require a large amount of storage devices. So, it makes them as a requirement for choosing as an optimization problem.

Option	Number/type
Optimization Techniques	Genetic Algorithms
Number of variables	3
Total number of generations	50
Population size	100; 500
Cross over	arithmetic
Cross over probability	0.01
Mutation	uniform
Mutation probability	0.1
Fitness scaling	Rank
Fitness function	Mean square error
Selection function	Stochastic uniform

Table 1: - parameters of Genetic Algorithms

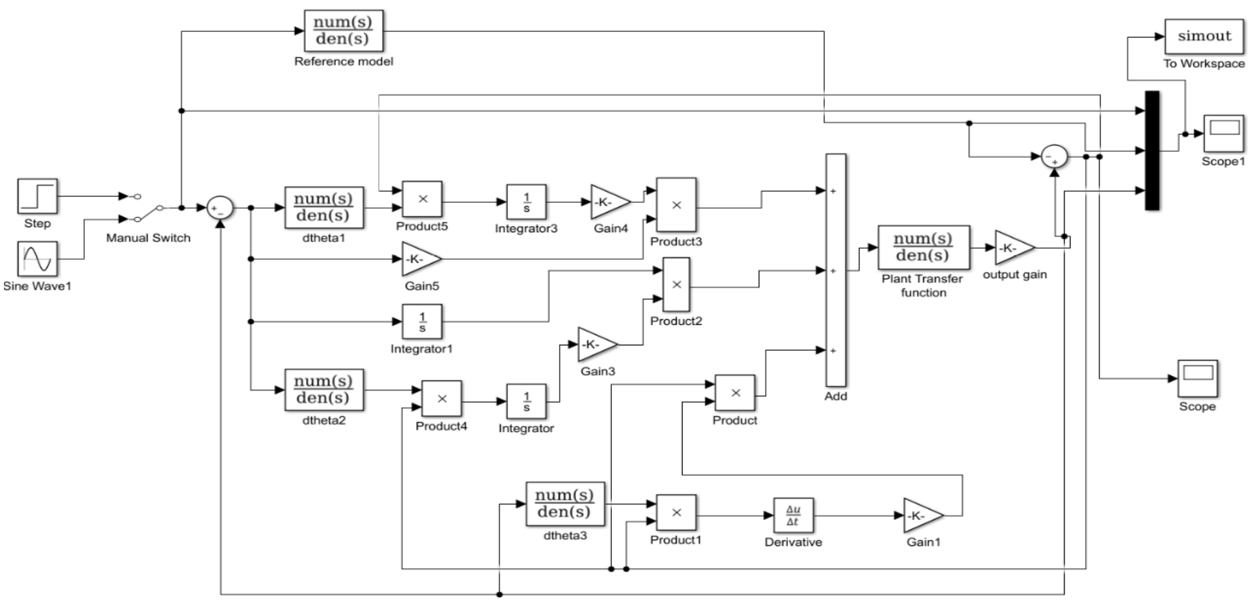

Figure 10: - Simulink model of an adaptive PID controller with MRAC.

