Nash Equilibrium Revisited
Nash Equilibrium Practices and Limits

HADDAD Nader
University Of Oxford, Department Of Economics Oxford, United Kingdom

Abstract:- The static or dynamic notions of equilibrium proposed in game theory can be justified from two perspectives. From an educational point of view, equilibrium results from the sole reasoning of hyper-intelligent players who have a common knowledge of the structure of the game and their respective rationalities. If the rationalizable equilibrium or the correlated equilibrium are easily justified, the Nash equilibrium is obtained only under very drastic conditions; as for the perfect equilibrium, its justification is very sensitive to the hypotheses made. From the evolutionist point of view, balances result from the convergence of a process of learning or evolution of players in limited rationality, but observing the past course of the game. The Nash equilibrium, at least in pure strategies, is often obtained as an asymptotic state and some of its refinements can even be selected; the perfect balance is also justified under very extensive conditions.

I. INTRODUCTION

Like Walrassian balance of economic theory, game theory is based on various notions of equilibrium, each of which reflects the way in which rational actors coordinate their actions on a state with a certain stability. However, any notion of equilibrium is postulated above the game by the modeller, a state of equilibrium being subject to the necessary condition that, if the actors are there, they perceive no interest to deviate unilaterally. On the other hand, no concrete process of achieving a state of equilibrium, which would be based solely on the deliberations and actions of actors without external intervention, is described by the modeller. By analogy with the Walrassian commissioner who provides equilibrium prices to economic agents, one can certainly introduce a fictitious entity, the 'Nashian regulator', which calculates a state of equilibrium of play and suggests the actors to adopt it. The actors must actually adopt it, which is the case only if they have good reason to believe that their opponents will adopt it too, the condition of stability postulated at equilibrium not being necessarily sufficient. The purpose of the "cognitive economy" is to study the beliefs and reasoning that economic actors mobilize to adapt to dynamic situations of mutual interaction (Walliser, 2000). One of his major themes is to report on concrete processes by which actors, endowed with both instrumental and cognitive rationality, are likely to coordinate their own forces on a state of equilibrium. The processes 2 exhibited must allow, in the same movement, to justify such or such notion of equilibrium posited a priori, and to select this or that equilibrium state associated in case of multiplicity of states.

A first approach aims to give "epistemic justifications" to balances, that is to say, to base balances solely on the reasoning of autonomous actors with extremely strong rationality. A second approach aims at giving "evolutionary justifications" to balances, that is to say, to make balances appear as asymptotic states of dynamic processes between actors with a very limited rationality. The first section deals with the epistemic justifications of the only usual notions of static equilibrium (Nash equilibrium, rationalizable equilibrium, correlated equilibrium). The second section deals with the epistemic justifications of the dominant notion of dynamic equilibrium (perfect balance in sub-games) by returning to the "paradox of the retroduction" (backward induction paradox). The third section discusses evolutionary justifications of equilibrium concepts, both static and dynamic, for various learning and evolution processes. Each section is divided into three parts: the first is devoted to the necessary analytical tools, the second to their application to the games considered and the third to the statement of the main results.

In a repeated game, we say that a pair of winnings.

II. EPISTEMIC JUSTIFICATIONS OF STATIC EQUILIBRIA

➢ *Logical Principles*

The formalization of the structure of the beliefs of an actor is carried out within the framework of the epistemic logic (a variety of modal logic), under the two syntactic and semantic forms of which one can show the equivalence. In syntax, the actor's physical universe of reference is described by "propositions" and the knowledge of the actor about it is translated by a "belief operator" which indicates whether he knows or does not know such or such proposal. In semantics, the physico-psychic states of the universe (associating material properties and beliefs of the actors) are described by "possible worlds" and the knowledge of the actor is described by an "area of accessibility", indicating in all world (the one being singled out as the real world) which are the worlds between which he can not discriminate. The transition from syntax to semantics is simply done by associating with each proposition an event, subset of worlds where it is true. Moreover, an actor knows a proposition in a certain world if the associated event is true in all the worlds accessible from this world. The syntactic representation makes it possible to define a demanding set of axioms to which the actor's beliefs about his environment and himself (self-hierarchical beliefs) are subject. These are the axioms of logical omniscience (the actor knows all the consequences of what he knows), of truthfulness (what the actor knows is true), of positive introspection (the actor knows what he knows) and negative introspection (the actor knows what he does not know). These properties have semantics counterparts in the form of properties to which the accessibility domains are subject or the associated accessibility relation (one world is connected to another if it is in its accessibility domain). In fact, logical omniscience is automatically satisfied in the semantics retained while the other three properties refer respectively to the reflexivity, transitivity and euclidianity of the accessibility relation.

Regarding truthfulness, the most controversial property, knowledge is called "knowledge" when it is true (in the sense of the modeller) and "belief" when it is likely to be false.

