

The Final Euclidean Solution for the Trisection of Random Acute Angle

First Ever Presentation in the History of Geometry

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(Dedicated to the memory of my beloved parents Spiros & Stavroula.)

Abstract:- A brief introduction about the Euclidean solution's "impossibility" of the trisection problem...

The historic problem of Euclidean trisection for a random acute angle, was involved humanity, from 6th century BC without any interruption until the late 19th century, by not finding a satisfied solution according to Euclidean Geometry.

The trisection is an equal achievement of making the "impossible" into possible, because there is a huge list of names, that includes the greatest genius mathematicians of all times, such as: Hippocrates of Chios, Archimedes, Nicomedes, Descartes, Pascal and Lagrange that all failed to give a satisfied solution according to Euclidean Geometry!

Never the less non-Euclidean solutions have been presented in the past, such as the Archimedes's Neusis method, that requires a measured straight edge with ruler.

The ancient Greeks found that certain angles could be trisected rather easily. The problem of trisecting a right angle is a relatively simple process. But the real trisection problem emerges, when we have to deal with an unknown acute angle.

Furthermore Pierre Wantzel's theorem of trisection impossibility, presented in the mid 19th century (1837), gave birth to more speculations about the already existing myth of the problem.

Since then almost two centuries have passed, and now in 21st century, things have changed dramatically. We all realise that future overcomes the limitations of the past and what remained "impossible", now becomes possible. The most difficult achievement for human intellectuality, always remains this: to make something that is simple to become simpler! And whenever this is happened as a fact, is regarded by all historians as a milestone. This very motto is the basis (or the inspiration if you ou want) of the newly arrived final solution of random acute angle's trisection, presented in this paper.!

As we have already mentioned, the limitations of the past becomes the offspring for the future!

This future has just arrived as present time and after so many centuries since antiquity, the Euclidean solution for angle's trisection is a fact.

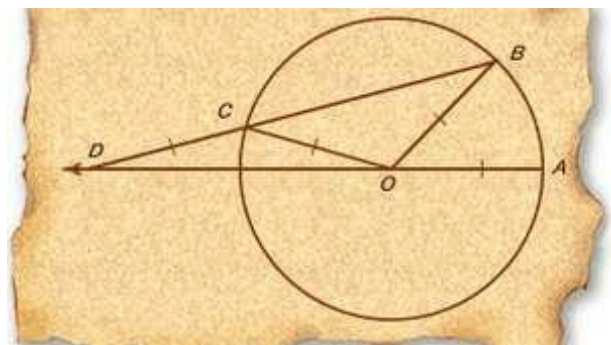
2500 years of "impossibility" have just ended and by doing so, this fact arrises new hopes to scientific researcher, amateurs or professionals, for even greater accomplishment in the future of humanity.

I. ARCHIMEDES' NON-EUCLEIDIAN METHOD OF TRISECTION

(ARTICLE TAKEN FROM BRITANNICA ENCYCLOPEDIA)

Written by: J.L. Heilbron/Senior Research Fellow at the University of Oxford, England. Author of *Geometry Civilized* and *The Sun in the Church* among others. (See Article History)

Euclid's insistence (c. 300 BC) on using only unmarked straightedge and compass for geometric constructions did not inhibit the imagination of his successors. Archimedes (c. 285–212/211 BC) made use of neusis (the sliding and maneuvering of a measured length, or marked straightedge) to solve one of the great problems of ancient geometry: constructing an angle that is one-third the size of a given angle.



Archimedes' method of angle trisection.

Encyclopedia Britannica, Inc.

For more information: www.britannica.com

II. TOWARDS A NEW EUCLIDIAN SOLUTION OF TRISECTING: THE THEOREM OF A CENTRAL ANGLE VERTICAL TO AN EQUAL INSCRIBED ONE.

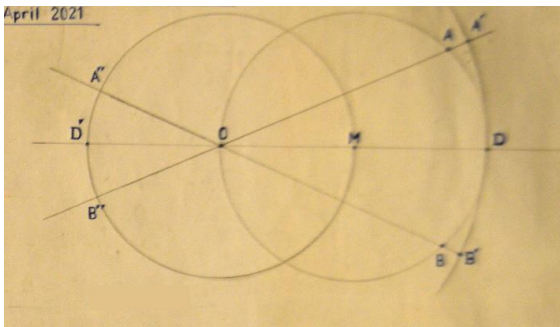


Diagram: The theorem of central and equal inscribed, angles vertical one to each other

We have a random angle A'OB' and we draw the bisector OD. We draw it as Inscribed angle of a circle where M is the midpoint of bisector OD and MD the radius of the circle. The new inscribed angle is AOB, and we will work with this from now on.

Then from center O we extend the angle sides A'O & B'O. We tranvert the new vertical angle, to a central one and we name it A''OB'', by drawing a circle with center point O and radius OA''=r=MD. Also OD' is the extension of bisector OD and also is bisector for angle A''OB'' too. That means that D'D is the bisector of both vertical angles.

