ISSN No:-2456-2165

# The Comparative Effects of Analytical and Synthetic Approaches on Students' Conceptual Understanding of Derivatives: The Case of Dr. Abdulmejid Hussein College of Teachers Education, Jijiga, Ethiopia.

**BY**

**PRINCIPAL INVESTIGATOR: MAMUSH ENDALE KELECHA**
**BSc and MEd in Mathematics from Addis Ababa and Haramaya Universities Ethiopia, respectively**
**(Mathematics Lecturer in JigjigaUniversity )**

**CO-INVESTIGATOR: TEWODROS DEMISSIE WORKU**
**MEd in Mathematics Education from Addis Abeba University Ethiopia**
**( Mathematics Lecturer in Jigjiga University )**

**February, 2021**
**Jigjiga, Ethiopia**

ISSN No:-2456-2165


## ABSTRACT

**This study sought to investigate the effect of analytic and synthetic teaching-learning approaches on understanding of concepts of derivative in introductory calculus course. The subjects of this study were 2nd year mathematics students in Dr. Abdulmejid Hussein College of Teachers Education in Somali Regional State. A total of Eight-two students were participated in two equal sections, each section contained 41 students, in the study. The students in the two sections were arbitrarily assigned as control group and experimental group andtaught by synthetic approach and analytic approach, respectively.**

**As tools of data collection, Achievement Tests were used for the pre-test, the post-test and the delayed post-test to serve as data for student performance. The pre-test was given just before the experiment began and the post-test was given immediately upon completion of the experiment. The two groups were also tested again after three weeks for the delayed post-test to see which of the approaches resulted in better long term learning. The means of the aggregate mean performance scores were compared by the use of independent sample t-test at 05probability level. Results revealed that there was no significant mean difference between the two sections on the pre-test and delayed post-test scores but there was a significant mean difference between the post-test scores, in favor of the section taught by the analytic teaching approach. Preference towards the teaching method used was also investigated. The result of the five-point scale preference Questionnaire revealed that most of the students in the analytic group preferred the analytic approach whereas the synthetic group students preferred the synthetic approach. The preference of the students towards the respective teaching approach used was also compared using t-test. It showed a significant mean difference. The effect of the students' preference towards the respective methods on their performance was also analyzed using Linear Regression and obtained no significant effect. A systematic classroom observation check list for eight successive periods also supported this result. It was recommended that course instructors in mathematics should implement the analytic approach as often as necessary to tap into its many advantages in maximizing students' educational outcome in mathematics regardless of their preference.**


***Keywords:-****Analytical Method, Synthetic Method, Teaching of Derivative Concepts.*

ISSN No:-2456-2165

# I. INTRODUCTION

## 1.1 Background

Mathematics plays a key role in the advancement of Sciences, Technology, Industries, Medicine and Social Sciences as a whole. Even, computer itself is a machine built up on the principle of Mathematics and its invention helps to bring a paradigm shift in data communication and processing efficiently which in turn contributes to accelerate economic revolution of the ever growing society. However, many students turn out to be very miserable and inattentive in a mathematics class after being taught a topic and discover they could not memorize or recall such a concept with ease.

Of greater concern to teachers is that this attitude is carried over by the students to college or university level. Gordon (1997) found that many college students have a weak grasp of mathematical concepts, and that students weak in algebra in upper secondary school are not only weak in mathematics but also have a negative attitude toward the subject. These students only memorize algorithms or procedures and do not understand the meaning behind the procedures used (Gordon, ibid). In fact some students do not realize there is meaning in mathematics and that concepts exist for every procedure applied. They believe that solving mathematics problems means to run many operations using symbols without meaning (Oaks, 1992).

The reason for this difficulty may vary but this could sometimes be related to the teaching method being used to explain such topics. Udeinya and Okabiah (1991) blamed poor performance of students in mathematics on poor methods and approaches to teaching which has reduced the level of motivation. Harbor-Peters (2001) asserted that the issue of poor performance in mathematics examinations was due to problem of teaching methods. There has also been an increasing awareness by those concerned with mathematics education that the conventional methods of teaching mathematics, has not been very successful. For effective teaching to take place, the skillful mathematics Instructor needs to use many different methods and techniques at his disposal. A carefully designed teaching method can make teaching and learning effective (Chianson, M.M 2008).

An instructor of mathematics has a variety of methods and techniques which s/he might use in her/his everyday classroom teaching. The main objective of these methods is to make teaching learning process more interactive and effective. There are various kinds of teaching methods and techniques by which a teacher educator can select the most relevant one keeping in view the needs of the learner and its relevance to the contents. Some of the commonly suggested and fairly used teaching methods of mathematics courses in college of teachers' education are project method, heuristic method, inductive and deductive methods and analytical and Synthetic methods (Revised Curriculum June 2009; Addis Ababa, Ethiopia). To grasp on the subject matter of mathematics it is necessary to use multiple methods in the teaching learning process at all level. It is generally observed that the students solve a question or set of questions quickly by using a technique or formula already learnt in the class but they have no understanding about the hidden logic that how it done. Prior is the deductive or synthetic approach to find a solution for a question but later is the inductive reasoning of analytic approach.

Mathematics learning for understanding is not easy. Many students fail to understand the concepts taught to them. They solve problems by memorizing formulae and procedures teachers have taught them. The students merely put the required figures into the formula to arrive at the answer (Miller, 1992).

Lack of understanding in mathematics often can make students lose interest in the subject and affect their mathematics achievement. The ability to use procedures and understand concepts in mathematics are two things necessary in learning mathematics (Hiebert and Carpenter, 1992). Understanding in mathematics learning generally involves actions to know concepts and principles related to the procedures and relating or creating meaningful relationships between existing concepts and newly-learnt concepts (Boroody and Ginsburg, 1990).

Here in this study, the researcher planned to overhaul which method is more appropriate to teach some selected topics from high level mathematics course offered in teacher education institute. That is, focus of this study is to carefully examine the comparative effectiveness of the analytical and synthetic methods of teaching mathematics courses of the Department of Mathematics in Dr.Abdulmejid Hussein Memorial College of Teachers Education (Dr. A.H-CTE).

The topics that will be discussed during the study are attached in appendices (*See appendixVI*); under the title derivatives there are topics to be covered during class discussion in the research which are intended to address the learning objectives stated as follows. At the end of the discussion of the derivative and its application each student should be able to: State and give examples of important theorems such as the mean value theorem, the fundamental theorem of calculus etc.; Apply the different techniques of differential calculus to investigate the analytic and geometric properties of elementary functions; Formulate and solve properties of the elementary functions; and Formulate and solve practical problems in related rates and extreme values.

**1.2 Statements of the problem**

Nowadays, in the 21$^{th}$ century, Ethiopia has recognized that the development of the country very much depends on the development of science and technology, and hence on Science and Mathematics education. Ethiopia has recently designed a strategy through which 70% of the university/Tertiary level education enrollment would be in science and technology. This scenario has created a unique and challenging situation whereby Science and Mathematics education is put to the spotlight.On the other hand Science and Mathematics education faces numerous challenges that call for immediate improvements (Eshetu, 2009).

In mathematics education as in all areas of education, the problem is quantity: the quantity of facts the students must learn, the quantity of procedures the students must master, and the quantity of concepts the students must understand (Glynna, 1993). According to his contention, the student must acquire this quantity of knowledge within constraints of his or her time and abilities. However, the instructor's understanding of learning theory and effective teaching strategies facilitates or hinders the student's acquisitions of knowledge; as does student management of his or her learning. According to Patel (2003) many people seem to think that mathematics is, for some unknown reason, a purely deductive science. He added that this is not very surprising: mathematics is, in our educational system, often presented as a purely deductive science. Students are not asked to invent or contrive their own mathematical concepts; instead, they are given certain mathematical concepts. Students are no asked to formulate strict and easy definitions of these mathematical concepts; instead, they are given certain definitions.

Knowledge about how to teach mathematics differs in important ways from content knowledge possessed by professionals in other mathematics-related disciplines (Hill et al., 2008). Mathematics teachers must know not only the content they teach, but also how students' knowledge of mathematics is developed and structured; how to manage internal and external representations of mathematical concepts; how to make students' understanding of mathematics visible; and how to diagnose student misunderstandings and misconceptions, correct them, and guide them in reconstructing complex conceptual knowledge of mathematics (Fennema and Franke, 1992; Darling-Hammond, 1999; Cohen and Hill, 2000; Ball et al., 2001).

*"Central to raising student achievement in mathematics is improving the quality of mathematics teaching. Students who receive high-quality instruction experience greater and more persistent achievement gains than their peers who receive lower-quality instruction (Rivkin et al., 2005; Wright et al., 1997)."*

The analytical and synthetic teaching approaches are among the teaching and learning methodologies which possess sound psychological principles that can overcome the limitations of the existing system of

instruction at college level. Thus, these methods need to be examined as to its applicability in teaching the course.

According to Rubio and Valle (2004), the empirical research shows that the use of the analysis ("numerical analysis") of the analytical method of numerical exploration fosters the development of the student's ability to establish and produce meanings for: a) the numerical relationships between the unknowns; b) the relationships between them and the data; and; c) the comparison between two quantities which represent the same in the problem, that is, that they are equivalent regarding their meaning.

However, there is really no basis as to which method is more favorable to the students: Learning the topic analytically or synthetically? Few studies have been made on the comparative effects of analytical and synthetic teaching approaches on students' understanding of mathematics concepts. These results do not show an overwhelming superiority of either method.

The method under consideration is analytic and synthetic approach of teaching mathematics. Analysis and synthesis, as scientific methods, always go hand in hand; they complement one another. Every synthesis is built upon the results of a preceding analysis, and every analysis requires a subsequent synthesis in order to verify and correct its results. In this context, to regard one method as being inherently better than the other, is meaningless. There are, however, important situations in which one method can be regarded as more suitablethan the other. This concerns the question of which method is most appropriate as the primary method or chief point of departure for the study of a given system or object of scientific inquiry (Ritchey, 1996).

Based on the researcher's observation from his more than six years of teaching at Dr.Abdulmejid Hussein College of Teachers Education (Dr. A.H-CTE) and formal and informal discussion with his partners he understood that teacher-educators are more familiar and apply usually the synthetic method than the analytical method. And also, as indicated in Dr. A.H-CTE Business Processing Reengineering (BPR) study paper only15% of the instructors employ participatory approach; this shows most instructors in the college use the conventional teaching approach; The Performance Gap is85%.From this point of view, the researcher became interested in comparing these two methods, namely; analytical and synthetic teaching methods. The researcher also hoped to get results that would clearly support one of the approaches and to find out the most suitable ways to teach mathematics in general and the concepts of Derivatives in particular.