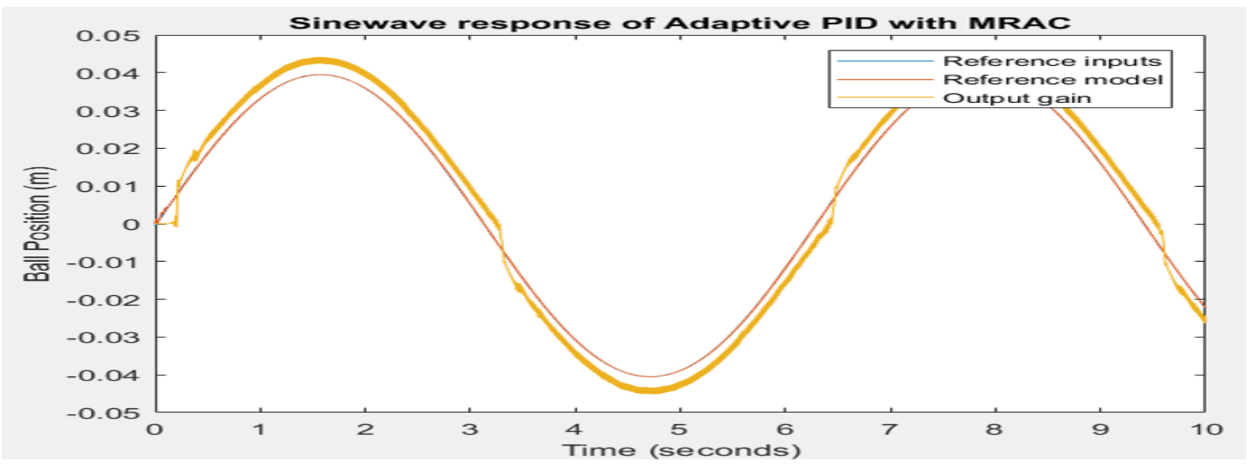

Figure 11: - Simulation result of an adaptive PID controller with MRAC controller.

Figure 11: - Indicates the Simulink model for a magnetic ball levitation system using adaptation mechanisms after choosing the adaptation gains by trial-and-error method. It is clearly shown in figure that the sinusoidal signal output for the process and got a good tracking to the reference point and nearer to the response of the reference model. i.e., $x_e = 0.04$

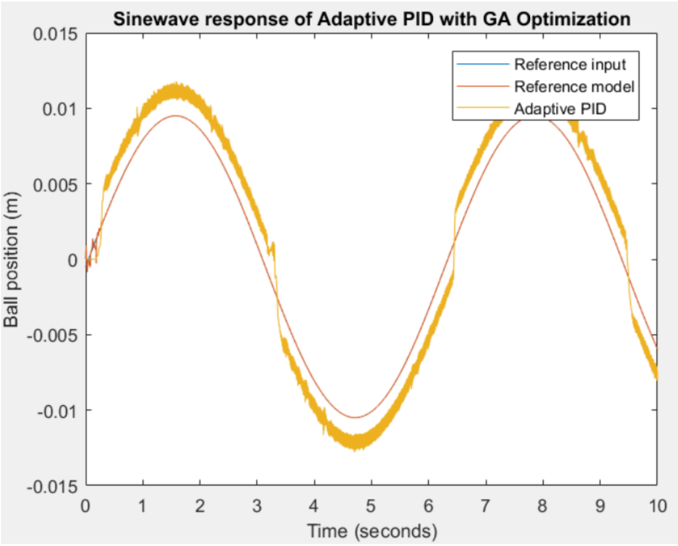

Figure 12: - Simulation result of an adaptive PID controller with GA optimization techniques.

Figure 12: - Indicates the Simulink model for a magnetic ball levitation system using adaptation mechanisms after choosing the adaptation gains by GA optimization methods. It is clearly shown in figure that the sinusoidal signal output for the process and got the best tracking to the reference point and nearest to the response of the Reference model. i.e., $x_e = 0.01$

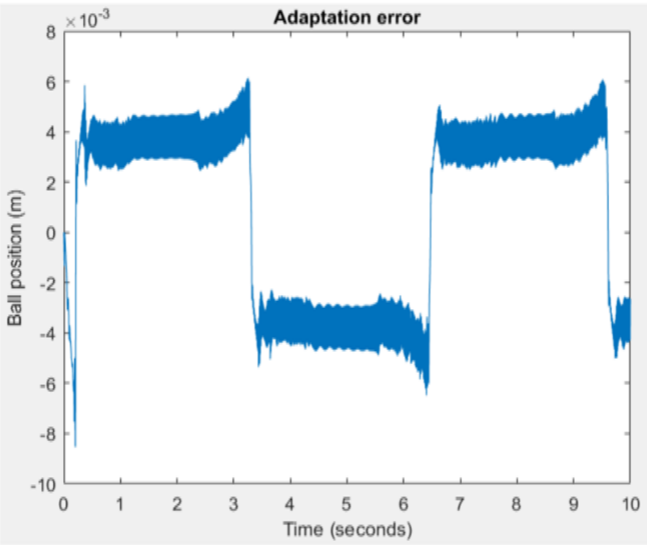

Figure 13: - adaptation error for an adaptive PID with MRAC controller

Figure 13: - Indicates that it is the property of the adaptation error (the difference between the plant output and the reference model output) through the adaptation mechanisms, and the adaptation error equals 0.006 and this is the case when the adaptation gains are selected by trial-and-error methods.

