# A Mapping Framework for Mapping from Subsets of Complex Matrix Spaces of Order m by n to the Subsets of Complex Matrix Spaces of Order J, Where J≥2

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**Abstract:- The present article extends the formalism proposed in [2] and presents a framework for mapping from subsets of the complex Matrix space $M_{m\times n}(C)$ characterized by a set of Global Mass and Global Alignment factor, denoted as $\bar{M}(r,c)$, to the subset of the Matrix space $M_{J\times J}(C)$, where $J \geq 2$, of all Hermitian, unit trace, positive definite and positive semi definite matrices of order *J*, denoted as $\hat{S}[M_{J\times J}(C)]$. The article presents the Mathematical formalism and illustrates the same with suitable numerical examples.**

***Keywords:-*** *Global Mass Factor of a Matrix, Global Alignment Factor of a Matrix, Effective Global Mass Factor of a Matrix, Hermitian Matrices, Positive Definite Matrices, Positive Semi definite Matrices.*

**Notations**

- $C$ denotes the complex number field
- $N$ denotes the set of all natural numbers
- $M_{m\times n}(C)$ denotes the Complex Matrix space of Matrices of order m by n
- $M_{J\times J}(C)$ denotes the Complex Matrix space of Matrices of order '*J*'
- $R(A)$ denotes the Global Mass Factor associated with the matrix $A_{m\times n}$
- $C(A)$ denotes the Global Alignment Factor associated with the matrix $A_{m\times n}$
- $R_0(A)$ denotes the Effective Global Mass Factor associated with the matrix $A_{m\times n}$
- $|c|$ denotes the modulus of the complex number $c$
- $\{|e_1\rangle, |e_2\rangle, ...., |e_m\rangle\}$ denotes the standard Orthonormal basis in $C^m$ and $\{|f_1\rangle, |f_2\rangle, ...., |f_n\rangle\}$ denotes the standard Orthonormal basis in $C^n$
- $c^{\bullet}$ denotes the complex conjugate of the complex number $c$
- $X^H$ denotes the Hermitian conjugate of the matrix $X$
- $\bar{M}(r,c)$ Is a subset of the Complex Matrix space $M_{m\times n}(C)$ characterized by the numerical values of the Global Mass factor and Global alignment factor, $r$ and $c$, respectively.
- $|V\rangle = \begin{bmatrix} v_1 \\ v_2 \\ . \\ . \\ v_p \end{bmatrix}_{p\times 1}, \langle V| = [v_1^{\bullet} \quad v_2^{\bullet} \quad . \quad . \quad v_p^{\bullet}]_{1\times p}, B = \left[b_{ij}\right]_{p\times p}, \langle V|B|V\rangle = \sum_{i=1}^{p}\sum_{j=1}^{p} b_{ij} v_i^{\bullet} v_j$

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# I. INTRODUCTION

In this research article, the formalism for the mapping scheme from subsets $\bar{M}(r,c)$ of the complex Matrix Space $M_{m\times n}(C)$ to subset $\hat{S}[M_{J\times J}(C)]$ of the Matrix Space $M_{J\times J}(C)$, for any arbitrary order '$J$ '$(J\in N, J\geq 2)$, is presented. This is achieved through appropriate block-partitioning structure of the vectors $\left|\theta_{xy}\right\rangle$ and $\left|\phi_{xy}\right\rangle$ associated with the $(x,y)$ degree of freedom of the matrix $A_{m\times n}$, for the cases when $J$ = even and J = odd. The dimensionality of the vectors $\left|\theta_{xy}\right\rangle$ and $\left|\phi_{xy}\right\rangle$ is dictated by the chosen value of '$J$ ', its elements are determined by the modulus terms and phase terms interrelationships corresponding to the matrix $A_{m\times n}$, where $A_{m\times n}\in\bar{M}(r,c)$.

The article discuses the numerical examples that were used in [2] for illustration of the mapping scheme in context of the $J$ = 2 case, and provides here the corresponding results for the cases $J$ = 3 and $J$ = 4.