**Research Question:** In the course of the study, the following research questions were formulated and this study hopedanswer:

1. Which method of teaching (analysis and synthesis) is more effective for students to understand the concepts of derivatives? i.e., effect of the two methods?
2. What is the preference/inclination of the students towards the method of teaching used?
3. Do the students' preference/inclination towards the method used affect their achievement?
4. What is the long term (retention) comparative effect of the analysis and synthesis teaching approaches on the students' learning of the concepts of derivatives?

### 1.3. Objectives of the Study

#### 1.3.1. General objective of the study

The general objective of this study was aimed at investigating an instructional strategy that is better suited to effectively yield students' understanding of the concepts of derivatives, specifically, to compare the effects of analytic and synthetic teaching approaches on the students' understanding of the concepts of derivatives.

### 1.3.2. Specific objectives of the study

The researcher intended to explore the following specific research objectives through this limited but highly devoted study:

1. To identify the comparative effects of the two methods on students' understanding of the concepts of derivatives.
2. To examine the preference of the students towards the analytical and synthetic teaching methods used.
3. To investigate the effects of the students' preference towards the analytical and synthetic teaching methods used on their achievement.
4. To compare the long term (retention) effects of the analytical and synthetic teaching methods on the students' learning of the concepts of derivatives.

## 1.4. Hypothesis

To guide the investigation, the following null hypotheses were formulated and tested in this research:

**$H0_1$**. There is no significant difference between the mean achievement scores of the students taught through the analytic and synthetic teaching approaches.

**$H0_2$**. There is no significant difference between the mean preferences scores of students towards the respective method used, namely; the analytic and synthetic teaching approaches.

**$H0_3$**. The students preference towards the respective method used has no significant effect on their achievement.

**$H0_4$**. There is no significant mean difference between analytic and synthetic teaching approaches on the students' retention of the concepts of derivative.

## 1.5. Significance of the Study

There is a strong desire and need to change the way we teach mathematics. The importance of a study that introduces and measures the achievement outcomes for a teaching strategy is enhanced by the fact that efforts have been made every time to update a curriculum and instructional methods to positively affect students' experiences in mathematics. The harmonized curriculum of Ethiopian Higher Education is one indication of this fact.

This study sought to find out the teaching strategy that could increase the conceptual understanding of second year major mathematics diploma students on the concepts of derivative.

Accordingly, the results of this study may have the following significant ramifications on the promotion of the teaching and learning process of mathematics.

- It may help to determine more suitable method of teaching mathematics with respect to the analytic and synthetic approaches.
- The study may provide valuable information to the academic officials and mathematics instructors concerning designing of effective teaching strategies as to preference for analytic and synthetic approaches.
- It may help future researches to identify the relative effectiveness of the analytic and synthetic teaching approaches for better long term learning of mathematics.
- The study may also add to the limited number of empirical studies on analytic and synthetic teaching approaches in advanced mathematics topics.
- Because of the nature of the problem, the study may have ramification on other undergraduate mathematics programs as well as secondary analytic and synthetic teaching approaches were fairly long time ago. So this study can bring a fresh insight about the approaches in a new context.
- The results of this study may serve as a baseline study for further study on this area.

## 1.6. Scope of the study

This study was delimited to second year major mathematics students at Dr. A.H-CTE and to the content elements of derivatives. The main objective is to determine the comparative effectiveness of analytic and

synthetic teaching approaches on students' understanding by considering the concepts of derivatives. The research mainly focused on the comparative study of the two methods rather than other methods mentioned in the curriculum of mathematics courses for diploma program due to the more appropriateness of the methods to teach concept of derivatives. This is because, some studies recommend analytic and synthetic approaches to teach algebra and geometry subjects and the concepts of algebra and geometry are commonly there to teach the concept of derivatives.

The results of this study should have been more conclusive and can have convincing power, if it had included representatives from other sisterly college of teacher's education and other topics from the diploma program course syllabuses. But due to time and budget constraints and little access of resources, the study were limited to only second major mathematics students at Dr. A.H-CTE and to the content elements of derivatives. Moreover, the absence of related research works on the comparative effect of Analytic and Synthetic strategies on students' achievement in Ethiopia was another limiting factor of this research. Thus, the researcher had to relay mainly on the study materials and experiences of other countries sources. All these might have affected the outcome of this research.

### 1.7. Definition of Terms

**Achievement**: the relative performance of the analytical and synthetic group students on the concepts of derivatives determined by the post-test.

**Analysis/**separation into components: "the separation of something into its constituents in order to find out what it contains, to examine individual parts, or to study the structure of the whole " (Ritchey, 1996).

**Inductive reasoning**: Generalizing from a set of examples; "logical thought process that attempts to reach conclusions by reasoning from specific instances or cases to general rules, principles, laws or conditions" (Glynna, 1993).

**Analytical Approach**: According to Trowbridge (1986), it proceeds from unknown to known facts. In this method the problem is analysed to find out the relations. A statement is analysed into simpler statements and then truth is discovered. It is based on inductive reasoning and critical thinking. All the related facts are analysed to seek help in proceeding to the known conclusion.

**Analytical Approach:** In this study, it is considered as a non-conventional approach since its Student–centered Approach; students are active participants; Facilitates students' conceptual understanding, gives opportunity to develop the skills of thinking and reasoning; Student gains confidence and understanding; Method suits the learner and the subject.

**Example**: It is a fact or a thing which represents a general rule of derivative and applications of derivative that the instructor presented to start with and the students discover the general rule or principles or the instructor used to clarify the general rules or principles of the derivative and applications of derivatives. It took of the forms objects, pictures, models, simple diagrams, illustrations, or verbal instances of the happenings being discussed.

**Preference/Inclination**: the view that one person, object, or course of action is more desirable than another or a choice based on such a view. That means the students' teaching or learning preference with respect to the analytic or synthetic approach as identified by the preference questionnaire.

**Generalization**: It is central to both analytical and synthetic teaching approaches. Generalization is used in this study in the sense of that it is the task of summing up in a single statement characteristics, traits, features which are alike or unlike.

**Synthesis**: combining of various components into whole: "the process of combining different ideas, influences, or objects into a new whole" (Ritchey, 1996).

**Deductive reasoning**: Applying a generalization to a particular instance; " logical thought process that attempts to reach conclusions by reasoning from general rules, principles, laws or conditions to specific instances or cases" (Glynna, 1993)

**Synthetic Approach:** It is just opposite to the analytical method. In this method we proceed from known to unknown as synthesis means combing together various parts. In mathematics various facts are collected and combined to find out the result which is unknown (Rehman, 2000). According to Katozai (2002), it is the process of putting together known bits of information to reach the point where unknown information because obvious and true.

**Synthetic Approach:** In this study, it is considered as a conventional approach since its Teacher–centered method, students are passive listeners; Students rely on rote memory; No opportunity to develop the skills of thinking and reasoning, understanding is hampered; Students lack confidence to do other type of sums.

**Retention/permanence:** The relative long term learning of the analytic and synthetic group. students determined by the delayed post-test.
**Understanding**: Something an individual has achieved when he or she handles a concept as a part of mental network.
**Teacher-Educator**: It stands for an instructor teaching in higher institute.
**Teacher:** It stands for teachers who are teaching in primary and secondary schools.

## II. SYNTHESIS OF THE RELATED LITERATURE

The related literatures are used to develop a framework that serve as a guide in conducting of the study. And also, the following basic issues of the study with respect to the analytic and synthetic approaches are captured from the related literature to get clear image of in accomplishing the planned objectives of this study.

"Synthesis ... is the process in which we begin from principles and [proceed to] build up theorems and problems.... while analysis is the process in which we begin with a given conclusion or proposed problem and seek the principles by which we may demonstrate the conclusion or solve the problem."

### I. Procedure

| Approach | Description | Procedure |
|---|---|---|
| Synthesis | Begin with Cause infer Effect | the laws and principles → (Buildup) theorems and problems |
| Analysis | Begin with Effect infer Cause | theorems and problems → (Seek) the laws and principles Or conclusion or proposed problem |

## II. Experimentation

| System | Stimuli | Response |
|---|---|---|
| Teaching Approaches and Concepts of Derivative | Synthesis/Analysis Approaches | Preference/Retention/Achievement |

(*Source: Richey, 1996*)

The following conceptual frame work is drawn to get insight of the right path that leads towards the final destination of this study.

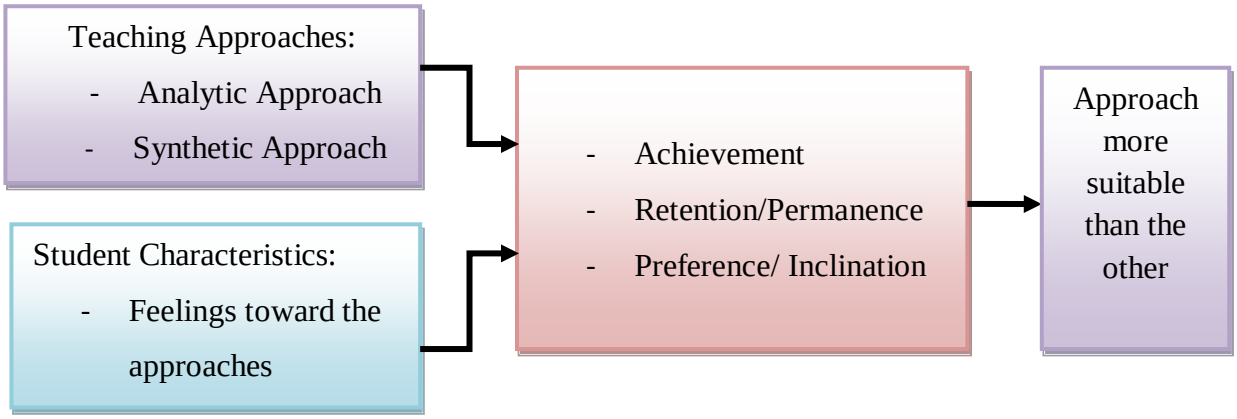


Figure 2. Framework of the study

## III. RESEARCH METHODOLOGY

This section discusses the method employed, subjects of the study, research design, tools used to collect data, treatments and method of data analysis and interpretation to address each research question.