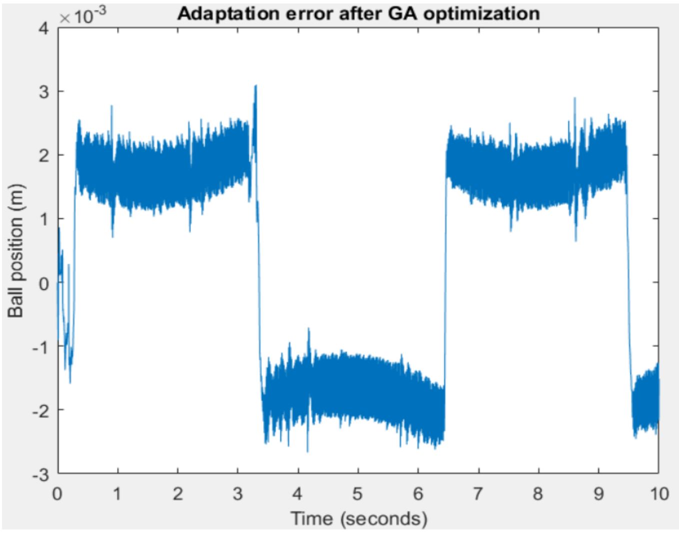

Figure 14: - adaptation error for an adaptive PID with GA optimization techniques.

Figure 14: - Indicates that it is the property of the adaptation error (the difference between the plant output and the reference model output) through the adaptation mechanisms, and the adaptation error equals 0.002 and this is the case when the adaptation gains are selected by GA optimization methods. it shows that the value is approximately deviated from zero.

Optimal Linear Quadratic Regulator

In a recent time, conventional optimal control theory was stated to develop an optimal state feedback controller which is called Linear Quadratic regulator (LQR) and it minimizes the time required in the locus of the tips of the state vector (state trajectories) to the process while sustaining minimum control effort. Linear quadratic regulators design is to be defined for the development of the Linear Quadratic Gaussian/Loop Transfer Recovery (LQG/LTR) design procedure.

Based on the analytical approach the weighting matrices are given as: -

$$\frac{q_1}{r} = \frac{1}{B_{31}^2}((\zeta\omega_n^3)^2 - A^2_{31}) \quad \ldots\ldots 25$$

$$\frac{q_2}{r} = \frac{2A_{31}(A_{33}-2\zeta\omega_n)+\zeta\omega_n^3A_{33}-3\zeta^2\omega_n^4+(2\zeta^2+1)^2\omega_n^4+A_{32}((2\zeta^2+1)\omega_n^2+1)}{B_{31}^2} \quad \ldots\ldots 26$$

$$\frac{q_3}{r} = \frac{-2\,(A_{32}+(2\zeta^2+1)\omega_n^2)-A_{33}^2+9\zeta^2\omega_n^2}{B_{31}^2} \quad \ldots\ldots 4.3.30$$

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Since the input for the system is the voltage applied to the coil, the value of R matrix to be selected is B'*B. Then, the corresponding Q matrix found by means of the formulated procedure is: -

$$Q = \begin{pmatrix} 7393000 & 0 & 0 \\ 0 & -16040 & 0 \\ 0 & 0 & -0.124 \end{pmatrix}$$

So, from the weighting matrix selection above you can find the best controller gain which satisfies the given time domain specification as:

$$K = \begin{pmatrix} -275.4210 & -6.0566 & 0.8724 \end{pmatrix}$$

And the corresponding transformation matrices computed from MATLAB is given as: -

$$p = 1.0e^{+06} * \begin{pmatrix} 8.4707 & 0.1914 & -0.0275 \\ 0.1914 & 0.0042 & -0.0006 \\ -0.0275 & -0.0006 & 0.0001 \end{pmatrix}$$