The previous beliefs have been defined in a propositional (or set-theoretic) framework, in which knowledge is expressed in all or nothing, the actor knowing or not knowing a certain proposition. In semantics, when all the properties are simultaneously satisfied, the actor has a partitioning "information structure" on the worlds, in the sense that the accessibility domains form a partition on all the possible worlds. Alternatively, beliefs can be defined in a probabilistic framework, in which knowledge is more nuanced, the actor knowing this time a proposal with a certain probability. In semantics, given the generalized properties of knowledge, the actor has a distribution of probabilities in each world on all of the worlds. In practice, a mixed semantic framework tends to impose itself, defined on the one hand by an initial probability distribution (prior) on the worlds, common to all the actors and translating an objective public information, and on the other hand by a partition of information, specific to each of the actors and translating his private information. Moreover, even in a context of near certainty (Monderer-Samet, 1989, Börgers, 1994, Brandenburger, 1992), the transition from the set-theoretic framework to the probabilistic framework can be achieved by considering a range of beliefs (Stalnaker, 1996). At one of the poles, the actor has a set knowledge that nothing can question (no contradictory message is likely to occur). At the other pole, the actor is endowed with a belief (belief with probability 1) which can nevertheless prove to be false (a message can come as a surprise). In a first intermediate situation, the actor has a belief in the real world that what he believes is true (no surprise is possible in the real world). In the second intermediate situation, the actor has a 1-belief 'robust compared to the truth', in the sense that his belief is confirmed if he receives a true message in the world considered (a surprise then becomes possible, even in the real world). In probabilistic cases, when a revision of the beliefs is necessary for a non-contradictory message, the revision rule supposedly used is none other than the Bayes rule. In a syntactic context this time multi-actors, they will adopt cross- beliefs (hetero-hierarchical beliefs) of the type 'I know you know that I know ...'. The inter-individual distribution of these beliefs makes, by increasing force, the shared belief ('everyone knows X') to the common belief ('everyone knows X, knows that the other knows X and so on until infinity '), the latter introduced by Lewis (1969). Each is symbolized by an autonomous belief operator, endowed with remarkable properties deduced from the axioms of the individual operators. In semantics, distributed beliefs are expressed through accessibility relationships obtained simply from individual accessibility relationships. In particular, if the individual beliefs are partitioned, the common belief is also translated by an information partition, namely the finest partition among the coarser partitions than those of the actors. By moving from a set-theoretic framework to a probabilistic framework, common belief is again more or less demanding, from common knowledge to common belief with intermediate positions (Stalnaker, 1996).

➢ *Hypotheses*

In a game context (static or dynamic), the beliefs of each player are of different types and are affected by a more or less strong uncertainty, expressed in a set or probabilistic form. "Structural beliefs" relate to the structure of the game, ie the decision-making "characteristics" of other players (opportunities, 4 beliefs, preferences) and the rationality that drives them; but the player is supposed to know his own characteristics. "Factual beliefs" relate to the past course of the game, that is, the past actions of other players; the player knows again his own past actions. "Strategic beliefs" refer to the future course of the game, that is, the anticipated future actions of other players, and are called "guesswork"; the player knows again his own intentions of action. Since the players are in a situation of strategic interaction (the effects of a player's action depend on those of the others), the players are naturally engaged in a system of crossed beliefs about their respective future actions, themselves rooted in a system of beliefs crossed over their respective characteristics. For a static game, a player's opportunities are defined by a set of actions (or pure strategies) that he can mobilize and his preferences by a utility function that depends both on his own action and his that of others. As for the beliefs of a player, they relate as much to the previous elements of others as to the beliefs of others; they are in fact summed up by the "type" of others, chosen from a set of possible types. The actor also adopts a 'Bayesian rationality', that is, he chooses the decision rule that maximizes his expectation of utility, given his beliefs about the type of others. He makes individual choices for the various sources of uncertainty without resorting to external mediation, all external influences being more or less faithfully integrated into his beliefs. The state defined by the joint actions of the players (or rather by their joint intentions of action) appears then as a 'balance of beliefs', in the sense that no player is incited to modify his beliefs in such a state. In a semantic framework, we consider a universal space of uncertainty where possible worlds incorporate combinations of types of players as their intentions of action; a player is still supposed to know his own type. Since each player has a mixed information structure on the worlds (the probability a priori being common to them or not), all the uncertainties are thus assumed to be probabilizable (and even partially shared) in the last instance. In addition, each player is led to choose a strategy (or decision rule), each rule defining the action that it implements in each of the possible worlds; the player is supposed to know his own chosen action, so that he must be

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identical in two worlds between which he can not discriminate. The rationality (ex post) of the player is defined by the choice of the strategy maximizing the hope of utility of the player in front of the envisaged strategies of others. Each player finally defines a conjecture, namely a distribution of probabilities on the actions of others; any mixed strategy of a player is then interpretable as a probabilistic belief of others about his own actions.

- *Results*

The first result - unexpectedly expected - asserts that, under the assumption of common knowledge of the game structure and the (Bayesian) rationality of the players, iteratively strongly dominated strategies are eliminated (Tan- Werlang, 1988). A strategy of an actor is said (strongly) dominated when there is another strategy of this actor which gives him a (strictly) usefulness for all the strategies of his opponents. A strategy is iteratively dominated if it is eliminated in the following process applied to the starting game: first we eliminate the dominated strategies for each player, then in the residual game, we eliminate again the dominated strategies and so on. . The sequential elimination of the dominated strategies in the epistemic process is explained by the fact that each player will eliminate his dominated strategies, knows that the other will do so much and so on. Of course, the sequential elimination of dominated strategies generally leaves a large number of issues, between which selection can be made only with additional conditions. A second result states that if, in addition to the common knowledge of the structure of the game and the rationality of the players, one postulates the common knowledge of the independence of the players, one obtains a "rationalizable equilibrium" (Bernheim, 1984, 1986 Pearce, 1984). The rationalizable equilibrium is defined by considering that each player defines his best answer to the anticipated strategy of the others, itself anticipated as a better answer to the opposing strategies, and so on until closing. It is built by iterated elimination of the lower strategies of the players, a lower strategy being a strategy that is never a better answer. The achievement of the balance is explained by the fact that the knowledge of the independence of the players (their intentions of action are not correlated) makes it possible to break down the conjectures on the set of the strategies in conjectures relating to each of the strategies. Any rationalizable strategy is iteratively non-dominated, but not the other way around; there is identity only in the case of two players because the condition of independence between players is then automatically satisfied. A third result states that if, in addition to the common knowledge of the structure of the game and the rationality of the players, it is postulated that the beliefs of the players on their respective strategies result from a distribution a priori common, we obtain a "balance correlated "(Aumann, 1987). A correlated equilibrium is characterized by a certain distribution of probabilities on all the (pure) issues of the game; an external entity, the correlator, draws a game outcome in accordance with this probability distribution, and indicates to each player the corresponding strategy; each player then has an interest in following the correlator's recommendation if the others do it. From the epistemic point of view, this time the players are precoordinated by their beliefs a priori common worlds, the latter reflecting exogenous states of nature conditioning the possible outcomes. Here again, a strategy of a correlated equilibrium is iteratively non-dominated, but not the opposite; on the other hand, it is not directly comparable to a rationalizable strategy. A fourth wave of results focuses on the Nash equilibrium, the usual balance defined by the fact that each strategy of a player is the best response to the balance strategies of others. For reduced two-player games, a first result (Tan-Werlang, 1988, Brandenburger-Dekel, 1989), soon weakened (Aumann-Brandenburger, 1995), asserts that, under the assumption of shared knowledge of the structure of the game , the rationality of the players and conjectures of the players, the conjectures constitute a Nash equilibrium (in mixed strategies). For games with any number of players, a more demanding result (AumannBrandenburger, 1995) asserts that, under the assumption of shared knowledge of the structure of the game and the rationality of the players, under the assumption that the beliefs of players result of a distribution a priori common and that their conjectures are of common knowledge, the conjectures of all the players on the same player agree and still define a Nash equilibrium.