To analyse the comparative effectiveness of both the analytical and synthetic methods, an experiment was conducted in the class of second year Mathematics students in section A and B in the Dr. A.H-CTE.

Formerly, it was proposed two instructors to teach both the control and experimental groups, but upon the advisor comment the researcher himself taught both groups by using analytic and synthetic. It was thought that no two individuals have absolutely equal efficiency and effectiveness in applying same method and illustrating his/her knowledge to deliver a lesson in two different classes as far as human nature concerns.

### 3.1. Description of the Study Area

Dr. Abdulmejid Hussein College of Teachers' Education (Dr. A.H-CTE) is governmental memorial Teachers Education Institute (TEI) which is after called the famous former Ethiopian ambassador to UN, Dr. Abdulmejid Hussein. It is located at Jig-jiga town in Somali regional state of eastern Ethiopia. Jig-jiga town presently serves as the seat of the Somali Region Government State. It is located 638 km east of Addis-Ababa, close to the Somalia border. The official language is Af-Somalia, but Oromiffa, Guragigna and Amharic are also spoken. Dr. A.H-CTE has established since 1999 with the mission to produce active and creative, knowledge proficiency, skillful and responsible primary school teachers who can actively participate in the building of education system particularly in the region and the country in general.

### 3.2. Subjects of the Study

The target population of the study was Second year students attending Mathematics as a major study area in Dr. A.H-CTE found in Jig-Jiga town of Somali National Regional State, Ethiopia. The target population in each section was 41 and they were a total population of 82 students (See table-1). They enrolled for the course Introduction to Calculus (Math162) during the research time (the academic year 2019/20). This course was offered in four credit hours and five contact hours per week. To conduct the study, the population of students kept in their original sections as they were, but the sections were assigned into control and experimental groups by drawing to avoid the biasness. The table below summarizes the number of students that took part in the study.

Table 1. Second Year Mathematics Students in each Section taking part in the study

| **Departments** | **Section** | **# of Students** | | |
|---|---|---|---|---|
| | | **M** | **F** | **Sub-total** |
| Mathematics | A | 41 | - | 41 |
| Mathematics | B | 41 | - | 41 |
| **Total** | | **82** | **-** | **82** |

### 3.3. Research Design

**Design of the Study**

The research design was experimental research that consists of independent and dependent variables. The independent variables were analytic and synthetic teaching approaches (instruction methods). The dependent variables were students'conceptual understanding of derivatives and the students' preference towards the two methods applied in the study. For this purpose two equivalent groups were formed. A Pre–DAT (Pre Derivative Achievement Test), Post-DAT (Post Derivative Achievement Test) and RET-DAT (Retention Derivative Achievement Test; delayed post-test) experimental-control group design was used in this research.

The control group was taught by synthetic method while experimental group was taught by analytical method and both the groups were kept away from each other so that one could not influence the performance of other. The duration of the classroom teaching was fifty minutes with Cr.Hr./Ct. Hr.= 4/5 and seven weeks for whole experiment. Design of the experimental procedure is depicted by the following table.

Table 2. The research design: Pretest- posttest and delayed post-test group design

| **Section** | **Group Name** | **Test-1** | **Treatment** | **Test-2** | **Test-3** | **Questionnaire** |
|---|---|---|---|---|---|---|
| **C** | Control group | Pre–DAT | Synthetic Approach | Post-DAT | RET-DAT | Preference |
| **E** | Experimental group | Pre–DAT | Analytical Approach | Post-DAT | RET-DAT | |
| **TIME** ⟶ | | | | | | |

The table shows the research design: **Pretest- posttest and delayed post-test group design.Section C and E represents the analytical and Synthetic groups, respectively.** The course content for both the groups is same but the only difference between the two groups was the method of teaching.

### 3.4 Instrumentation

As seen in the framework (see table above), there were three major issues to be addressed in the study: **achievement**, **preference** and **retention**. Hence, instruments were designed make sure that data was

collected to address the issues. All the data used in this research was primary data. Since there were no standardized instruments regarding the research design, the tools used in this research were developed by the researcher along with the mathematics department staff members at Dr. A.H-CTE and considering the theories and principles of analysis and synthesis approaches reviewed from the related literature. All the instruments used to collect the data were pilot tested to establish the reliability and mathematics department staff members ascertained their validity before being used for data collection.

**Tests**

Tests were constructed to know the prior knowledge before the experiment, the relative achievement and long term learning after the treatments of the students in the two groups. The detail explanations of the different tests are presented as follows.

i. **Pre-DAT** :-This is a test that was given to the two groups to check the prior knowledge of the students about the concepts of derivatives and its applications and to equalize the two groups on their previous knowledge about the given contents (see Appendix-I). It was prepared by the researcher in collaboration with the mathematics department staff members. The total number of question items was 14 of each question item is worth three points and the total possible maximum point is 42 (see Appendix –). The pre-test was pilot tested on 20 third year students of mathematics department at Dr.A.H-CTE to establish its reliability using Kuder-Richardson reliability coefficient or Cronbach alpha, α. It was also validated by group of mathematics department staff members at Dr. A.H-CTE. A researcher make Pre DAT was distributed among the students to analyse the performance of students before the experiment.

ii. **Post-DAT:-**This is a test that was given to the two groups after completing the treatment to assess their comparative achievements of the covered concepts in the experiment (see Appendix-). The Post-DAT was prepared by the researcher in collaboration with the mathematics department staff members at Dr. A.H-CTE which covers the whole unit to be discussed during the experiment. The total number of question was 18 of each question item is worth three points and the total possible maximum point is 54 (see Appendix-VII). It was pilot tested on 24 second year students of mathematics department at Harar College of Teachers Education (**HCTE**). Its reliability was calculated using Kuder-Richardson reliability coefficient or Cronbach alpha, α. A table of specification regarding the objectives of the syllabus according to the Bloom's taxonomy of educational objectives (1956) was used to check content validity of the test. Difficulty level and way of constructing the statements of the test items was also validated by the group of mathematics instruction. The research advisor also commented it regarding its type and composition. When we use a table of specifications for the objectives, this becomes a matter of matching test items with the specified objectives in the table. After conducting the experiment, a researcher make post testwas distributed among the students of two groups.

iii. **Ret DAT/Delayed post-Test**:- The delayed post- test was administered after the experiment had been completed about three weeks later together with the mid-term examination of the course compiled with items from the subsequent derivative chapter to see the comparative long term/retention effects of the analysis and synthesis teaching approaches on the students' learning of the concepts of derivatives (see Appendix-). Basically, the fundamental structures of the Post-DAT and Ret DAT (delayed post-test) are the same. In order to avoid memory effect, the two tests were not exactly the same, but only some difference among the question items. The delayed post-test was a part of the students' course examination. It was to measure whether there was any difference in the students' learning between the teaching experiment with the post-test and later acquisition during the course or during self study.

iv. **The Questionnaire**:-This was used to measure the student's preference with respect to the teaching method used (the analysis or synthesis approach) during the experiment (see appendix-). It was also used to investigate whether the students understanding of the concepts they taught was affected by their preference towards the respective method of teaching used for each group. The questionnaire was

developed by the researcher based on the principles of analysis or synthesis learning style preference. Efforts had been made in designing the questionnaire so as to exactly express the students learning and teaching preference corresponding to the analysis and synthesis approaches. It has 22 items and constructed in 5-point Likert scale from strongly disagree (1) to strongly agree (5). It was pilot tested for 24 second year mathematics department students at HCTE and its reliability was estimated using Kuder-Richardson reliability coefficient or Cronbach alpha, $\alpha$. The contents of the questionnaire were validated by group of mathematics instructors.

v. **Systematic Classroom Observation Checklist:** -This was used to collect information about the students' active involvement in the teaching and learning process in the two groups through check lists. A check list was prepared by the researcher together with the staff members of mathematics department at Dr. A.H-CTE in such a way that the researcher can observe students participations, attaining the lesson, asking questions, giving a response for the asked questions, their interest to learn etc in the two groups and it consisted of 10 items. The class room observation was taken place for eight successive classes in both groups by the researcher and each item in the check list was observed eight times.

### 3.5. Treatment

Leibniz was among the first to define analysis and synthesis as modern methodological concepts: "*Synthesis* ... is the process in which we begin from principles and [proceed to] build up theorems and problems, ... while *analysis* is the process in which we begin with a given conclusion or proposed problem and seek the principles by which we may demonstrate the conclusion or solve the problem."(Cited by Ritchey, 1991)

Experimental group was treated by analytical approach, and students were more active participant in the teaching–learning process. And also, Control group was treated by synthetic approach; this is the conventional way of teaching mostly encourages rigor memorization of formulas/ principles or etc, specifically a more teacher-driven lesson, textbook problems solved by their teacher. The role of the student just they receive information from their teacher.

The two classes were receive instruction on the same material: Definition of derivatives, Differentiable functions and differentiation on intervals, Derivatives of combination of functions, The chain Rule, Higher derivatives, Implicit differentiation, Related rates, Max-Min theorem (Extreme value theorem), Graphical discussion of the hypothesis of the max-min theorem, Rolle's and the Mean value theorem, Applications, Monotonic functions, The first and second derivative tests, Extreme value problems. The contents are taken from the Harmonize Curriculum for Linear Diploma program for College of Teachers Education in Mathematics of Ethiopia (2009). The same textbook is used in both classes and the same assignments and exercises are given. Both classes received 3 and ½ weeks instruction on the unit. There was four contact/class hours per week. The length of each class time was 50 minutes per- period, four days per week and the overall experiment was completed in seven weeks.

### 3.6. Methods of Data Analysis

This section describes us how the quantitative data which are collected by the researcher are summarized or described. The quantitative data was collected, coded, tabulated, analyzed, described and interpreted subjecting to both descriptive and inferential statistics.

For the related analysis of test results of the students descriptive statistics (such as mean, standard deviation etc.) was used. Data was also analyzed using inferential statistics to test the stated null hypotheses and according to the collected data appropriate statistical tools and computer software package (*i.e.* **SPSS**16.0 for windows) was used.

**The Pre-DAT:** The independent sample t-test was utilized to examine whether there is a significant mean difference between the experimental group and the control group scores of the test. A t-test comparison between the two groups based on their Pre-DAT results was performed to check whether there is any statically significant prior knowledge difference about the concepts of derivative.

**The post-DAT:** - Here to analyze the results of the Post DAT the researcher made two ways of analysis. First the results of post-DAT of both groups were compared by using t-test to see which method lead to better immediate learning of the given concepts. Second ANOVA is also performed to statistically control the pre-test and to refine further the result of the post-test for safe conclusion about the relative effectiveness of the analytical and synthetic teaching approaches.