# II. MATHEMATICAL FRAMEWORK AND ASSOCIATED ANALYSIS

The following set of results, stated in [1] and [2], forms the platform for the formalism described in this research article:

- $A\in M_{m\times n}(C), A=\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}\left|e_i\right\rangle\left\langle f_j\right|$ , $a_{ij}=r_{ij}c_{ij}$ , $r_{ij}=\left|a_{ij}\right|$ , $c_{ij}\in C$ , $\left|c_{ij}\right|=1$ , we consider the following convention that in the case of zero matrix elements of matrix $A$: $a_{ij}=0\Rightarrow r_{ij}=0, c_{ij}=1$
- $R(A)=\sum_{i=1}^{m}\sum_{j=1}^{n}r_{ij}$ , $R_0(A)=\sum_{i=1}^{m}\sum_{j=1}^{n}r_{ij}(1-\exp(-r_{ij}))$
- $C(A)=c_{11}c_{12}....c_{1n}c_{21}c_{22}....c_{2n}.........c_{m1}c_{m2}....c_{mn}=\prod_{i=1}^{m}\prod_{j=1}^{n}c_{ij}$ , $C(A)\in C, \left|C(A)\right|=1, \forall A\in M_{m\times n}(C)$
- $\bar{M}(r,c)\subset M_{m\times n}(C), \bar{M}(r,c)=\{A\in M_{m\times n}(C)\mid A\neq 0_{m\times n}, R(A)=r, C(A)=c\}$ , where we have the condition: $r>0, c\in C, \left|c\right|=1$
- $\lambda_{ij}=(\frac{r_{ij}}{r_0})(1-\exp(-r_{ij}))$ , where $r_0=\sum_{i=1}^{m}\sum_{j=1}^{n}r_{ij}(1-\exp(-r_{ij}))$ , is the numerical realization of the Effective Global Mass factor $R_0(A)$
- $\mu_{ij}=(\frac{r_{ij}}{r})$ , where $r=\sum_{i=1}^{m}\sum_{j=1}^{n}r_{ij}$ , is the numerical realization of the Global Mass factor $R(A)$
- $\lambda_{ij}\geq 0, \forall i=1,2,..m; j=1,2,....,n$ and we have: $\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}=1$
- $\mu_{ij}\geq 0, \forall i=1,2,..m; j=1,2,....,n$ and we have: $\sum_{i=1}^{m}\sum_{j=1}^{n}\mu_{ij}=1$
- $\alpha_i=c_{i1}c_{i2}.....c_{in}$ , $\beta_j=c_{1j}c_{2j}.....c_{mj}$ , Therefore, $\alpha_i\beta_j=c_{i1}c_{i2}.....c_{in}c_{1j}c_{2j}.....c_{mj}$ , $i=1,2,..m; j=1,2,....,n$ , we have: $\alpha_i\beta_j\in C, \left|\alpha_i\beta_j\right|=1, \forall i=1,2,....m; j=1,2,....,n$
- $\hat{S}[M_{J\times J}(C)]\subset M_{J\times J}(C)$ , such that :

$$\hat{S}[M_{J\times J}(C)]=\{Q\in M_{J\times J}(C)\mid Q^H=Q, Trace(Q)=1, \left\langle v\right|Q\left|v\right\rangle\geq 0, or, \left\langle v\right|Q\left|v\right\rangle>0, \forall\left|v\right\rangle\in C^J, \left|v\right\rangle\neq 0_{J\times 1}\}$$

Case 1: (Even $J$)

- $J = 2P$, where : $P = 1,2,3,....$

- We define : $\left|\theta_{xy}\right\rangle_{J\times1} = (\frac{1}{\sqrt{P}})\begin{bmatrix}\left|\theta_{xy}(+)\right\rangle_{P\times1} \\ \left|\theta_{xy}(-)\right\rangle_{P\times1}\end{bmatrix}_{(2P)\times1}$ , where we have :

$$\left|\theta_{xy}(+)\right\rangle_{P\times1} = \begin{bmatrix}\sqrt{\lambda_{xy}}(\alpha_x\beta_y)(\alpha_x\beta_y)....(\alpha_x\beta_y) \\ . \\ . \\ \sqrt{\lambda_{xy}}(\alpha_x\beta_y)(\alpha_x\beta_y) \\ \sqrt{\lambda_{xy}}(\alpha_x\beta_y)\end{bmatrix}_{P\times1},$$

$$\left|\theta_{xy}(-)\right\rangle_{P\times1} = \begin{bmatrix}\sqrt{1-\lambda_{xy}}(\alpha_x\beta_y)^{\bullet}(\alpha_x\beta_y)^{\bullet}....(\alpha_x\beta_y)^{\bullet} \\ . \\ . \\ \sqrt{1-\lambda_{xy}}(\alpha_x\beta_y)^{\bullet}(\alpha_x\beta_y)^{\bullet} \\ \sqrt{1-\lambda_{xy}}(\alpha_x\beta_y)^{\bullet}\end{bmatrix}_{P\times1}$$