The Nash equilibrium appears both as a weakening of the rationalizable equilibrium (the beliefs on the actions of others close at the second level) and the correlated equilibrium (the probabilities assigned to the outcomes are decomposed into probabilities on the equilibrium. strategies of each player). However, the results obtained are very restrictive because they assume that the players can know the conjectures of others, which are not immutable structural characteristics of these players, but cyclical beliefs whose origin 6 is again not described. In the case of more than two players, even more stringent additional conditions are needed to ensure that the conjunctions of two players on the same third are identical. The difficulty of epistemic coordination between the players comes from the fact that the Nash equilibrium is fundamentally based on an interaction loop between these players. It appears in fact as a self-fulfilling balance, namely that the expectations of players on their strategies (Nash) cause their achievements. If the problem of the definition of a notion of equilibrium is treated through the preceding results, it remains the problem of the selection of a state of equilibrium in the (frequent) case of multiplicity of the associated states. For the rationalizable equilibrium, as each player selects independently one of his rationalizable strategies, no coordination on this or that state is possible. For the correlated equilibrium, it is the fictional correlator (or in its absence, the distribution of probabilities a priori common to both players) that selects a particular state. For the Nash equilibrium, knowing (without specifying how) the conjectures of the other defines another state among others. However, in coordination games (for which all equilibrium issues are utility-equivalent for players), the selection can be made through "conventions" that are common knowledge between players1. These conventions relate directly to 'focal states' of the game (Schelling, 1960), whose salience still reflects off-model cultural phenomena. They concern more ambitiously criteria of choice between issues, such as the symmetry criterion or the Pareto-optimality criterion. Finally, if the game is repeated, it is its very story that is likely to reveal salient characters (Crawford-Haller, 1990).

III. EPISTEMIC FOUNDATIONS OF DYNAMIC EQUILIBRIA

➢ *Logical principles*

When one places oneself in a dynamic frame, one poses an additional problem, that of the genesis and the evolution of the beliefs of the actor. In fact, the epistemic logic does not deal head-on with the problem of the formation of beliefs, but confines itself to the problem of the revision of an a priori belief (the origin of which is not specified). Traditionally, there are two contexts of revision, characterized by the type of message that passes from an initial belief to a final belief about a universe of reference. In the context of (revising), the message comes to clarify or even to invalidate the initial belief concerning this universe considered as fixed. In the context of "updating", the message indicates in which direction the universe considered this time changes. A third context, that of "focalisation" (focusing), concerns a message relating to an object drawn at random within a universe made up of a population of objects, but it is reducible to a principle of rectification associated with a projection principle. In syntax, axioms related to each of the two main contexts have been developed, first for propositional beliefs (Alchourron- GärdenforsMakinson, 1985, Katsuno-Mendelzon, 1992). A family of axioms is common to both contexts. Thus, the axiom of success asserts that the final belief must validate the message, which assigns a priority to the (supposedly true) message about the (possibly false) initial belief. Likewise, the axiom of inclusion asserts that the part of the initial belief compatible with the message is preserved in the final belief, which reflects a weak (we keep what can be). Other axioms are specific to each context. For rectification, the axiom of preservation asserts that, if the message is compatible with the initial belief, the final belief is limited to their common part, which reflects a strong 'conservation principle' (one changes as little as possible what must be changed). For the actualization, the axiom of monotony expresses that, for a given message, if the initial belief is weakened, the final belief is weakened, which again reflects a principle of minimal change. In semantics, revision rules are inferred from axiomatics by representation theorems. For rectification, everything happens as if there is a set of concentric crowns around the initial belief, reflecting worlds further and further away from this initial belief. The final belief is then simply obtained as an intersection between the message and the first crown that intersects the message. Revised in syntax, this rule of revision amounts to endowing each proposition with a "degree of epistemic rooting", adding the message to the initial belief, and removing from the system thus constituted the least ingrained propositions until it restores its logical coherence. For the actualization, everything happens as if there is a set of concentric crowns around each world, reflecting worlds more and more distant from this world. The final belief is obtained here as the union, for each of the worlds of the initial belief, of the intersections of the message with the first crown that intersects the message. This approach can be extended from a set-theoretic framework to a probabilistic framework, a message (always set-up) passing from a distribution of prior probabilities to a posterior distribution (Walliser-Zwirn, 2002). In syntax, the previously defined axioms can receive a weak (in terms of support of probability distributions) or strong (in terms of numerical values attributed to probabilities) transcription. In semantics, revision rules of probability distributions are then derived from axioms by representation theorems. For rectification, with a weak transcription of axioms, the proper rule is the generalized conditioning rule. The most usual rule for game theorists, the Bayes conditioning rule, is singled out only for a very strong transcription of axioms; it receives no epistemic justification (as opposed to its decisional justification through bets made by the actor) in a very restrictive context. For the actualization, with a weak transcription of axioms, the proper rule is the generalized imaging rule proposed by Lewis.