**The preference rated scale**: The mean difference of the students' preference scope on the rated scale was compared by using t-test. Linear Regression analysis was used to determine the effect of the students' preference towards the respective teaching approach used on their performance in the post-test. It was also analyzed using Mann-Whitney U-test to compare the two groups on their mean score difference of each item.

**Classroom observation:** The systematic classroom observation was analyzed using frequency distribution method.

**The delayed post-test**: The researcher also looked at the results of the delayed post-test to see which group had reached higher scores by the time. The results of the post-test and the delayed post-test were also compared applying the paired t-test to see which approach leads to better long-term acquisition with in groups.

### 3.6.1. Reliability calculation

Reliability is the internal consistency of the scores obtained. A test considered meaningful if it produces consistent or reliable result. The reliability of the test would be calculated by calculating coefficient alpha ($0 \leq \alpha \approx 1$) which is a measure of internal consistency of an exam. This coefficient ($\alpha$) is a general form of Kuder-Richardson reliability coefficient (Fraenkel&Wallen, 1996) with has the form:

$$\alpha = \frac{k}{K-1}\left[\frac{1-\sum pq}{\delta 2}\right]$$

Where:

**α** is Kuder-Richardson reliability coefficient

**p** is the proportion of students passing giving the correct answer

**q**is the proportion of students that did not give the correct answer

**δ** is the variance of the total score on the assessment

**K** is the number of items on the test

## IV. RESULTS AND DISCUSSSIONS

In this chapter results of the data collected from the students by means of the different data collecting tools: tests, questionnaire and systematic classroom observation are presented, analyzed, interpreted and discussed in detail using both descriptive and inferential statistics. The results obtained from the tests, the questionnaire and the systematic classroom observation check lists are presented.

### 4.1. Analysis and Interpretation on Achievement

The pretest and posttest were used to measure the students' performance on the concepts of derivatives before and after the treatment. The pre-test was given before the experiment had begun and the post-test was

given upon the completion of the experiment to see which method resulted in better achievement of the students on the learned concepts.

Under this section, both descriptive and inferential statistics were used to answer the research questions and to test the stated null hypothesis based on the results obtained from the pre-test and post-test.

### 4.1.1. Descriptive Statistics

Descriptive statistics is presented here to identify the summarized means, standard deviations and standard error means of the students' achievement on the concepts of derivatives as measured by the pretest and posttest of both groups. Since the maximum score values from the pre-test and post-test were different, the results are converted in to percentage to see the mean gain difference of the two groups.

**Table 1. Descriptive statistics related to the percentage of scores on the pretest and posttest in both groups**

| Test | Group | N | Mean% | SD | S.E.M |
|---|---|---|---|---|---|
| Pretest | Analytical | 41 | 20.79 | 8.84 | 1.35 |
| | Synthetic | 41 | 20.10 | 7.16 | 1.09 |
| Posttest | Analytical | 41 | 56.56 | 12.36 | 1.89 |
| | Synthetic | 41 | 50.81 | 12.29 | 1.87 |
| Difference | Analytical | 41 | 35.77 | 15.48 | 2.36 |
| | Synthetic | 41 | 30.71 | 7.24 | 2.35 |

SD=Standard Deviation S.E.D=Standard Error Mean

As can be seen from Table 1, the mean scores of the students on the pre-test in both groups were almost the same with the analytic group scored a mean of 20.79 percent with standard deviation 8.84 where as the synthetic group scored a mean of 20.10 percent with standard deviation 7. 16. This result indicates that the students' achievement on the concepts of derivatives before the treatment was the same. Similarly, the means score of the analytic group on the post-test was 56.56 percent with standard deviation 12.36 and the mean score of the synthetic group on the post-test was 50.81 percent with standard deviation 12.29. In the post-test, the means score of the analytic group was higher than the synthetic group. A mean score difference between the pre-test and the post-test was also observed in both groups (35.77 percent for analytic and 30.71 percent of synthetic ) in which theanalytic group students demonstrated better improvement than the synthetic group students.

The standard deviation of the scores in the pre-test of both groups was very small as compared to the standard deviation of the scores in the post-test of both groups. This indicates that there was no great variation among the students on their prior knowledge on the given concepts. But after the experiment, the actual learning took place hence the variation of scores among the students.

### 4.1.2. The pre-test and post-analysis using t-test

In this section, the independent sample t-test was used to check the existence of significance mean difference between the students in the analytic and synthetic groups based on the pre-test and post-test results at probability level of $\alpha = 0.05$. Mean achievement and mean difference scores of the students in each group were analyzed and interpreted based on the t-test results (see Table 2).

According to Morgan, et. at (2004), when investigating the difference between two unrelated or independent groups (in this case the analytic and synthetic groups) on an approximately normal dependent variable, it is appropriate to choose an independent samples t-test if the following assumptions are not markedly violated.

1. The variance of the dependent variable in the two populations is equal.
2. The dependent variable is normally distributed within each population.

3. The data are independent (scores of one participant are not related systematically to scores of others).

**Table 2.Independent Samples t-test Comparison between Analytic and Synthetic Groups on the pre-test**

| Group | N | Mean | SD | t | MD | P | df |
|---|---|---|---|---|---|---|---|
| Analytical | 41 | 8.73 | 3.58 | .384 | .29 | .703 | 80 |
| Synthetic | 41 | 8.44 | 2.96 | | | | |

Based on the analyzed data results displayed in Table 2, the students' prior knowledge about the concepts of derivatives was identified by the pre-test which was given just before the experiment had been implemented. Independent sample t-test was used to determine significance of the mean difference scores of the two groups, and it was interpreted as follows:

The mean score of students in the analytic group was 8.73 (SD = 3.58), and the mean score of students in the synthetic group was 8.44 (SD = 2.94) at (t= .403, MD = .29, P >.05). This shows that students in the two groups had no significant prior knowledge difference on the concepts of derivatives. Results from the pre-test suggest that the two groups were equivalent with respect to their prior knowledge on the selected concepts.

The null hypothesis regarding the first research question was stated as:
$H0_1$. There is no significant difference between the mean achievement scores of the students taught through the analytic and synthetic teaching approaches.

**Table 3. Independent Samples t-test Comparison between analytic and synthetic Groups on Post –test**

| Group | N | Mean | SD | t | MD | P | df |
|---|---|---|---|---|---|---|---|
| Analytical | 41 | 30.54 | 6.43 | 1.942 | 3.1 | .039 | 80 |
| Synthetic | 41 | 27.44 | 7.24 | | | | |

In order to determine the comparative effects of the analytic and synthetic teaching approaches on the students' understanding of the concepts of derivatives and achievement test was given to the students in the two groups few days later after the experiment had been completed and their mean score difference was compared using t-test (see Table 3). The mean scores of the students in the analytic group was 30.54 (SD = 6.43).

And the mean scores of the students from the synthetic group was 27.44 (SD=7.24). The results in Table 3 convey that, even if the difference is not to that much exaggerated, there is a significant mean difference (MD=3.1) in the scores of the students between the analytic and synthetic group at P<.05. Hence, the stated null hypothesis is rejected and the alternative hypothesis is confirmed. From this result it can be concluded that analytic teaching approach has greater effect than the syntheticapproach on the students' understanding of the concepts of derivatives. Students taught by the analytic teaching approach performed significantly better than students taught by the synthetic teaching approach.

### 4.1.3. Analysis of Covariance (ANCOVA) for the Pretest and Posttest using Pretest as Covariance

The analysis of covariance was used to assess whether the mean difference between the analytic and synthetic groups on the post-test score was significant when the scores in the pre-test was statically controlled. The post-test was as dependent variable, the treatment used with respect to both groups as fixed factor and the pretest as covariate.

ISSN No:-2456-2165

**Table 4. ANCOVA- analysis of the pre-test and post test using the pre-test as covariance**

| Source | Dependant Variable | Type III Sum of Square | df | Mean square | F | Sig. |
|---|---|---|---|---|---|---|
| Corrected Model | | 787.507(a) | 2 | 393.753 | 2.576 | .086 |
| Intercept | | 37734.726 | 1 | 37734.726 | 246.903 | .000 |
| Group | Post-test | 657.718 | 1 | 657.718 | | .041 |
| Pre-test | | 150.809 | 1 | 150.809 | | |
| .323 | | | | | | |
| Error | | 12685.051 | 79 | | | |
| Total | | 258664.000 | 82 | | | |
| Corrected Total | | 13472.558 | 81 | | | |

Significance is at $\alpha = .05$

Table 4 indicates that the treatment (both analytic and synthetic approaches) has highly significant effect F (1, 79) = 4.304, P < .05. However, the covariate (pre-test) is no longer significant F (1, 79) = .987, P > .05. After controlling the difference on scores of the pre-test, there was still a significant difference between the students in the analytic and synthetic groups in their score in the post-test. This result suggested that performance on the post-test was significantly and uniquely influenced by the treatment after accounting its effect on the pre-test score.

### 4.2. Analysis and Interpretation on Preference

Through this questionnaire the researcher sought to identify the students' attitudinal dispositions towards the analytic or synthetic teaching-learning approaches accompanied by the given treatments. He wanted to know how the students experience (feel) about their learning preference with respect to the analytic and synthetic approaches.

A questionnaire, prepared by the researcher, was used to assess the students' teaching-learning preference with respect to the analytic or synthetic approaches. It comprised of 22 items of which 11 times are for analytic approach and 11 items are for synthetic approach (see Appendix- IV). So that from this arrangement of the items the students can choose their preference without any hesitation between the given alternatives.

Since the analytic and synthetic approaches are pair-wise opposite dimensions, the items are arranged pair-wise and compiled in one paper. They were rated in five-point Likert-type scale: Strongly Agree, Agree, Natural, Disagree, and Strongly Disagree. The items were coded as Strongly Agree=1, Agree=2, Neutral=3, Disagree=4 and Strongly Disagree=5 for the items with the synthetic approach connotation. But the way of coding for the items with the analytic approach implications were reverse. This way of coding has the implication that if a student is strongly agree with the statement for the synthetic approach implication, then she/he is strongly disagree with the statement which has analytic approach implication with the same idea.

The scores of the twenty two statements were summed to get a total score for each student in both groups and the descriptive Statistics for the total score is presented in table 5. In reporting and interpreting the descriptive statistics presentation of the preference rated scale towards the analytic or synthetic teaching-learning preference score was classified as follows.