- We define : $\left|\phi_{xy}\right\rangle_{J\times1} = (\frac{1}{\sqrt{P}})\begin{bmatrix}\left|\phi_{xy}(+)\right\rangle_{P\times1} \\ \left|\phi_{xy}(-)\right\rangle_{P\times1}\end{bmatrix}_{(2P)\times1}$ , where we have :

$$\left|\phi_{xy}(+)\right\rangle_{P\times1} = \begin{bmatrix}\sqrt{\mu_{xy}}(\alpha_x\beta_y)(\alpha_x\beta_y)....(\alpha_x\beta_y) \\ . \\ . \\ \sqrt{\mu_{xy}}(\alpha_x\beta_y)(\alpha_x\beta_y) \\ \sqrt{\mu_{xy}}(\alpha_x\beta_y)\end{bmatrix}_{P\times1},$$

$$\left|\phi_{xy}(-)\right\rangle_{P\times1} = \begin{bmatrix}\sqrt{1-\mu_{xy}}(\alpha_x\beta_y)^{\bullet}(\alpha_x\beta_y)^{\bullet}....(\alpha_x\beta_y)^{\bullet} \\ . \\ . \\ \sqrt{1-\mu_{xy}}(\alpha_x\beta_y)^{\bullet}(\alpha_x\beta_y)^{\bullet} \\ \sqrt{1-\mu_{xy}}(\alpha_x\beta_y)^{\bullet}\end{bmatrix}_{P\times1}$$

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- $\langle \theta_{xy} | \theta_{xy} \rangle = 1$ , $\langle \phi_{xy} | \phi_{xy} \rangle = 1$ , $\forall \quad x = 1,2,....,m$ , $y = 1,2,....,n$

Case 2: (Odd *J*)

- $J = 2P + 1, P = 1,2,3,....$

- We define : $|\theta_{xy}\rangle_{J\times 1} = (\frac{1}{\sqrt{P+1}}) \begin{bmatrix} |\theta_{xy}(+)\rangle_{P\times 1} \\ (1)_{1\times 1} \\ |\theta_{xy}(-)\rangle_{P\times 1} \end{bmatrix}_{(2P+1)\times 1}$ , where we have :

$$|\theta_{xy}(+)\rangle_{P\times 1} = \begin{bmatrix} \sqrt{\lambda_{xy}}(\alpha_x\beta_y)(\alpha_x\beta_y)....(\alpha_x\beta_y) \\ . \\ . \\ \sqrt{\lambda_{xy}}(\alpha_x\beta_y)(\alpha_x\beta_y) \\ \sqrt{\lambda_{xy}}(\alpha_x\beta_y) \end{bmatrix}_{P\times 1} ,$$

$$|\theta_{xy}(-)\rangle_{P\times 1} = \begin{bmatrix} \sqrt{1-\lambda_{xy}}(\alpha_x\beta_y)^{\bullet}(\alpha_x\beta_y)^{\bullet}....(\alpha_x\beta_y)^{\bullet} \\ . \\ . \\ \sqrt{1-\lambda_{xy}}(\alpha_x\beta_y)^{\bullet}(\alpha_x\beta_y)^{\bullet} \\ \sqrt{1-\lambda_{xy}}(\alpha_x\beta_y)^{\bullet} \end{bmatrix}_{P\times 1}$$

- We define : $|\phi_{xy}\rangle_{J\times 1} = (\frac{1}{\sqrt{P+1}}) \begin{bmatrix} |\phi_{xy}(+)\rangle_{P\times 1} \\ (1)_{1\times 1} \\ |\phi_{xy}(-)\rangle_{P\times 1} \end{bmatrix}_{(2P+1)\times 1}$ , where we have :

$$|\phi_{xy}(+)\rangle_{P\times 1} = \begin{bmatrix} \sqrt{\mu_{xy}}(\alpha_x\beta_y)(\alpha_x\beta_y)....(\alpha_x\beta_y) \\ . \\ . \\ \sqrt{\mu_{xy}}(\alpha_x\beta_y)(\alpha_x\beta_y) \\ \sqrt{\mu_{xy}}(\alpha_x\beta_y) \end{bmatrix}_{P\times 1} ,$$