The revision of beliefs can be considered as the fundamental form of reasoning to the extent that various other forms of reasoning can be reduced to it (Walliser-Zwirn- Zwirn, 2002). This is the case of the non-monotonic reasoning of the type 'of the facts A, we normally infer the facts B', which weakens the classical deduction by considering exceptions and which can be reinterpreted as a revision in a context of rectification. It is the same with the abductive reasoning of the type 'of the facts A, one abducts the hypothesis B' which is a form of inverted explanation and which can also be reinterpreted in two ways in a context of rectification. This is especially true of the conditional reasoning, of the type 'if the antecedent A was the case, then the consequent B would be true', which is a reinforcement of the material implication and which can be reinterpreted this time in a context of discount. It is based on a physical transformation of the universe to make the antecedent true, even if this transformation remains virtual. 8 In syntax, conditional reasoning distinguishes a prof - tual proposition whose antecedent is true of a counterfactual proposition whose antecedent is false. A conditional proposition A> B differs from the material implication A → B in that the latter is (wrongly) true as soon as its antecedent is false. Conditional reasoning is subject to a series of axioms of which we can give some examples. The axiom of reflexivity asserts that a conditional whose consequence is identical to the antecedent is always valid. The axiom of infra-classicality asserts that a condition whose antecedent is true coincides with the intersection of the antecedent and the consequent. The axiom of conservative monotony asserts that if two conditioners of the same antecedent are valid, one can construct a valid conditional by taking as antecedent the conjunction of the common antecedent and the consequence of the one and consequently the consequence of the other. . Additional axioms have a more 'topological' role (and right, or left). In semantics, conditional reasoning is translated by a selection function which, in each world, associates with each hypothetical event H a retained event K, possibly reduced to a single world (Stalnaker, 1968). The bridge between syntax and semantics is then realized by the following condition: a conditional is valid in a world if, for this world, all the worlds retained for its antecedent validate its consequent. Using the only topological axioms, the selection function can be derived from a set of concentric crowns grouping worlds more and more distant from the world considered (Lewis, 1973). A conditional is then valid if, in the worlds closest to the considered world where the antecedent is true, the consequent is also true. The more substantial axioms then correspond to properties assigned to the selection function (or world crowns). The property of "satisfaction" requires the event selected from a world and a hypothetical event to be contained in the latter. The property of "maintenance" requires, for a world situated in the hypothetical event, to be located in the event selected for this world and this hypothetical event. A variant (Samet, 1996) creates a more direct link between conventional reasoning and conditional reasoning. In syntax, we add to the usual belief operator a hypothetical belief operator (parameterized by a hypothetical proposition); the second is reduced to the first when the hypothetical proposition is tautology. Both operators are subject to a list of axioms that both generalize the properties of the first (logical omniscience, positive and negative introspection) and combine them. In semantics, we again define, by leaning on the score associated with the usual belief, a hypothesis transformation function that associates with any world and any hypothetical event a retained event. The hypothesis transformation function then satisfies the two classic properties of satisfaction and maintenance. It can be shown (Halpern, 1999) that one can reduce oneself to conventional conditional reasoning by interpreting the hypothetical belief operator as 'if the antecedent is accepted, the consequent is known' (a proposition is said to be accepted if it is not known that it is false).

➢ *Assumptions*

Conditionals were first used in decision theory in a configuration where actions are likely to influence the states of nature. The two proposed decision criteria have in common the retention of the utility expectancy maximization rule, but differ in the probability to affect the states (Gibbard-Harper, 1978). The "theory of the evidential decision" retains the probability of the conditional state to the action, expressing a simple probabilistic correlation without causal dependence between action and state 9 (due for example to a common factor which influences the two entities). The "theory of causal decision" retains the probability of the conditional 'if such action, then such state', expressing this time a direct causality between action and state. These two theories have been mobilized in the analysis of the 'Newcomb problem' to show that the first leads the actor to take a box (action maximizing the expected utility) while the second leads him to take two boxes (dominant action). The conditionals were early mentioned, if not really used, to deal with games in extensive form, expressed in the form of a game tree (describing the possible successive strokes and utilities resulting from any trajectory). Selten-Leopold (1982) point out that this is what would happen outside the equilibrium trajectory of the game that justifies the choice of the trajectory itself, and therefore discuss the relevance of various theories of conditionalities. . Harsanyi-Selten (1988) observe that a strategy must be expressed by a conditional rather than a material implication. These reflections have been pursued more recently by attempting to describe by conditionalities various aspects of the game. The works are restricted to the class of generic games, namely games without equalities between the issues for the same player (or such that, when there is equality for one player, there is also equality for the others) and with perfect information (each player knows at any time where he is in the tree of the game). In addition, the structure of the game is considered as common knowledge. First, conditional reasoning can be applied to the structure of the game, ie to the game tree. It indicates that in any intermediate node, the player must retain one action and only one, and that in any terminal node, the player receives a certain utility; it indicates in reverse that any node can be reached only if a specific move has been played previously. In fact, conditional reasoning does not differ here from material implication and only translates physical constraints and utilitarian rules related to gambling. Moreover, conditional reasoning is applicable to the definition of a strategy. a player, namely the action that player would play in any intermediate node of the game tree. It can be a factual condition if the node is actually on the trajectory followed, or a counterfactual if the node is outside this path. To limit oneself to a material implication is insufficient here because it would allow any action in a non-equilibrium node. By combining the two previous uses, conditional reasoning is used to unambiguously translate the rationality of the player, always analyzed from a strictly individual point of view. This rationality is apprehended both in each intermediate node of the game tree and in each of the possible worlds. Since the possible worlds here again reflect the types of players (essentially their beliefs), the strategies retained by the players are naturally conditional on these worlds. The rationality of a player is more precisely defined by considering the strategies of the other players as fixed by his beliefs and by examining if it has interest to deviate from his own strategy, according to what happens downstream of the considered node (consequentialist principle). But it nevertheless receives many alternative definitions, more or less strong, depending on the individual criterion chosen, the type of node where it is actually evaluated and the beliefs attributed to the player in this node.