The average score of each student for all the items in the rated scale was calculated. Then, if the average score of the student is above 3 it was labeled as "proponent of analytic", if it is below 3 "proponent

of synthetic" and if it is 3 "neutral". Based on this result, to describe the frequency of number of students who responded for the rated scales of the preference a cross tabulation was used.

Under this section it was analyzed extensively in both qualitative and quantitative way to compare the mean scores preference rated scale with respect to each learning preferences of the two groups. The total mean score of the students' preference for each item in the two groups was also compared by U-test.

**Table 5. Preference *Group Cross tabulation**

| Group | Preference | | | Total |
|---|---|---|---|---|
| | Analytic | Neutral | Synthetic | |
| Analytic | 41 | 0 | 0 | 41 |
| Synthetic | 8 | 3 | 30 | 41 |
| Total | 49 | 3 | 30 | 82 |

Based on the result displayed in Table 5, it is observed that 41 students from the analytic group preferred the analytic approach and no student from the analytic group preferred the synthetic approach. Three students from the synthetic group responded "Neutral". Thirty (30) students from the synthetic group preferred the synthetic approach and eight students from the synthetic group preferred the analytic approach. The result showed that the intervention during the experiment created a great influence on the students' preference with respect to the analytic and synthetic approach. From this it can be concluded that both the analytic and synthetic teaching methods were suitable approaches for the students to learn the concepts of derivatives.

**Table 6. Analysis of the preference rated scale scores using frequency, percentage and U-test**

| Item | Group | No. | Mean | U- test |
|---|---|---|---|---|
| X1 | Analytic | 41 | 4.2927 | .026 * |
| | Synthetic | 41 | 3.8293 | |
| X2 | Analytic | 41 | 2.7561 | .000 * |
| | Synthetic | 41 | 1.6341 | |
| X3 | Analytic | 41 | 4.4634 | .039 * |
| | Synthetic | 41 | 4.1951 | |
| X4 | Analytic | 41 | 2.6341 | .000 * |
| | Synthetic | 41 | 1.3415 | |
| X5 | Analytic | 41 | 4.5610 | .020 * |
| | Synthetic | 41 | 4.2927 | |
| X6 | Analytic | 41 | 2.8049 | .000 * |
| | Synthetic | 41 | 1.5854 | |
| X7 | Analytic | 41 | 4.4878 | .303 |
| | Synthetic | 41 | 4.3659 | |
| X8 | Analytic | 41 | 2.7317 | .000 * |
| | Synthetic | 41 | 1.4390 | |
| X9 | Analytic | 41 | 4.3659 | .660 |
| | Synthetic | 41 | 4.2927 | |
| X10 | Analytic | 41 | 2.8293 | .000 * |
| | Synthetic | 41 | 1.4634 | |
| X11 | Analytic | 41 | 4.5610 | .457 |
| | Synthetic | 41 | 4.4634 | |
| X12 | Analytic | 41 | 2.5610 | .000 * |

| | | | | |
|---|---|---|---|---|
| | Synthetic | 41 | 1.6341 | |
| X13 | Analytic | 41 | 4.5610 | .004 * |
| | Synthetic | 41 | 4.0732 | |
| X14 | Analytic | 41 | 2.5122 | .000 * |
| | Synthetic | 41 | 1.4878 | |
| X15 | Analytic | 41 | 4.5366 | .088 |
| | Synthetic | 41 | 4.3415 | |
| X16 | Analytic | 41 | 2.7073 | .000 * |
| | Synthetic | 41 | 1.7317 | |
| X17 | Analytic | 41 | 4.5854 | .022 * |
| | Synthetic | 41 | 4.2683 | |
| X18 | Analytic | 41 | 2.6829 | .000 * |
| | Synthetic | 41 | 1.3171 | |
| X19 | Analytic | 41 | 4.5610 | .022 * |
| | Synthetic | 41 | 4.2439 | |
| X20 | Analytic | 41 | 2.5610 | .000 * |
| | Synthetic | 41 | 1.5366 | |
| X21 | Analytic | 41 | 4.5122 | .070 |
| | Synthetic | 41 | 4.2439 | |
| X22 | Analytic | 41 | 3.1463 | .000 * |
| | Synthetic | 41 | 1.6098 | |
| **Grand Total** | **Analytic**<br>**Synthetic** | | **3.6098**<br>**2.8814** | |

Source: Data collected and analyzed

Since the analytical and synthetic approaches are pair-wise opposite dimensions, the 22 items analyzed pair-wise as disclosed in Table 6 and interpreted as follows.

**Items X1 and X2**

Table 6 indicates that the mean score of the students in both groups for item 1 which says "When I solve problems on derivatives, I often just see the solutions but then I have to struggle to figure out the steps to get to them" was inclined to the analytic approach connotation of the rated scale of the statement. The mean score of the students to the rated scale in the synthetic group exceeds the mean score of students in the analytic group (Mean =3.39 for analytic and Mean = 3.66 for synthetic groups). The students reflected their strong agreement by indicating their response to the reverse statement of this item (X2) the mean score for the rated scale to be 3.32 for the analytic group and 3.32 for the synthetic group. This shows both groups scored equal mean for the statement which is inclined to synthetic approach implication. The nonparametric independent sample of Mann-Whitney U-test also reveals that there is no significant mean score difference between the analytic and synthetic group students (U = .000 for X 1 and U = 1.000 for X2). The students in both groups have the judgment that they have to figure out all necessary steps in solving mathematical problems so as to accept the solution is correct.

Regarding to this situation, Prince and Felder (2006) stated that a better way to motivate students is analytic/inductive teaching method, in which the instructor begins with specific, such as experimental data to interpret, a case study to analyze, or a complex real world problem to solve. Students grappling with these challenges quickly recognize the need for facts, skills and a conceptual understanding, at which points the teacher provides and helps students figure things out for themselves. Cornu (1991) assured that students seem to be very pragmatic when they learn about limits. They focus on the problem solving not so much on the theory. This shows that it is needed to direct efforts tothose activities which favor students' deep

comprehension of mathematical concepts rather than to the mere transmission of formal definitions and using them blindly to solve problems.

**ItemsX3 and X4**

The mean score of the students response for X3, which says "It is more important tome that an instructor lay out the material in clear sequential steps", was3.44 for analytic and 3.56 for synthetic groups (Table 6) and for its reverse statement was3.37 for analytic and2.61 for synthetic groups (item X 4). From this it can be observed that students in both groups put their judgment that it would be better for them if an instructor presented his material in clear sequential steps. But the responses of the students for the syntheticconnotation of the statement was observed a considerable difference between the analytic and synthetic group students in which the synthetic group students need an instructor to give them an over-all picture of the materials first. The U-test shows that mean score difference of the students in both groups has no significant difference for theanalytic connotation of the statement (U= .059), but there is significant difference for thesynthetic connotation of the statement (U = .000). The reason for observing considerable numbers of students in the synthetic group agreed with both dimension of the statement might be due to the synthetic intervention during the experiment. In light of this, when the concept of derivatives considered in the analytic treatment students meet notion of derivative in an informal intuitive way where the tasks are from situations where they can easily see the outcome. This can create a feeling of control and the students think they know what the concept is about even if they could not solve in more demanding task where they would have to master meaning of the formal definition.

**ItemsX5 and X6**

The mean score of the students for item 5 was 3.34 for analytic and 3.02 for synthetic group (Table 6) and for its reverse statement was 3.66 for analytic and 2.40 for synthetic group (item X6). The U-test ascertained that there is no significant mean score difference between the analytic and synthetic group students for the analytic approach connotation of the statement in which the synthetic group students responded a neutral preference, but a significant mean difference was observed for the deductive/synthetic connotation of the statement (U = .000 for item X5 and U = .000 for item X6). This result shows that the students in the analytic group reflected their tendency towards the idea of the statement that calls for detail information, while the students in the synthetic group was eager to know the big picture of a given information, missing its detail. Regarding these approaches, Felder (1988) suggested that a student favoring the sequential learning style would respond well to an instructor who presents information in a step-by-step fashion. It also follows that a student favoring the global learning style would respond well to an instructor who presents information in a holistic (big-picture) fashion.

**ItemsX7 and X 8**

The mean score of the students for X7, which says "When I have to perform a task on derivative, I prefer to come up with new way of doing it", was 3.61 for analytic and 3.10 for synthetic groups (Table 6) and for its reverse statement 3.51 was for analytic and 2.49 for synthetic (item X8). The students in both groups responded in supporting the analytic connotation of the statement, but a significant mean difference was observed in their response for the synthetic connotation of the statement by which the analytic group students oppose the analytic connotation and the synthetic group students support the synthetic connotation of the statement (U = .000 for X7 and U= .000 for X8).

For the synthetic implication of the statement by which the analytic group students oppose the synthetic implication and the synthetic group students support the analytic implication of the statement. This has the implication that the students are more satisfied if they involved in an innovative way of solving a mathematical problems rather than solving a problem by substituting to the existing formula. It can have the implication that a highly structured presentation of the contents arose students' interest too.

**ItemsX9 and X10**

ISSN No:-2456-2165

Students in the analytic and synthetic groups responded for X9, which says "I prefer contents that emphasize concrete material (facts, data)", of mean 4.02 and 3.90 respectively (Table 6) and on its reverse statement was 3.37 for analytic and3.37 for syntheticfor the rated scale of the statement. Both groups scored equal mean for the statement inclined to synthetic implication (X10). Students in both groups arose their interests when they were provided with concrete materials, i.e. facts, data. The nonparametric U-test compilation reveals that there is no significant mean difference in both dimension of the statement (U = .000 for X 9) and there is no significance mean difference between the analytic and synthetic group students ( U = 1.000 for item 10). Regarding the idea of this item, Applebaum (2001) expressed the power of inductive/analytic method of teaching to develop students' insight learning. According to his contention the advantage of analytic method of teaching lies in its maximum vicinity and in the fact that it develops intuition and creates condition for the insight and impressing rise by providing concrete materials. The analytic method of teaching can considerably activate the students' creative activity.

**Item X11 and X12**

The students' response for X11 reflect a remarkable difference on their standpoint regarding the relative presentation of example first then the generalization or the generalization first then example by the mean scoring of 3.83 the analytic and 2.44 the synthetic groups (Table 6) and 3.49 the analytic and 2.15 the synthetic groups (X12). The non-parametric independent sample of Mann-Whitney U-test also reveals that there is significance mean score difference between the analytic and synthetic group students (U=.000 for X11) and there is no significance mean difference between the analytic and synthetic group students (U= 1.000 for X12). The students in the analytic group preferred deriving the rules for themselves from examples where as the students in the synthetic group favored being given the rule first. The result of this item conveys that the students in both groups were highly influenced by the intervention of the methods used. The students in the synthetic group were interested with examples after the presentation of the generalization to sense it in particular case.