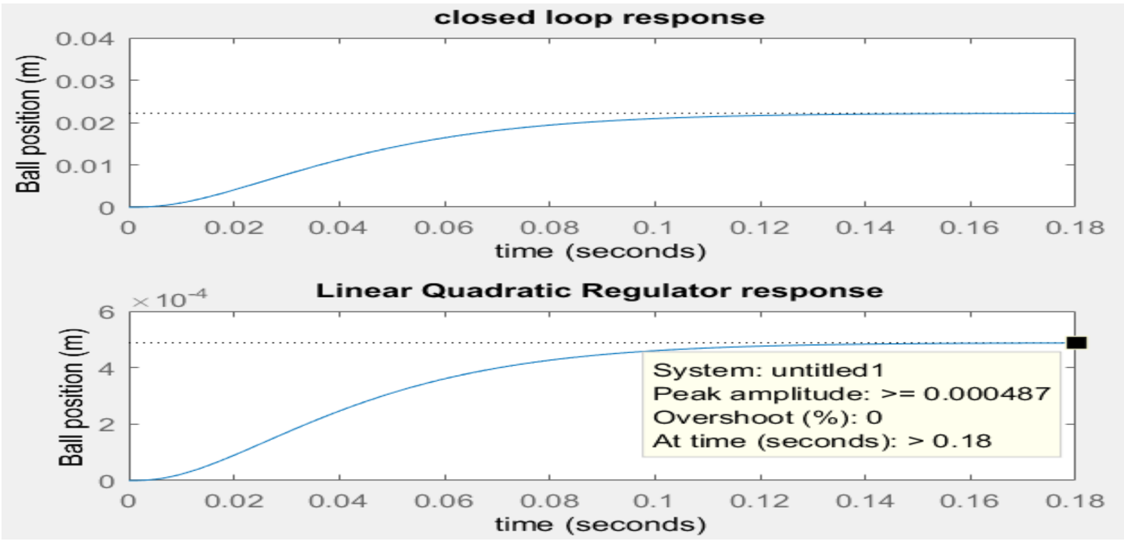

Figure 15: - Closed loop response of the linear quadratic regulator of the system.

The figure given above indicates that the step response of the system, and you can have selected that the time domain specification which is the angular frequency and the damping ratio of the response was 0.55 and 60.61 $rad/_{sec}$, respectively and you can deduce that the design criteria is satisfactory. And also, the steady state error value of the ball position yields zero.