First of all, expressed in set-theoretic terms, rationality can be content with saying that the player is not aware that he can do better with another action or want to say that the player is fully aware that he can not do better. In probabilistic terms, it can be expressed by the fact that the player almost certainly knows that he can not improve his utility expectancy. This last hypothesis is intermediate between the two preceding ones. Then, the rationality is called 'substantial' (Aumann, 1995) if it is defined in any node, conditionally to the attainment of this node, that is to say through a conditional. It is called 'material' if it is only defined at the nodes reached by the equilibrium trajectory, that is to say by a material implication. Material rationality is naturally weaker than substantive rationality. Finally, rationality is defined ex ante with the beliefs available at the beginning of the game or ex post with the beliefs available at the time of play. The second is obviously stronger than the first. In semantics, conditional reasoning will be translated by a selection function defined in any world, with the hypothetical event of reaching a certain node. The general conditions imposed on this function therefore receive a more concrete interpretation, in relation to the strategies followed by the players in this world (Halpern, 2001). The satisfaction property indicates that the strategies adopted in the worlds closest to the world considered lead to a trajectory that passes through the considered node. The sustaining property indicates that if the considered node is reached by the strategies relating to the world considered, the nearest world is reduced to this world. In addition, an additional property of "uniqueness" imposes that, on the subtree starting at the considered node, the strategies relating to the world considered and to the nearest world coincide; it ensures that the strategies dictate exactly what the players will do if the node is actually reached.

- *Results*

For a game in extensive form and with perfect information, the fundamental notion of balance is that of perfect balance (of sub-games). This notion is stronger than that of Nash equilibrium, which remains however applicable if the game in extensive form is brought back in normal form thanks to the concept of strategy. The perfect balance has the advantage of existing and being unique for any generic game because it is obtained through a constructive procedure, the backward induction procedure. In a terminal node, the player who has the trait chooses his best action; in a previous node, the player who has the trait chooses his best action, taking into account that of the next; the procedure continues from node to node to the initial node. While this notion of equilibrium initially seemed somewhat paradoxical, the paradox gradually dissolved when the perfect equilibrium was justified or reversed by a variety of results, which are based on epistemic assumptions that differ only in very subtle way. Binmore (1987) considers the possibility of justifying the perfect balance by the common belief of the rationality of the players, and highlights the 'paradox of the' backward induction paradox '. This paradox is based on the fact that the reasoning of the player must be exercised in both directions of the arrow of time: in the opposite direction to define the strategies according to the beliefs (instrumental rationality), in the direct sense to revise beliefs based on the information collected (cognitive rationality). It expresses that, since the rationality of the actor is based on its action in out-of-equilibrium nodes, it is necessary to concretely envisage the possibility of a deviation out of the equilibrium trajectory and to examine the revised beliefs of the actor at the nodes. deviants. However, the player's beliefs in a non-equilibrium node are no longer compatible with the common knowledge of the rationality that served to define equilibrium, at least if the other structural hypotheses are supposed to be preserved (Reny, 1992).

A first result (Aumann, 1995) shows, however, that if the players have a common knowledge of their respective rationalities, the strategies chosen determine a perfect balance. It is based on the assumption that the rationality of the player is set low, substantial and ex ante; no revision of beliefs and no conditional is considered. The result is a fortiori preserved with a stronger form of rationality, in particular Bayesian rationality or ex post rationality. However, rationality is supposed to be common knowledge in the strong sense (it is never called into question by virtue of the principle of truth) and this knowledge can not be weakened. The result indicates that, under the assumption of common knowledge of rationality in any node, no player can deviate from his trajectory of perfect balance under penalty of violating it. We are close to the theorem of virtual works in mechanics, which indicates which trajectory will follow a material system in the set of virtual trajectories, but these are never realized. This result triggered a controversy between Aumann (1996) and Binmore (1996), the first recital that we can maintain the hypothesis of common knowledge of rationality against wind and tides, the second stating that we can not to forget to consider and interpret what is happening out of balance. If it is in a non-equilibrium knot, a player must choose, in the set of assumptions that justify the balance, which must be questioned (Walliser, 1996). This hypothesis may result from the degree of epistemic rooting that the player attributes to each of them, a degree that may depend on the game in question (in the chess game, even if perfect balance strategies are not known, deviations will be attributed to defects in the cognitive rationality of the opponent rather than a lack of knowledge of the structure of the game). The degrees of rooting epistemic and more generally the rules of revision of the beliefs are then part of the characteristics of the players and can themselves be the object of a common knowledge. It can be shown that, according to the hypothesis questioned in the revision, it is the perfect equilibrium or other notions of equilibrium that will prove to be epistemically justified. The first hypothesis that can be considered is that of "no desire" of the players, assumption that the actions retained by a player are actually those he implements. It is easy to see that if we question this hypothesis in the form of a 'trembling hand', namely that every player is affected by successive random and independent tremors when he implements his intentions of action (Selten, 1975), the perfect balance will remain. Indeed, any deviation from others will then be interpreted by a player as purely accidental and revealing nothing about the future behavior of this player. A second hypothesis concerns the common knowledge of the structure of the game. Kreps et al. (1982) have shown (in an unofficial epistemic framework) that, if a player has uncertainty about the type of other players, one can obtain alternatives to perfect balance. Any deviation from others is then interpreted by a player by the fact that he plays a different game than the real game or that he believes that a third party plays a different game. The most sensitive hypothesis, however, is that of common knowledge of the rationality of the players because it does not allow any surprise. If we only consider a common belief that what agents believe is true, the perfect equilibrium remains because no surprises are considered (Stalnaker, 1996). On the other hand, if one weakens again in a common belief with robustness compared to the truth, the perfect balance is generally no longer guaranteed (Stalnaker, 1996). Finally, if there is simply 1-common belief, perfect balance is a fortiori not guaranteed because a surprise is explicitly considered. For example, Ben 12 Porath (1992) shows that, if there is a common belief of rationality at the beginning of the game, the outcome is that obtained by a step of elimination of weakly dominated strategies, then by elimination sequential highly dominated strategies; Stalnaker (1998) obtains a similar notion of equilibrium when it is common knowledge that the players are rational and apply a 'rationalization principle'. On the other hand, in a game of length m, if the perfect equilibrium remains justified with a common m-knowledge of rationality (Bicchieri, 1988), it is no longer guaranteed by a common ε- belief.