**Items X13 and X14**

The students' response for X13, which says "When I learn derivative, I prefer to learn the rules as I go along", reflected in the way that they were treated by scoring of mean 3.85 for the analytic group and 2.71 for the synthetic group students (Table 6) and on its reverse statement was 3.29 for analytic and 2.56synthetic (X14) for the rated scale of the statement. It is obtained a significant mean difference between the two groups in both dimensions of the statement (U = .000 for X 13 and U = .000 for X 14). Both analytic and synthetic groups preferred the statement inclined to the respective approach in which they were taught.

**Items X15 and X16**

The students in both groups responded for the statement "I prefer to learn the concepts of l derivatives in the sequence of Specific examples, General rule, Practice" (X15) of mean 3.78 for analytic and 2.51 for synthetic group (Table 6). The students in the analytic group gave a neutral response for the reverse of this statement (X16) with mean score 3.02where as the students in the synthetic group supported it with mean score 2.32. In both dimensions of the statements, there is a significant mean difference between the two groups (U = .000 for X15 and U = .000 for X16). This indicates that the students in both groups were influenced by the given treatments.

**Items X17 and X18**

For X 17, which says "I understand the concepts of derivatives better when the teacher presents a whole range of examples", the students I both groups favored it with mean scores 3.90 for the analytic and 3.37 for the synthetic group (Table 6). Even if both groups supported the given statement, a significant mean difference is observed on their score on the rated scale (U = .022 for X17). The students reflected their opposition for the synthetic connotation of the statement (X18) with mean score 3.34 for the analytic group and 3.05 for the synthetic group. Again, a significance mean difference is obtained between the two groups (U= .000 for X18) for rated scale statement inclined to synthetic implication. Example in mathematics

education provides some thing as raw material for inductive reasoning, as a particular instances for a generality, and creates an environment for practice.

According to Watson and Mason (2005) ideas, key feature of examples is that they are chosen from a range of possibilities and it is vital that learners appreciate that range. According to these authors contention whenever students encounter a mathematical statement that is not immediately obvious, the natural thing to do is to construct or call upon an example so as to see the general through intimate experience of the particular. Examples in mathematics context help the students to detect and express a structured essence which lies behind several apparently different situations and out of these can arise a new unifying concept and an associated collection of definitions and theorems.

**ItemsX19 and X20**

The students in the analytic and synthetic groups scored of mean 3.85 and3.00 respectively on X 19 (Table 6), which says "I develop a clear image about the idea of the content, when I proceed from specific to general", and their mean scores on the reverse idea of the item is 3.07 for analytic and 2.44 for synthetic group (item X20). Significant mean difference is detected on both items,X19 (U =.000) and X 20 (U = .000.070) between the two groups.

**ItemsX21 and X22**

For X 21, which says "I feel most engaged, when I provide with more examples, practical problems and observational data to interpret", scored a mean of 3.66 for the analytic group and 2.51 for the synthetic group (Table 6). The analytic group students preferred the given statement but the synthetic group students did not. For the synthetic dimension of the statement, the synthetic group students preferred but the analytic group students did not by scoring mean of 2.34 for the synthetic group and 3.63 for the analytic group (X22). In both dimensions of the statement, a significant mean difference is obtained between the two groups (U =.000 for X21 and U = .000 for X22)

Generally, the students in both groups biased for their preference towards the approaches in which they were treated in the experiment. The grand mean of the total mean scores of each items ascertained this fact (Grand mean = 3.54 for analytic and Grand mean = 2.86 for synthetic)

The null hypothesis regarding the students in both group means score difference on the preference rated scale was stated and tested as follows.

$H0_2$. There is no significant difference between the mean preference-scores of students towards the respective method used.

**Table 7. Independent t-test comparison of mean scores on preference rated scale towards the analytic or synthetic approaches in both groups.**

| Group | N | Mean | SD | t | MD | P | df |
|---|---|---|---|---|---|---|---|
| Analytical | 41 | 79.41 | 2.49 | 21.501 | 16.02 | .000 | 80 |
| Synthetic | 41 | 63.39 | 3.67 | | | | |

As it can be observed from Table 7, the mean scores of the students in the analytic group on the preference rated scale was 78.09 (SD = 6.36) and the mean scores of the students in the synthetic group on the preference rated scale was 63.19 (SD = 2.05). Table 7 conveys that there is significant mean difference between the students' score on the preference rated scale towards the analytic or synthetic approaches (t = 14.63, MD = 14.91, P = .000).

As it can be observed from Table 7, the means scores on the students' preference show that the analytic group students preferred the analytic approach where as the synthetic group students preferred the synthetic

approach. This result agrees with result presented in Table 5 result reveals that the students in both groups favored the respective method implemented in the treatment.

### 4.3. Analysis and Interpretation on Preference and Achievement

A null hypothesis regarding the third research question was stated and tested as follows:
H03. The students' preference towards the respective method used has no significant effect on their achievement.

The Linear Regression analysis here is used to determine where he teaching methodologies used with respect to both groups could significantly predict the achievement of the students in the post test over and above the effect of the students' preference towards the respective methodologies. In other words, it is intended to identify whether the students' preference towards the respective method used was significantly affected or not for the students' mean difference scores in the post test.

In order to investigate the extent to which the preference variation towards the respective approaches employed was attributable in performance of the post-test, Linear Regression was performed.

**Table 8. Summary Table for Regression of Preference on Post-test**

| Mode 1 | R | R Square | Adjusted R Square | Std. Error of the Estimate | Change Statistics | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | R Square Change | F Change | df1 | df2 | Sig. F Change |
| 1 | .179(a) | .032 | .02 | 6.714 | .032 | 2.76 | 1 | 80 | .100 |

The result in Table 8 illustrate the proportion of variance explained by the preference variable on the post-test variable. The preference of the students towards the approaches accounted for 2.0% of the total variability in performance in the post-test as measured by the Adjusted R Squared and this was not significant contribution (F ( 1, 80) = 2.7666, P = .100). As a result of the Regression analyses, the students' preference towards the respective method used has no significant effect on their achievement. Hence, the stated null hypothesis is supported.

### 4.4. Analysis and Interpretation on the Delayed Post-test

A delayed post-test was administered within three weeks time of completing the experiment to compare the long term effects of the analytic and synthetic teaching approaches on the students' learning the concepts of derivatives. It was given together with the students' mid-term course exam. In the immediate post-test a slight preference was found in favor of the analytic approach. But the means or of the synthetic group students was not far behind.

Both descriptive and inferential statistics was used to analyze the results from the delayed post-test. The mean, standard deviation and standard error is summarized by the descriptive statistics. An inferential statistic was used to answer the research question regarding to the comparative effects of the analytic and synthetic teaching approaches for long term learning of the students. The independent sample t-test was used to compare the mean score difference of the analytic and synthetic group students. A paired t-test was also performed to determine which method resulted in better long term learning within the group.

**Table 9. Descriptive Statistics for the delayed post-test**

| Test | Group | N | Mean | SD | SE |
|---|---|---|---|---|---|
| Delayed Post-test | Analytical | 41 | 33.93 | 5.58 | .87 |
| | Synthetic | 41 | 33.90 | 5.71 | .89 |

SD= Standard Deviation S.E = Standard Error

Based on the results displayed in Table 9, the mean scores of the analytic group students was 33.93 (SD = 5.58) and the mean scores of the synthetic group students was 33.90 (SD = 5.71). This shows that the students in the two groups reached the same level of understanding at this time.

The null hypothesis regarding to the comparative long term effects of the analytic and synthetic teaching approaches on the students' learning was stated and tested as follows.

$H0_4$. There is no significant mean difference between analytic and synthetic teaching approaches on the students' retention of the concepts of derivatives.

**Table 10. Independent Samples t-test Comparison between analytic and synthetic Groups on the delayed post-test**

| Group | N | Mean | SD | t | MD | P | df |
|---|---|---|---|---|---|---|---|
| Analytical | 41 | 33.93 | 3.58 | .023 | .024 | .982 | 80 |
| Synthetic | 41 | 33.90 | 2.96 | | | | |

Table 10 displays the means score of the students in the two groups. The students mean score in both groups according to the delayed post-test was very high (33.93) for analytic and (33.90) for synthetic groups as expected. The mean difference score in the two groups was not statistically significant ( P> .05). The result of this test convey a permissible reason from the natural of the used contents as the students met in the subsequent chapters with deep conceptual structures and more practical applications. When students learn the application of differentiation and integrations chapters they introduced exclusively using the concepts of limits and continuity. As a result, the stated null hypothesis is supported.

In addition to the aforementioned reason since the delayed post-test was given along with the normal schedule of mid-term examination for the course, the students were well prepared and did better. As a result of this the students in the two groups reached the same level of performance on the concepts of derivatives after they completed the subsequent unit of the course. Therefore, the study seems remained as inconclusive of the superiority of one of the approaches in the long term learning by considering the concepts of derivatives. Based on this result, the analytic and synthetic teaching approaches are equally effective for long term learning of the students on the concepts of derivatives.