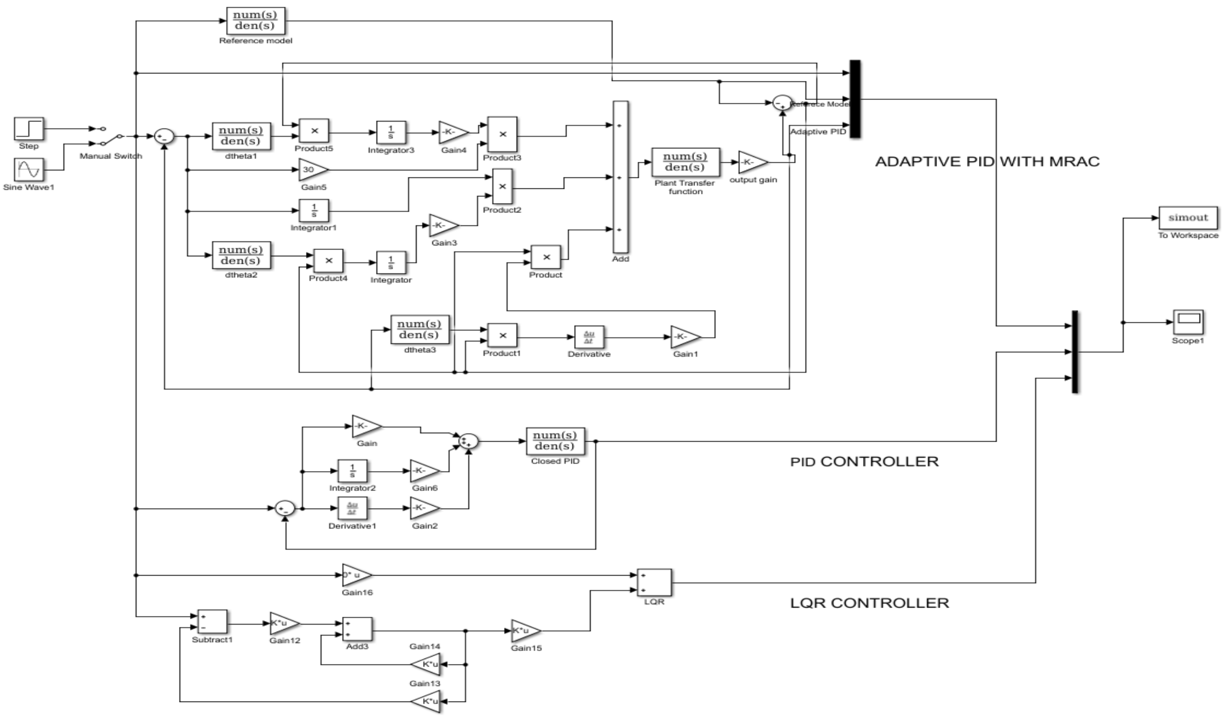

Figure 16: - Simulink model of an Adaptive PID with MRAC, a PID and a LQR controller for the system.

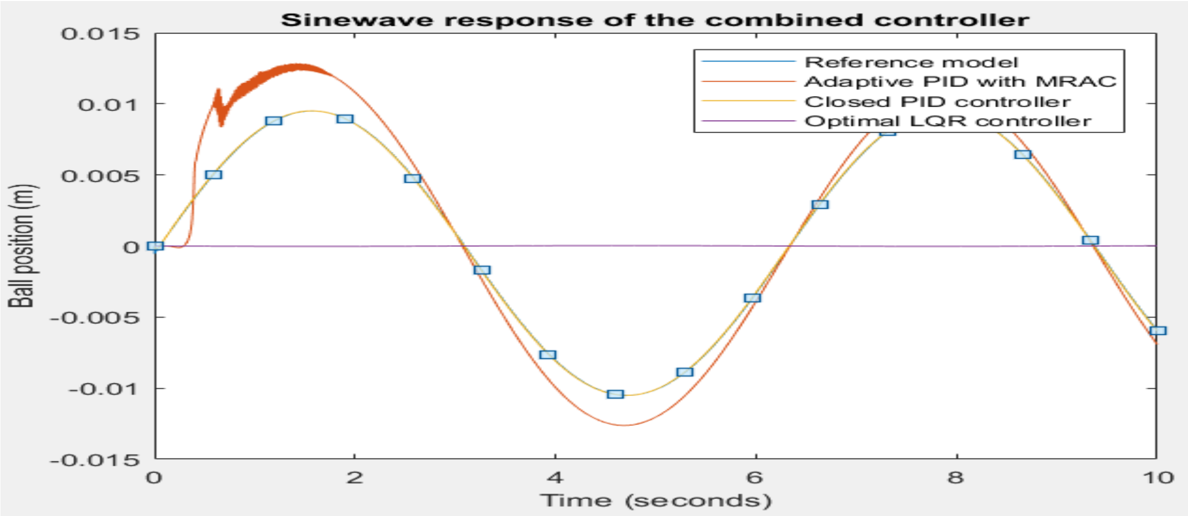

Figure 17: - Simulation Results of an Adaptive PID with MRAC, a PID and a LQR controller of the system.

From the overall simulation result as shown in figure (17) we can conclude that a linear quadratic regulator was the best controller mechanism to control the required position of the given system.

IV. RESULT AND DISCUSION

In this section we can see the results and discussion of the controller depends on the comparative analysis from the Simulation result found in MATLAB/Simulink.

The simulation results and discussion of PID controller is given as follows: - This was done to the control techniques of a magnetic ball levitation system depends on a linear feedback controller which is a PID controller to adjust the parameter gain and obtained as $K_p = -4392$, $K_i = -3.398e04$ and $K_d = -52.305$ resulting characteristics performance of raising time, settling time, overshoot, steady state error and peak value as 0.0227 sec, 0.0606 sec, 55.7 %, 0 % and 1.56m in y-axis respectively, Based on the characteristics performance stated above and the simulation result as given in the control design we can conclude that the PID controller could stabilize the given system.