We can finally look at the hypothesis of rationality itself, despite the strong degree of rooting towards it. The basic result justifying perfect equilibrium does not survive if substantive rationality is replaced by material rationality, except for certain particular classes of play (Aumann, 1998). More precisely, if one evaluates rationality not in each world considered, but in the closest world in the sense of a selection function, the result is invalidated (Stalnaker, 1998). The selection function, assumed to be identical for all players and satisfying the three previously stated conditions, serves in fact to revise the beliefs when a non-equilibrium node is reached. Even if, in the world considered and in the nearest world, the downstream strategies coincide, the beliefs will then differ (Halpern, 2001). A similar result (Samet, 1996) shows that, if one introduces a conditional reasoning with the help of an operator of hypothetical beliefs, the perfect equilibrium is no longer guaranteed. It is restored only under very strong additional conditions (hypothetical sequential beliefs). A hypothesis related to that of rationality of the players is that of independence between the choices of the players, considered by Stalnaker in two aspects. The hypothesis of causal independence, inherent in the idea of non-cooperative play, simply considers that players play independently. The hypothesis of epistemic independence, which is more demanding, expresses that what one player learns about another (especially what that other person thinks about a third party) does not affect what he thinks of a third person. Causal independence does not lead to epistemic independence, the former does not prevent beliefs from possibly being correlated. Seen in this light, Aumann (1995) justifies the perfect balance by a hypothesis of causal independence, associated with a hypothesis of reinforced epistemic independence, namely the independence between successive strokes of the same player (insensitive to messages received). Stalnaker (1998) shows, however, that if the players' beliefs satisfy a condition of epistemic independence between players, but differ at each node, the proper concept of equilibrium remains that of perfect equilibrium. The condition of epistemic independence in fact plays the same role as a fourth property (together with those of satisfaction, maintenance and uniqueness) assigned to the selection function (Halpern, 2001). This indicates that, if a first world is accessible from a world retained by the selection function, there is a second world accessible from the world considered such that the strategies in these two worlds are identical on the subtree beginning to the world under consideration. This very demanding property is interpreted by the fact that in an intermediate node, the player keeps the same beliefs about the others in the world considered and in the nearest world. As a consequence, the strategies that a player considers for the worlds selected by the selection function are a subset of those envisaged in the world considered. If, as has been stated, the first three properties of the selection function are not enough to guarantee the perfect equilibrium, the fourth allows again to ensure it.

IV. DYNAMIC JUSTIFICATIONS OF EQUILIBRIA

➢ *General Principles*

Evolutionary game theory considers a basic game that is played sequentially on a generally infinite set of periods. It is based on five major modeling principles. Only the principle of satisfaction is common with classical game theory and explains the immutable characteristics of the players. It details the structure of the basic game, namely the possible actions of each player and the utilities resulting from the combination of their actions. Depending on whether the base game itself is static or dynamic, these features will be represented as a game matrix or game tree. It also expresses how players aggregate utilities gained from successive game occurrences. (in general by updating the successive utilities). The other four principles deal with the dynamic, generally random process of repeating the basic game, and describe both the physical interactions between the players and the psychic reasoning that drives them. The principle of confrontation specifies the nature of the interactions between the players, in particular their modes of encounter. The basic game can be played by both singular players and player populations. When the game is asymmetrical, we introduce as many populations as players of the basic game (multi-population game). When the game is symmetrical, we can do the same (multi-population game) or consider that the players are only one and the same population (single-population game). Each player can potentially meet any other individual (global interactions) or only those located in his 'interaction neighborhood' (local interactions). In fact, individuals are often located on a network (linear, planar) on which the interaction neighborhoods are geometrically defined. In each period, the interactions between individuals can be systematic or only a sample of individuals is active, each individual meeting another sample of individuals from his neighborhood. The information principle describes the information collected by each player on both the structure of the game and its past. The structural information is often very small, the player knowing his possibilities of action, but not necessarily his preferences and a fortiori the characteristics of others. Factual information is richer and comes in two types. On the one hand, he can observe past actions played by other players (his own being known to him). On the other hand, it can observe the utilities that it obtains with the actions that it has implemented (and more exceptionally the utilities obtained by others). All this information is collected in its 'information neighborhood', usually included in its interaction neighborhood. Again, he can fully observe the information or receive information only from a random sample of players. Finally, as will be seen later, he obtains the information either as a spontaneous by-product of the course of the game (passive experimentation), or voluntarily by moving his 'normal' action (active experimentation). The evaluation principle focuses on the interpretation and processing of information in order to obtain condensed information for action. On the one hand, the player can focus his attention on the statistical distribution of past actions of his opponents to identify some invariants. He will then use these retrospective indicators to form anticipations (usually probabilistic) on the future actions of others. On the other hand, the player can calculate indexes relating to the past performance of his own strategies (or of all strategies). It will then assume that these indicators remain valid in the future and possibly compare them to normative aspiration thresholds, themselves progressive (increasing or decreasing according to whether or not they have been reached in the past). More rarely, he will try to reveal, from the utilities he himself has experienced, his own preferences and even, from the actions observed of others, certain structural characteristics of others, in particular their preferences. The decision principle explains the choice rules adopted by each player, based on the previous aggregated information. There are two types of behavior, which take into account the strategic dimension of the information that it is likely to acquire. Exploitation behavior consists in using in the best interests of the already existing information. Exploration behavior consists of moving the previous action more or less randomly to test opponents and acquire original information.