**Table 11. Difference between the post-test in both participant groups**

**I. Analytic Group**

**Paired Samples Statistics**

| | | Mean | N | Std. Deviation | Std. Error Mean |
|---|---|---|---|---|---|
| Pair 1 | Ret.-DAT(54) | 33.9268 | 41 | 5.58297 | .87191 |
| | Post-DAT(54) | 30.5366 | 41 | 6.43466 | 1.00493 |

**Paired Samples Test**

| | | Paired Differences | | | | | t | Df | Sig. (2-tailed) |
|---|---|---|---|---|---|---|---|---|---|
| | | Mean | Std. Deviation | Std. Error Mean | 95% Confidence Interval of the Difference | | | | |
| | | | | | Lower | Upper | | | |
| Pair 1 | Ret.-DAT(54) - Post-DAT(54) | 3.39024 | 8.80022 | 1.37436 | .61255 | 6.16794 | 2.467 | 40 | .018 |

## II. Synthetic group

**Paired Samples Statistics**

| | | Mean | N | Std. Deviation | Std. Error Mean |
|---|---|---|---|---|---|
| Pair 1 | Ret.-DAT(54) | 33.9024 | 41 | 5.70879 | .89156 |
| | Post-DAT(54) | 27.4390 | 41 | 7.23895 | 1.13053 |

**Paired Samples Test**

| | | Paired Differences | | | | | t | Df | Sig. (2-tailed) |
|---|---|---|---|---|---|---|---|---|---|
| | | Mean | Std. Deviation | Std. Error Mean | 95% Confidence Interval of the Difference | | | | |
| | | | | | Lower | Upper | | | |
| Pair 1 | Ret.-DAT(54) - Post-DAT(54) | 6.46341 | 8.28884 | 1.29450 | 3.84713 | 9.07970 | 4.993 | 40 | .000 |

The two post-tests were also compared to each other with in the group to see which one of them resulted in better long term learning. Table 11 presents the difference between the groups in the post-tests. Paired samples t-test was utilized to see the comparative effects of the methods in case of the immediate and long term learning within the group. As Table 11 indicates, the differences between the first and second post-tests of the analytic group was significant ( P = .002). The students in the analytic group exhibited much difference in long term learning in the given content elements. Similarly, the synthetic group students demonstrated a significant difference in the two post-test scores in favor of the delayed post-test (P = .000).

### 4.4. The Dominant Method

As could be seen from the presentation of the approaches and from the results of the analyzed data, it seems difficult to pick out which one of the methods is more dominate for the students to learn mathematical

concepts. In this study it was obtained that the students in the analytic group performed significantly better than the students in the synthetic group on the post test.

According to the result of this study, the analytic method played more dominate role than the synthetic approach as per the students' achievement.

The result obtained from the survey indicates that both methods equally created a favorable condition for learning the concepts of derivatives. The students in the analytic group actually prefer the analytic approach and the students in the synthetic group prefer the synthetic approach as the students in the analytic group wanted to derive the rules themselves while the students in the synthetic group wanted the rules to be explained to them. By having to work out the rules by themselves, the students in the analytic group need to use more time to actually understand them, thus, the rules can be easy to remember. On the other hand, it seems that if the students are already aware of the concepts used in mathematics teaching, the synthetic approach might help them to internalize the rules as they have the tools to compare the new information to their previous knowledge about the concept.

The students in both groups did not show significant difference in long term learning as determine by the delayed post-test. Based on this result, the two methods are equally important for the long term learning of the given concepts. Hence, in terms of the retention level either method did not show superiority of the other.

To get supportive evidence for the results obtained from the collected data, a classroom observation was conducted and it is analyzed below.

**4.5.1. Analysis of systematic classroom observation results**

Systematic classroom observations check lists were prepared and conducted for eight successive periods to gather information about the overall practices of the students in both groups. The observed practices for each checkpoint were summarized and a tally mark (/) was used to indicate the extent of presence of the checked activity under each column during the teaching and learning process. For indicating to what extent each activity was practiced in each class was labeled based on the judgment that the indicated percent of the total number of students was used.

- Most = indicates above 75% of the total students in each group demonstrated the observed behavior.
- Many = indicates from 59% to 75% of the total students in each group demonstrated the observed behavior.
- Some = indicates from 25% to 50% of the total students in each group demonstrated the observed behavior.
- Few = indicates below 25% of the total students in each group demonstrated the observed behavior.
- None = was mean that the observed behavior was not demonstrated.

**The Students' over all classroom practice was observed using the following check points:**

- Students active involvement in the learning process
- Students' conscious effort to learn
- Students' ability in formulating and structuring a problem
- Completing the given classroom exercises and assignments
- Students curiosity to ask a question and to give a response
- Students ability to draw a conclusion based on the given information
- Students ability to solve problems using formulas and definitions
- Students' ability to justify a generalization using a counter example
- Students' ability to refuse a generalization using a counter example
- Students' involvement time on task

**Table 12. Analysis of systematic classroom observation by frequency distribution**

| Group | No | Most (>75%) | | Many (50-75%) | | Some (25-50%) | | Few (25%) | | None | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | f | % | f | % | f | % | f | % | f | % |
| Analytical | | 34 | 42.5 | 30 | 37.5 | 16 | 20 | 0 | 0 | 0 | 0 |
| Synthetic | | 20 | 25 | 32 | 40 | 26 | 32.5 | 2 | 2.5 | 0 | 0 |

Table 12 displays the results that 36 (45%) out of the total observed times of the eight periods each activity was practiced by above 75% of the total students in the analytic group, while 21 (26.25%) out of the total observed times of each activity in eight periods were practiced by above 75% of the total students in the synthetic group. Under the remaining columns of Table 12 the total sum of times of each activities practiced by the synthetic group exceed the total sum f times of each activity practiced by the analytic group. No activity was indicated under "None" in both groups and no activity was practiced by "Few" in the analytic groups.

This can have the implication that once the students being familiarized with the concepts of derivatives, they developed a curiosity that lead them to ask for an explanation of unclear ideas and to give a response regarding their perception of the content, to devote their time and energy in the learning task. This systematic class room observation checked that techniques that use an authentic (real-world, professionally relevant) situations and problems have the opportunity to provide contexts for the students actively involved in the teaching and learning process. The inductive approach was endowed with this power. The students in the inductive group were evoked their curiosity through the well structured examples used to introduce and to develop the formal definitions of the contents.

The students in the deductive group were also superior in completing the given classroom exercises and assignments and their ability to solve problems using formulas and definitions. According to Senk (1985) the analytic approach encourages to students to explore, to generalize, to refute a generalization, to formulate a generalization and explain it. Peard (1976) proposed that learners should be made to arrive at generalization through activities and if there is no counter example to it, it may be proved deductively.

The students' ability to justify a generalization using a counter example was greater in the inductive group, when the generalization was formulated by themselves through explorations and considering different cases. But it was observed in the synthetic group the students' wide spread difficulties in exploring examples and in constructing and exposing their problems. It was observed through the check lists that when the concepts of derivatives were presented by representing concrete environmental facts, the students put efforts to understand fully the meaning of the concise formalized definitions of the concepts. Juter (2006) pointed out that students can devote a lot of every and time if they are exited and curious about the subject and if there is something that derives them to make sense of what they are doing with.

## V. SUMMAR, CONCLUSION AND RECOMMENDATION

After all presentation, analysis and interpretation of the data obtained during the research, this chapter contains a summary of the results, conclusions and recommendations.

### 5.1. Summary

This study was designed to examine the comparative effects of two teaching approaches, the analytic and syntheticon students' understanding of the concepts of derivatives from calculus-I.The pre-DAT, Post-DAT and delayed post-DAT equivalent group design was used in this study. The derivatives unit from calculus-I was the same for both the analytic and synthetic treatments. The analytic treatment followed basically an example rule paradigm in this treatment, numerous examples were worked by the instructor and students before a rule or principle was stated. Following the examples, the students were encouraged, via

questions, to formulate a principle or generalization of their won. The synthetic treatment was also basically a rule-example paradigm. A rule or principle was stated and explained for the students, after which several examples were worked by the instructor and students together in class. The rule or principle was written for the students by the instructor while it was being stated. The treatment was lasted for about three and one half weeks with four periods of teaching per week and fifty minutes per a period.

Previous research results do not show an overwhelming superiority of either method in mathematics achievement. In this study it was intended to uncover the relative effectiveness of these two methods on students' understanding by considering the concepts of derivatives. Eighty-two (82) second year mathematics department students at Dr. A.H-CTE were subjects of this study. Two groups were formed consisted of 41 students each. The independent variables for this study were the analytic and synthetic teaching approaches and the pre-test result and the students' preference were a covariate. The students' achievement and retention were the dependent variables.

A diagnostic pre-test was used as a covariate for equating the two groups. The overall achievement of the students was measured by the post-test. A questionnaire consisted 22 items was administered to examine the students' learning preference with respect to the analytic or synthetic approaches as well as to determine the effect of the students' preference towards the respective method used n their performance on the post-test. A systematic class room observation check list was prepared to observe the overall practices of the students in the class in both groups and it was conducted for eight successive periods. A delayed post-test was also given two both groups after three weeks of the experiment.

Results from the achievement post-test as well as the delayed post-test were analyzed using independent t-test and an analysis of covariance was also used to statistically control the pre-test score and to refine further the result of the post-test. Results from the questionnaires and the systematic classroom observations were analyzed extensively in a qualitative was as well as quantitatively. The students' preference towards the respective method was compare using t-test and its influence on their achievement was also analyzed using the Linear Regression.

The result of the t-test analysis revealed that the students in the two groups mean score difference on the pre-test was not significant (MD = .233, P > .05). This indicates that the students in the two groups were equivalent according to their prior knowledge about the concepts of derivatives.

The t-test mean difference comparison on the post- score of the students in the two groups ascertained that students taught by the analytic teaching approach performed significantly better than their counter part taught by the synthetic teaching approach (MD = 2.977, P < .05)

Most of the students in both groups preferred to the respective teaching approach implemented during the experiment. The analytic group students preferred the analytic approach where as the synthetic group students preferred the synthetic approaches. Based on the U-test results a significant mean difference is observed on the items 4, 6, 8, 11. 12, 13, 14, 15, 16, 19, 21, and 22. But there was no significant mean difference between the two groups on the items 1, 2, 3, 5, 7, 9, 10, 17, 18 and 20. Based on the grand mean, the students in both group also confirmed their agreement with the respective approach used ( 3.51 for the inductive and 2.87 for the deductive group) and their preference towards the respective method showed no significant effect on their performance on the post-test.

The students mean score in both groups according to the delayed post-test was very high (34.67) for analytic and (34.74) for synthetic groups. The mean difference score in the two groups was not statistically significant ( P> .05), but paired samples t-test comparison in case of the immediate and long term learning within the group showed that the differences between the post-test and delayed post-test of the analytic and

synthetic group was significant ( P = .002 for analytic and P = .000 for synthetic ) in favor of the long term learning.

### 5.2. Conclusion

The study reported preliminary findings on the relative effectiveness of the analytic and synthetic teaching approaches for the students' understanding by considering the derivative concepts from calculus-I. Based on the results of this study, the following conclusion can be drawn.

- The students in the analytic and synthetic groups were equivalent in their prior knowledge about the concepts of derivative before the experiment.
- The students taught by the analytic teaching approach performed significantly better than their counter part taught by the synthetic teaching approach in the immediate post-test.
- The analytic teaching approach reflected its favor for the students' understanding of the concepts of derivative.
- The students in the analytic group seemed more convinced by the generalization they formulated than by the rule given to them by the instructor.
- Both the analytic and synthetic teaching approaches appeal to the students a preferable method of teaching and learning approach for the concepts of derivative.
- Both the analytic and synthetic teaching approaches are equally effective for the long term learning of the students on the concepts of derivative.