$$\left|\phi_{xy}(-)\right\rangle_{P\times 1}=\begin{bmatrix}\sqrt{1-\mu_{xy}}(\alpha_x\beta_y)^{\bullet}(\alpha_x\beta_y)^{\bullet}....(\alpha_x\beta_y)^{\bullet}\\ .\\ .\\ \sqrt{1-\mu_{xy}}(\alpha_x\beta_y)^{\bullet}(\alpha_x\beta_y)^{\bullet}\\ \sqrt{1-\mu_{xy}}(\alpha_x\beta_y)^{\bullet}\end{bmatrix}_{P\times 1}$$

- $\left\langle\theta_{xy}\middle|\theta_{xy}\right\rangle=1,\left\langle\phi_{xy}\middle|\phi_{xy}\right\rangle=1,\forall x=1,2,....,m,y=1,2,....,n$

**Construction of** $\rho_{J\times J}$ :

$$[\rho(1)]_{J\times J}=\sum_{x=1}^{m}\sum_{y=1}^{n}\mu_{xy}(\left|\theta_{xy}\right\rangle\left\langle\theta_{xy}\right|)_{J\times J}\quad,\quad[\rho(2)]_{J\times J}=\sum_{x=1}^{m}\sum_{y=1}^{n}\lambda_{xy}(\left|\phi_{xy}\right\rangle\left\langle\phi_{xy}\right|)_{J\times J}$$

Where we have $J=2P$ or $J=2P+1,\ P=1,2,3,....$

$$\rho_{J\times J}=(\frac{1}{2})[\rho(1)]_{J\times J}+(\frac{1}{2})[\rho(2)]_{J\times J}=(\frac{1}{2})\sum_{x=1}^{m}\sum_{y=1}^{n}\mu_{xy}(\left|\theta_{xy}\right\rangle\left\langle\theta_{xy}\right|)_{J\times J}+(\frac{1}{2})\sum_{x=1}^{m}\sum_{y=1}^{n}\lambda_{xy}(\left|\phi_{xy}\right\rangle\left\langle\phi_{xy}\right|)_{J\times J}\quad\text{..... (eqn. 1)}$$

Clearly, $\rho_{J\times J}\in\hat{S}[M_{J\times J}(C)]$ ,

- The complete mapping scheme is described in terms of the Transformation $\hat{\Lambda}$ :

$\hat{\Lambda}:\bar{M}(r,c)\mapsto\hat{S}[M_{J\times J}(C)]$ , such that:

$$\hat{\Lambda}(\sum_{i=1}^{m}\sum_{j=1}^{n}r_{ij}c_{ij}(\left|e_i\right\rangle\left\langle f_j\right|)_{m\times n})=(\frac{1}{2})\sum_{x=1}^{m}\sum_{y=1}^{n}\mu_{xy}(\left|\theta_{xy}\right\rangle\left\langle\theta_{xy}\right|)_{J\times J}+(\frac{1}{2})\sum_{x=1}^{m}\sum_{y=1}^{n}\lambda_{xy}(\left|\phi_{xy}\right\rangle\left\langle\phi_{xy}\right|)_{J\times J}$$

..... (eqn.2)

**Numerical Examples**

**1)** $A_{2\times 3}=\begin{bmatrix}1&0&0\\0&1&0\end{bmatrix}$ , we have: $r(A)=2$ , $r_0(A)=2(1-\exp(-1))$ , $A\in\bar{M}(r=2,c=1)$

- $\lambda_{11}=\frac{1}{2},\lambda_{12}=0,\lambda_{13}=0,\lambda_{21}=0,\lambda_{22}=\frac{1}{2},\lambda_{23}=0$
- $\mu_{11}=\frac{1}{2},\mu_{12}=0,\mu_{13}=0,\mu_{21}=0,\mu_{22}=\frac{1}{2},\mu_{23}=0$
- $\alpha_i\beta_j=1,\forall i=1,2,....,m;\ j=1,2,....,n$

<u>J=3:</u>

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$$\hat{\Lambda}(A_{2\times3}) = \rho_{3\times3} = (\frac{1}{4})\begin{bmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 2 & \sqrt{2} \\ 1 & \sqrt{2} & 1 \end{bmatrix} = |v\rangle\langle v| \text{ , } \quad \text{where} \quad |v\rangle = (\frac{1}{2})\begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix}$$