The player must in fact make his decision between 'exploration' and 'exploitation', the second translating into aloss of utility in the short term in favor of a long-term utility gain (information investment). This arbitration (not optimal in general) is often directly inscribed in the rule of choice, which combines the two behaviors. In particular, the exploration must appear to be more important at the beginning of the process and the exploitation at the end of the process.

➢ *Capital Market equilibrium*

We note Qk (P, Fk) the demand of an investor with information Fk and X the global offer of risky assets. The equilibrium price is determined by the equilibrium condition of the risky asset market:

$$\sum_{k=1}^{N+M} Q_k \text{ (P, } F_k \text{)=X}$$

The equilibrium price depends on X and S. The functional form of this equilibrium price is determined precisely by the way in which the agents formulate their expectations (calculate E_k (V F_k) It is at this level that the hypothesis of rational expectations comes into play. H.4 It is assumed that agents have rational expectations. This means that the agents formulate their expectations on the basis of the functional relationship that is effectively established between the equilibrium price, X and S. The knowledge of this relation allows them to determine the joint distribution of Vet P. The information set of an uninformed agent (henceforth referred to as Fu) includes not only the observation of the equilibrium price but also the relation which links this price to the realization of X and S. This implies it is not possible to determine the demand functions and the equilibrium price separately. The demand functions obtained are the demand functions that are realized at equilibrium

➢ *Typology*

Three large families of evolutionary models are generally considered, which correspond to relatively contrasting illustrations of the aforementioned principles (Walliser, 1998). They attribute to the players information increasingly reduced and forms of rationality cognitive (matching between available means and objectives) and instrumental (adequacy between available information and beliefs adopted) increasingly weak (Walliser, 1989). Epistemic learning is based on a process of revising stakeholder beliefs about the strategies of their opponents. Behavioral learning is based on the reinforcement by the actors of their own best-performing strategies. The evolutionary process is based on a Lamarckian mechanism of natural selection of individuals forming populations. These three types of models can be combined in hybrid models combining various principles. In addition, the second and third families present a formal isomorphism, which can be reduced to the first two families, which are the most realistic. Epistemic learning assumes that the actors know their utility function and are able to observe their respective actions. Based on these observations, they revise their beliefs about the future strategies of others. Relying on their utility function, they choose their best response to these beliefs, by virtue of exploitative behavior. This better response can, however, be 'disturbed' by hazards, by virtue of the exploration behavior. The simplest model obeying this scheme is that of 'fictitious play'. Each player first calculates the observed frequency of use of each other's action; it then transforms this past frequency into a future probability of using an action; he finally chooses his best answer in the sense of the expectation of utility of each of his actions. The model of 'fictitious stochastic play' is a variant that assumes that the player no longer optimizes, but plays an action with increasing probability with its expectation of future utility.

Behavioral learning assumes that the actors only observe the performance (in terms of utility) related to their own past actions. They calculate an aggregate index of performance of each action, possibly compared to a suction threshold. They adopt a probabilistic choice, the probability of playing an action being increasing with its index; this choice thus simultaneously presents an exploration component (by the fact that any action always has a non-zero probability of being used) and exploitation (by the fact that the most powerful actions are played more often). They can also mimic the behavior of the most successful players if they observe their performance. The simplest model obeying this scheme is the CPR model. Each player is satisfied first to observe the utility obtained with the action he has implemented; it then calculates the cumulative utility obtained by this action since the beginning of the game;

Finally, he takes a probabilistic decision by choosing each action with a probability proportional to his index. The evolutionary process supposes that the actors no longer observe anything (except the actions of others if their strategies depend on it) and no longer hold beliefs. They have a fixed behavior, but form populations of individuals symbolizing the same player. They undergo a breeding process favoring the reproduction of those who obtain the best utility (assimilated to the 'fitness' of biologists), which reflects the behavior of exploitation; they may experience a mutation process that randomly changes their strategy, which indicates an exploration behavior. The simplest model obeying this scheme is the replicator model. In a multi-population or mono-population setting, individuals meet randomly; they reproduce proportionally to the utility they derive from their interactions. One variant, the stochastic replicator, also considers mutant individuals randomly introduced into the population at each period.