### 5.3.Recommendation

It is the concern of scholars in the field of education to create favorable situations for the improvement of the teaching learning process. This small study on the comparison of the relative effectiveness of the analysis and synthesis methods of teaching will contribute to the enhancement of mathematics instruction. Taking the results of this study into great consideration, the researcher has the following important recommendation to the stake holders in the education system.

- Course instructors in mathematics should implement the analysis approach as often as it is necessary to tap its many advantages in enhancing students' learning and maximizing achievement in mathematics.
- In order to avoid the mismatch between students' performance of learning style and teacher-educators teaching methods with respect to the analysis and synthesis approaches, it would be much better if teacher-educators follow analysis approach in an introductory courses and synthesis approach in advanced courses.
- The analysis method should be employed wisely and sparingly, restricted only rules that can be perceived and defined quickly; otherwise a lot of valuable class time can be wasted on futile and frustrating guessing.
- In this study, it was simply measured the overall achievement of the students on the concepts of derivatives. But, it would have been necessary to consider and classify the students' understanding with respect to different knowledge categories. Therefore, the researchers recommends that a further investigation needs to conduct for determining effects of the two methods on students' factual knowledge, procedural skills and conceptual understanding of derivatives.
- In the study, no factors was considered of which in one or the other way could affect the results obtained through the treatments implement in both the analytic and synthetic groups. Some of the factors one must consider along to those methodologies are sex, demographic back grounds of the students, their motivation to study mathematics courses in college for their future career, the students' ability level etc. must be included.

## 6. REFERENCES

[1]. Allen, N. B. R. 1992. A study of metacognitive skills as influenced by expressive writing in college introductory algebra classes. Doctoral dissertation, Louisiana State University. *Dissertation Abstracts Internationa*, 53, 432A.
[2]. Anderson , L.W., and Sosniak, L.A. (Eds.). 1994. Bloom's taxonomy: a forty-year retrospective. Ninety-third yearbook of the National Society for the Study of Education, Pt.2 ., Chicago , IL ., University of Chicago Press.
[3]. Ball, D. L., Lubienski, S. T., andMewborn, D. S. 2001. Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbookof research on teaching* (4th ed., pp. 433–456). New York: Macmillan.
[4]. Bloom, B., Englehart, M., Furst, E., Hill, W. &Krathwohl, D. 1956. Taxonomy of educational objectives: The classification of educational goals. Handbook I: Cognitive domain New York: Longmans.
[5]. Borasi, R. and Rose, B. J. 1989. Journal writing and mathematics instructions. *Educational*
[6]. *Studies in Mathematics*, 20, 347- 65.
[7]. Borko, H., Eisenhart, M., Brown, C. A., Underhill, R. G., Jones, D., andAgard, P. C. 1992. Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? Journal for Research in Mathematics Education, 23(3), 194-222.
[8]. Burton, L. and Morgan, C. 2000. Mathematicians writing. *Journal for Research in Mathematics Education*, 31, 420 - 453.
[9]. Carpenter, T. P., and Fennema, E.1992. Cognitively guided instruction: Building on the knowledge of students and teachers. *International Journal of Educational Research, 17*(5), 457–470.
[10]. Carpenter, T. P., Fennema, E., and Franke, M. L. 1996. Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *Elementary School Journal, 97*(1), 3–20.
[11]. Casado, M. 2000. Teaching methods in higher education: A student perspective. *Journal of Hospitality & Tourism Education, 12*, 65-70.
[12]. Chianson, M.M. 2008. Cooperative learning: in Kurumeh, M.S. andOpala, M.F (eds) *Innovative TeachingApproaches of Mathematics Education in the 21st Century. Vol. 1, Pp: 27-40*. Makurdi: Nigeria. AzabenPress.
[13]. Darling-Hammond, L. 1999. Teacher quality and student achievement: A review of state policy evidence. Seattle: University of Washington, Center for the Study of Teaching and Policy.
[14]. Eshetu , A. 2009. Science and mathematics secondary education in Ethiopia An Option paper presented at the Technical Workshop on Science and Mathematics Secondary Education in Africa (SEIA) Follow-up Program.
[15]. Fennema, E., andFranke, M. L. 1992. Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147–164). New York: Macmillan.
[16]. Ferrini-Mundy, J. and Lauten, D. 1993: Teaching and Learning Calculus. In: Wilson, P.S. (Ed.), *Research Ideas for the Classroom* (pp.155–176). NewYork: Macmillan.
[17]. Filloy, E. and Rubio, G. 1993. "Didactic Models, Cognition and Competence in the Solutions of Arithmetic and Algebra Word Problems in Hirabayashi", In. *et al.* (eds). *Proceedings ofthe 17th International Conference for the Psychology of Mathematics Education*, Vol.I, pp. 154-161. University of Tsukuba, Japan.
[18]. Glynna,R., 1993. A comparison of inductive and deductive teaching strategies utilizing graphical calculator capabilities. A Doctorial dissertation in Texas Tech University.
[19]. Gordon, S. 1997. *Functioning in the Real World: A Precalculus Experience*. New York: Addison Wesley.
[20]. Harbor Peters, V.F.A 2001. Unmasking Some Aversive Aspect of School Mathematics and Strategies for Averting them. *Inaugural Lecture*: 5 th, July.
[21]. Harel and Trgalova. 1996. Higher Mathematics Education. In: Bishop, A.J.et.al (eds). International Handbook of Mathematics Education. (pp 675-700). Netherlands
[22]. Hesson, M. and Shad, K. F. 2007. *A student –centered Learning Model,* American Journal of Applied Science **4**, 628-636.

[23]. Hiebert, J. and Carpenter, T. P. 1992. Learning and teaching with understanding. In D. Grouws (Ed).,*handbooks forresearch in mathematics teaching and learning* (pp. 65 -97). New York: MacMillan.
[24]. Hill, H.C., Schilling, S. G., and Ball, D.L. (2004). Developing measures of teachers' mathematics knowledge for teaching. Elementary School Journal, 105, 11-30.
[25]. Hunt, D., Haidet, P., Coverdale, J., and Richards, B. (2003). The effect of using team learning in an evidence-based medicine course for medical students. *Teaching and Learning in Medicine, 15*(2), 131-139.
[26]. Jones, T. (1990). *Towards a typology of educational uses of hypermedia*. In D.H. Norrie and H.-W. Six (Eds.) Lecture Notes in Computer Science. Proceedings of the 3rd International Conference on Computer Assisted Learning (pp. 265-76). Hagen, FRG.
[27]. Katozai, M.A. (2002). *Senior English Teacher: A Book for Preparation for PCS*. University Publishers'. Peshawar.
[28]. Khalid, T.M. (1976). *Education: An Introduction to Educational Psychology and History*, Anmol Publishers. Karachi.
[29]. LeGere, A. (1991). Collaboration and writing in the mathematics classroom. *Mathematics Teacher*, 21, pp. 166 - 71.
[30]. Lewy's Introduction to B. Riemann, *GesammelteMathematischeWerke*(Dover, New York, 1953); L. Steen (ed.) *Mathematics Today* (New York, 1985); D. Struik, *A Concise History of Mathematics* (New York, 1967).
[31]. Marwaha, P. (2009). The Teaching of Mathematics, Accessed on 20th of May, 2010. http://www.articlesbase.com/writing-articles/the-teaching-of-mathematics-1058849.html#ixzz0oS8BiIK3
[32]. Miller, L. D. (1992). Begin mathematics class with writing. *Mathematics Teacher*, 85(5), pp. 354 - 355.
[33]. MOE, (2009). Revised Mathematics course Catalogue for Diploma Teacher Training *Linear* Program; Addis Ababa, Ethiopia Revised Curriculum.
[34]. Mulugetaetal., (2002). Curriculum, Aims and Objectives in the Teaching of MathematicsAddis Ababa: Educational Media Agency Ministry of Education pub.
[35]. Oaks, A. and Rose, B. (1992). Writing as a tool for expanding student conception of mathematics. Paper presented at the 7th International Congress on Mathematics Education. Working Group 7: Language and Communication in the Classroom, Quebec.
[36]. Orton, A. (1983). Students' understanding of differentiation. Educational Studies in Mathematics, 14, 235 -250.
[37]. Patel, R., (2003). Innovations in teaching of Mathematics. Lecture way made college of education VallabhVidyyanagar.
[38]. Perkins, D. and Blythe, T. (1994). Putting understanding up front. *Educational Leadership*, 51(5), 4 -7.
[39]. Qualters, D. (2001). Do students want to be active? *The Journal of Scholarship of Teaching and Learning, 2*(1), 51-60.
[40]. Rehman, M. (2000). Teaching of Science and Mathematics, University of Peshawar. Peshawar.
[41]. Ritchey, T. (1991). Revised Version (1996). Analysis and Synthesis on Scientific Method- Based on a study by Bernhard Riemann, *System Research*, Vol. 8 No. 4. pp-21-24.
[42]. Rivkin, S. G., Hanushek, E. A., andKain, J. F. (2005). Teachers, schools, and academic achievement. *Econometrica, 73*(2), 417–458.
[43]. Rubio, G. and Valle, R.D. (2004). The Competent use of the Analytic Method in the Solution of Algebraic Word Problems. A didactical model based on a numerical approach with junior high students', *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*. Vol. 4.pp 129-136. Universidad NacionalAutónoma de México. CCH Sur.
[44]. Skemp, P. R. (1976). Relational Understanding and Instrumental Understanding. *Mathematics Teaching* **77**, 20–26.

[45]. Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching* 77, 20 – 26.
[46]. Skemp, R. R. (1979). *Intelligence, learning, and action*. New York: Wiley.
[47]. The Webster Comprehensive Dictionary. (1982). Encyclopedic ed. Vol. I & II. Ferguson Publishing Company. Chicago.
[48]. Trowbridge. (1986). *Becoming a Secondary School Science Teacher*, 4th ed. Merrill Publishing Company. Columbus.
[49]. Udeinya, C.S and Okabiah, O.S (1991): *Special methods of Teaching Science Subjects* Enugu: ABIC.
[50]. Wright, S. P., Horn, S. P., and Sanders, W. L. (1997). Teacher and classroom context effects on student achievement: Implications for teacher evaluation. *Journal of Personnel Evaluationin Education, 11*, 57–67.