*J=4:*

$$\hat{\Lambda}(A_{2\times3}) = \rho_{4\times4} = (\frac{1}{4})\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = |v\rangle\langle v| \text{ , } \quad \text{where} \quad |v\rangle = (\frac{1}{2})\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

**2)** $B_{2\times3} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ , we have: $r(B) = 2$ , $r_0(B) = 2(1 - \exp(-2))$ , $B \in \bar{M}(r = 2, c = 1)$

- $\lambda_{11} = 1, \lambda_{12} = 0, \lambda_{13} = 0, \lambda_{21} = 0, \lambda_{22} = 0, \lambda_{23} = 0$
- $\mu_{11} = 1, \mu_{12} = 0, \mu_{13} = 0, \mu_{21} = 0, \mu_{22} = 0, \mu_{23} = 0$
- $\alpha_i \beta_j = 1, \forall i = 1, 2, ...., m; \ j = 1, 2, ...., n$

*J=3:*

$$\hat{\Lambda}(B_{2\times3}) = \rho_{3\times3} = (\frac{1}{2})\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = |v\rangle\langle v| \text{ , } \quad \text{where} \quad |v\rangle = (\frac{1}{\sqrt{2}})\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

*J=4:*

$$\hat{\Lambda}(B_{2\times3}) = \rho_{4\times4} = (\frac{1}{2})\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = |v\rangle\langle v| \text{ , } \quad \text{where} \quad |v\rangle = (\frac{1}{\sqrt{2}})\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

**3)** $C_{2\times3} = \begin{bmatrix} -i & 0 & 0 \\ 0 & +i & 0 \end{bmatrix}$ , we have: $r(C) = 2$ , $r_0(C) = 2(1 - \exp(-1))$ , $C \in \bar{M}(r = 2, c = 1)$

- $\lambda_{11} = \frac{1}{2}, \lambda_{12} = 0, \lambda_{13} = 0, \lambda_{21} = 0, \lambda_{22} = \frac{1}{2}, \lambda_{23} = 0$
- $\mu_{11} = \frac{1}{2}, \mu_{12} = 0, \mu_{13} = 0, \mu_{21} = 0, \mu_{22} = \frac{1}{2}, \mu_{23} = 0$
- $\alpha_1\beta_1 = -1, \alpha_1\beta_2 = 1, \alpha_1\beta_3 = -i, \alpha_2\beta_1 = 1, \alpha_2\beta_2 = -1, \alpha_2\beta_3 = +i$

*J=3:*

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$$\hat{\Lambda}(C_{2\times 3}) = \rho_{3\times 3} = (\frac{1}{4})\begin{bmatrix} 1 & -\sqrt{2} & 1 \\ -\sqrt{2} & 2 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{bmatrix} = |v\rangle\langle v| \ , \quad \text{where } |v\rangle = (\frac{1}{2})\begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix}$$

*J=4:*

$$\hat{\Lambda}(C_{2\times 3}) = \rho_{4\times 4} = (\frac{1}{4})\begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix} = |v\rangle\langle v| \ , \quad \text{where } |v\rangle = (\frac{1}{2})\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

**4)** $D_{2\times 3} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ , we have: $r(D) = 6$ , $r_0(D) = 6(1-\exp(-1))$ , $D \in \bar{M}(r = 6, c = 1)$

- $\lambda_{ij} = (\frac{1}{6}), \mu_{ij} = (\frac{1}{6}), \forall i = 1,2,....,m;\ j = 1,2,....,n$
- $\alpha_i \beta_j = 1, \forall i = 1,2,....,m;\ j = 1,2,....,n$

*J=3:*

$$\hat{\Lambda}(D_{2\times 3}) = \rho_{3\times 3} = (\frac{1}{12})\begin{bmatrix} 1 & \sqrt{6} & \sqrt{5} \\ \sqrt{6} & 6 & \sqrt{30} \\ \sqrt{5} & \sqrt{30} & 5 \end{bmatrix} = |v\rangle\langle v| \ , \quad \text{where } |v\rangle = (\frac{1}{\sqrt{12}})\begin{bmatrix} 1 \\ \sqrt{6} \\ \sqrt{5} \end{bmatrix}$$