The simulation results and discussion of an Adaptive PID controller with MRAC is given as follows: - This was done to the control techniques of a magnetic ball levitation system depends on a non-linear feedback controller which is an Adaptive PID with MRAC controller to adjust the adaptation gain by trial-and-error method and taken as $\gamma_p = -0.09$, $\gamma_i = -0.08$ and $\gamma_d = 0.07$ resulting the performance characteristics of raising time, settling time, overshoot, steady state error and peak value as 0.01sec, 0.01sec, 4 % , 0% and 0.04 m in y-axis respectively. We can deduce that the sinusoidal signal output for sinusoidal input for the system and we got a good tracking reference to the set point and approach to the response of the reference model having a small oscillation.

The simulation results and discussion of Genetic Algorithms optimization techniques was given as follows: - This was done to the Optimization techniques of a magnetic ball levitation system depends on a genetic algorithms optimization technique to adjust the adaptation gain and taken as $\boldsymbol{\gamma_p = 14.5 * 10^{-10}}$, $\boldsymbol{\gamma_i = -1 * 10^{-10}}$ and $\boldsymbol{\gamma_d = 2 * 10^{-10}}$ resulting the performance characteristics of raising time, settling time, overshoot, steady state error and peak value as 0.01sec, 0.01sec, 1% , 0% and 0.01 m in y-axis respectively.

The simulation results and discussion of LQR controller is given as follows: - This is done to introduces the control techniques of a magnetic ball levitation system depending on an optimal controller which is a Linear Quadratic Regulator to find the optimum value by selecting the weighting matrices and resulting the performance characteristics of raising time, settling time, overshoot, steady state error and peak value as 0.072 sec, 0.012 sec, 0%, 0% and 0.0004m in the y-axis respectively. We can deduce that the step response of the system gives almost an optimum value which is nearest to the equilibrium point (0.01m).

property	PID Controller	Adaptive PID with MRAC Controller	Adaptive PID with GA Optimization	LQR Controller
O.S (%)	55.7	4	1	0
Settling time (s)	0.0606	0.01	0.01	0.012
Rising time (s)	0.0227	0.01	0.01	0.072
Peak amplitude (m)	1.56	0.04	0.01	0.0004
S.S.E (%)	0	0	0	0

Table 2: - Comparison b/n PID controller, Adaptive PID with MRAC Controller, Adaptive PID with GA Optimization techniques and LQR Controller.

From the time domain specification property mentioned above in table 2 we can conclude that the Linear Quadratic Regulators is better than those of PID Controller, an Adaptive PID with MRAC Controller and an Adaptive PID with GA Optimization techniques interms of overshoot, settling time and peak amplitude but the rising time was slightly different . It gives better results than the nearest one which is PID Controller, an Adaptive PID with MRAC Controller and an Adaptive PID with GA Optimization techniques that is almost zero in all of the time domain specifications and the peak value in the y-axis is 0.0004 m which is almost nearest to the equilibrium.

Lastly, it can conclude that from the above results as discussed in simulation results, from the characteristics specification and from comparison method stated in table (2) above a linear quadratic regulator was the best controller mechanism to find the required position of the system by improving the overshoot and peak amplitude.

V. CONCLUSION

We could have formulated the state space equations and the system input-output model for a magnetic ball levitation system based on Taylor series approximation. We know that the system is linearized around a specific equilibrium point (i.e., at 0.01m). We have proved that the system is unstable and inadequate in its performance characteristics around a certain equilibrium-point. In order to stabilize the system a control strategy i.e., a Proportional-Integral-Derivatives, Adaptive Proportional-Integral-Derivatives and a Linear Quadratic Regulators controller were developed on the system. Hence, for position control and stabilization of the magnetic ball levitation system an optimal linear quadratic regulator was better and the weighting matrix Q and R based on the analytical approach was chosen and are positive semidefinite and positive definite matrix respectively. So, a PID controller is improved by an Adaptive PID controller with MRAC and an Adaptive PID controller is more improved by a LQR controller; the characteristic performance gained by a LQR controller has obtained better when compared to a PID, an Adaptive PID and GA optimization.

Finally, the simulation result using the SIMULINK model and MATLAB Code on the MATLAB Software were developed and therefore, the requirements of the regulator have the ability to levitate the ball at the required position is to be stable.

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