- *Results*

We are now interested in the asymptotic states of the preceding processes, knowing that these processes can converge towards stationary states (point attractors) rather than converging (cyclic or chaotic attractors). We can notice that the problem of the selection of an equilibrium no longer arises because the trajectory of the system is always directed (at least in probability) towards a certain outcome (if it converges). The results obtained are valid only for particular classes of dynamic processes and specific categories of games. They are also related to the type of stability imposed on the asymptotic trajectories as well as the time scale at which one is located (long term in the absence of stochastic disturbances, very long term with disturbances). These results are not very robust to small modifications of the model specifications and are particularly sensitive to stochastic elements introduced on different elements of the models (interaction modes, information sampling, decision rules). The results primarily concern dynamics based on static basic games. In fact, the only truly general result is the elimination of heavily dominated strategies as well as iteratively heavily dominated strategies. This result has been demonstrated for quite extensive evolutionary processes including the standard replicator (Weibull, 1995) as well as for various learning processes. However, it is not maintained for strategies (iteratively) weakly dominated. Samuelson-Zhang (1992) has indeed shown that in case of evolutionary dynamics, with the dynamics of the replicator and even in the presence of noise, weakly dominated strategies can be preserved. This result applies to dynamic base games because they can be reduced to static games thanks to the notion of strategy. They assure that perfect balances, which are never strongly dominated, but are sometimes weakly dominated, are not systematically eliminated. If we look at Nash equilibria of static games, they appear as rest points of most dynamic processes (if we are initially there, we stay there). More ambitiously, Nash equilibria in strict pure strategies can be obtained as limit states in some processes because any deviation leads to a strict loss of utility. Thus, with the CPR rule (Laslier-Topol- Walliser, 2002), trajectories converge with a positive probability to any Nash equilibrium in pure strategies, if any exist. Similarly, with the dynamics of the standard replicator in a multipopulation frame, a state is asymptotically stable if and only if it is a strict Nash equilibrium. On the other hand, Nash equilibria in mixed strategies are much more difficult to obtain. Still with the CPR model, the convergence of the trajectories with positive probability towards such equilibria is realized only for particular classes of games, for example those with a unique equilibrium. For the stochastic fictitious play model, however, such convergence with global stability is assured (Hofbauer-Sandholm, 2001) for various classes of games (zero sum, potential, supermodular).

Refinements of the Nash equilibrium can sometimes be obtained, always for static basic games. Thus, Young (1993a) considers a model of stochastic fictitious play where each player is endowed with a limited memory, observes only a sample of the actions of others and chooses his action either as a better answer to these observations (with a certain probability) either randomly (with the complementary probability). It shows that, for 2×2 coordination games, the stochastically stable equilibria correspond to a selection of Nash equilibria in pure strategies, namely the 'risqudominants' equilibria. This selection is only due to the random disturbances that cause the trajectory of the system in the most extensive basins of attraction. A similar result is obtained by Kandori-Mailath-Rob (1993), who consider a limit process of fictitious stochastic play (each player defines his best response to the distribution of the shares of others over the previous period, with the possibility of a random deviation of the action). More recent results focus on static base games in which players are located on a network and have only local interactions. More precisely, they are distributed on a circle, a grid plan or a grid torus and interact or collect information only with their immediate neighbors. Ellison (1993) adopts the KandoriMailath-Rob (1993) model without disturbances and shows that one can only end up with a limit cycle or converge towards a Nash equilibrium. By adding a stochastic (spatial) sampling procedure to the Young's information, Durieu-Solal (2000) shows that limit cycles can be eliminated to keep only the Nash equilibria; we can even obtain the risk-dominant equilibrium if the players are able to experiment. In any case, one can asymptotically obtain various spatial structures, in particular a segmentation of the domain showing such equilibrium in certain zones and another in other zones.

Results were finally obtained for dynamic basic games (generic and with perfect information), namely a convergence towards the perfect (unique) balance. By relying on a particular epistemic learning process that allows for random mutations, Nöldeke-Samuelson (1993) has shown that any 'locally stable' outcome is a perfect equilibrium, although any perfect equilibrium could be found locally. stable with other balances. With regard to learning by reinforcement, it is now applied to each action relating to an arc of the game tree and no longer to the overall strategy, the global utility obtained following a track followed being affected. to each of its constituent actions (Pak, 2001). With the CPR model, the convergence of the process towards the perfect balance is guaranteed by the fact that each action is played an infinite number of times (Laslier-Walliser, 2002). Similar results have been obtained with evolutionary processes.

V. CONCLUSION

The results obtained from an eductive perspective and from an evolutionary perspective present, with regard to the dispersion of the contexts considered, both strong similarities and strong differences. Strategies (iteratively) strongly dominated are unanimously eliminated. In an evolutionary framework, pure and strict Nash balances are often justified, and even mixed equilibria for epistemic learning. On the other hand, in an educational framework, Nash equilibria can be obtained, whether pure or mixed, only under very drastic conditions. However, weaker notions of equilibrium are justified in an educational framework, such as the rationalizable equilibrium or the correlated equilibrium, which have no clear counterpart (currently) in an evolutionary framework. Finally, the perfect balance is obtained in both perspectives, possibly with other balances, in a way that is not robust in an educational setting, in a more robust way in an evolutionary framework. The results obtained are very sensitive to the details of the modeling and require a very fine formalization to specify the conditions of validity of each notion of equilibrium. Those made in an eductive perspective have even highlighted the existence of implicit rationality hypotheses playing a fundamental role in the conclusions. The work carried out has led to the development of traditional analytical tools and the mobilization of new tools, this quest is currently continuing. The eductive perspective can be enriched by new logical tools such as temporal logics while the evolutionist perspective is based on new theorems concerning stochastic processes applied to networks. The efforts undertaken finally make it possible to shed light on certain more concrete problems, if they can be expressed in terms of equilibrium of games. This is the problem of the genesis of institutions, which results from the conscious coordination of the actors or a partially unconscious learning process.

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