*J=4:*

$$\hat{\Lambda}(D_{2\times 3}) = \rho_{4\times 4} = (\frac{1}{12})\begin{bmatrix} 1 & 1 & \sqrt{5} & \sqrt{5} \\ 1 & 1 & \sqrt{5} & \sqrt{5} \\ \sqrt{5} & \sqrt{5} & 5 & 5 \\ \sqrt{5} & \sqrt{5} & 5 & 5 \end{bmatrix} = |v\rangle\langle v| \ , \quad \text{where } |v\rangle = (\frac{1}{\sqrt{12}})\begin{bmatrix} 1 \\ 1 \\ \sqrt{5} \\ \sqrt{5} \end{bmatrix}$$

**5)** $E_{2\times 3} = \begin{bmatrix} 1 & +i & -i \\ -i & 1 & +i \end{bmatrix}$ , we have: $r(E) = 6$ , $r_0(E) = 6(1-\exp(-1))$ , $E \in \bar{M}(r = 6, c = 1)$

- $\lambda_{ij} = (\frac{1}{6}), \mu_{ij} = (\frac{1}{6}), \forall i = 1,2,....,m;\ j = 1,2,....,n$
- $\alpha_1\beta_1 = -i, \alpha_1\beta_2 = +i, \alpha_1\beta_3 = 1, \alpha_2\beta_1 = -i, \alpha_2\beta_2 = +i, \alpha_2\beta_3 = 1$

*J=3:*

$$\hat{\Lambda}(E_{2\times3}) = \rho_{3\times3} = (\frac{1}{36})\begin{bmatrix} 3 & \sqrt{6} & -\sqrt{5} \\ \sqrt{6} & 18 & \sqrt{30} \\ -\sqrt{5} & \sqrt{30} & 15 \end{bmatrix},$$

Eigenvalues of $\rho_{3\times3}$: 0.616612, 0.333333, 0.050054 .... (up to 6 decimal places)

*J=4:*

$$\hat{\Lambda}(E_{2\times3}) = \rho_{4\times4} = (\frac{1}{36})\begin{bmatrix} 3 & 1 & 3\sqrt{5} & \sqrt{5} \\ 1 & 3 & \sqrt{5} & -\sqrt{5} \\ 3\sqrt{5} & \sqrt{5} & 15 & 5 \\ \sqrt{5} & -\sqrt{5} & 5 & 15 \end{bmatrix},$$

Eigenvalues of $\rho_{4\times4}$: 0.616612, 0.333333, 0.050054, 0 ..... (up to 6 decimal places)

## III. DISCUSSION AND CONCLUSION

The present article completes the research initiative undertaken in [2] and proposes a general framework to map a non-zero complex Matrix belonging to the Matrix space $M_{m\times n}(C)$ to unit trace, Hermitian, Positive definite/semi definite Matrices belonging to the matrix space $M_{J\times J}(C)$, for any $J \in N, J \geq 2$. This mapping is intricately dependant on the modulus terms $r_{xy}$ distribution and the phase terms $c_{xy}$ interrelationships associated with the matrix $A_{m\times n} \in \bar{M}(r,c)$.

In the Numerical examples 1 through 3, the matrices $A_{2\times3}, B_{2\times3}, C_{2\times3}$ belong to the subset $\bar{M}(r=2, c=1)$ of the complex matrix space $M_{2\times3}(C)$, they differ from each other either in terms of modulus term distribution or phase term interrelationships or both, we observe all of them are mapped to rank-1 positive semi definite matrices in $\hat{S}[M_{3\times3}(C)]$ and also in $\hat{S}[M_{4\times4}(C)]$.

In the Numerical examples 4 and 5, $D_{2\times3}$ and $E_{2\times3}$ belong to the subset $\bar{M}(r=6, c=1)$ of the matrix space $M_{2\times3}(C)$, they differ from each other in terms of phase term interrelationships. It is observed that $D_{2\times3}$ is mapped to rank-1 positive semi definite matrices in $\hat{S}[M_{3\times3}(C)]$ and also in $\hat{S}[M_{4\times4}(C)]$. However, $E_{2\times3}$ is mapped to positive definite matrix in $\hat{S}[M_{3\times3}(C)]$ and to a rank-3 positive semi-definite matrix in $\hat{S}[M_{4\times4}(C)]$.

In subsequent follow up studies, the mapping formalism will be analyzed further to understand its intricacies with greater depth and clarity and its scope of applicability in solving Theoretical and Computational problems.

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