SO FINALLY:
angle AOB=angle A''OB''
&
arch A''OB''=2*arch AOB

Now let us look at the use of DD' as a rotate line between the edges of both vertical angles. We will explore the ratio of the 4 random angles that are created by the rotation of DD'?

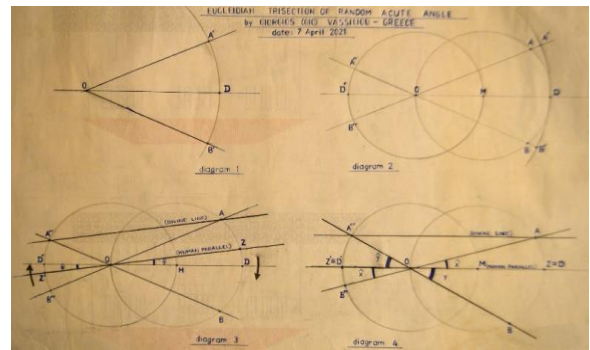
So we have:
 For every angle AOB=angle A''OB'' (vertical & so equal angles, A''OB'' is central angle to equal inscribed AOB angle)
 Also for EVERY LINE LIKE DD' that passes through center O & cuts both angles, the ratio of the four new created arches in vertical pairs are 2:1 so:

1. arch A''D'= 1/2arch DB or arch BD=2 arch A''D'
2. arch B''D'=1/2arch DA or arch DA=2 arch B''D'

SO THE FINAL FORM OF THE THEOREM IS THIS:
 When we have two vertical angles one to each other and one of them is the central

angle and the second is the equal inscribed of it, on an equal circle, then the arch of the inscribed angle is twice the length of the central one. Also for every rotating line (like DD') that passes from the center and cuts both angles, then the arch ratio for every pair of vertical angles is 2:1 for all cases.
THIS NEWLY PRESENTED GEOMETRICAL THEOREM, SERVES AS THE FOUNDATION FOR THE TRISECTION OF RANDOM ACUTE ANGLE.

III. THE FINAL EUCLIDIAN SOLUTION OF RANDOM ACUTE ANGLE'S TRISECTION (With the USE of compass & unmarked ruler)



Diagrams 1- 4: The four stages of trisection

«Δώσμει πα στο και ταν γαν κινάσω!»
"Give me a stable place and i will move the Earth!"
Archimedes of Syracuse

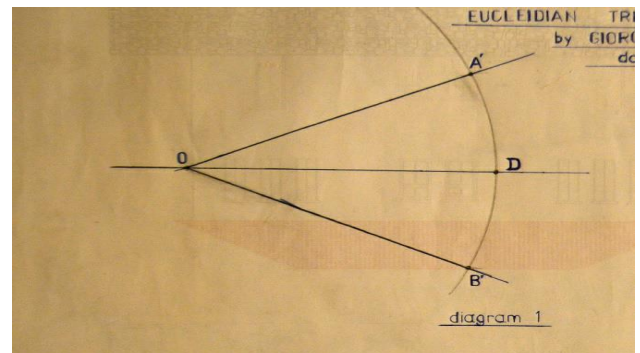


Diagram 1: First stage of trisection

GEOMETRICAL PROOF OF DIAGRAM 1

First we have drawn a random angle. Then we tranvert this angle to an inscribed one of a circle. We draw the circle (O, OA'). We see the points A' and B' are the intersections of angle's sides with the circle. We draw the bisector OD of angle A'OB'.

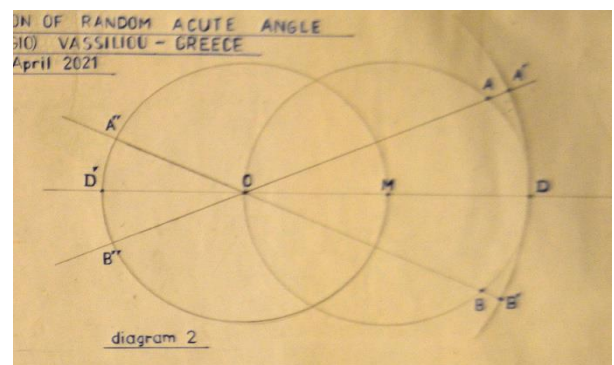


Diagram 2: Second stage of trisection

GEOMETRICAL PROOF OF DIAGRAM 2

From the center point M of OD we draw a circle (M, MD), and the intersections of the angle with the new circle are points A, B. Now we will work with angle AOB. Continuously we extend from point O the angle's sides AO, OB and we create a new central angle. Now we draw the

circle (O, OA") r=OA"=MD. The new central angle A"OB" (as an extension) is equal to angle AOB. Also D' is the bisector's OD extension point on angle A"OB". OD' is also the bisector of angle A"OB".

arch AB=3*arch A"Z'
 So we have...
 arch A"Z'=1/3 arch AB
 & arch AZ =arch A"Z'=1/3 arch AB

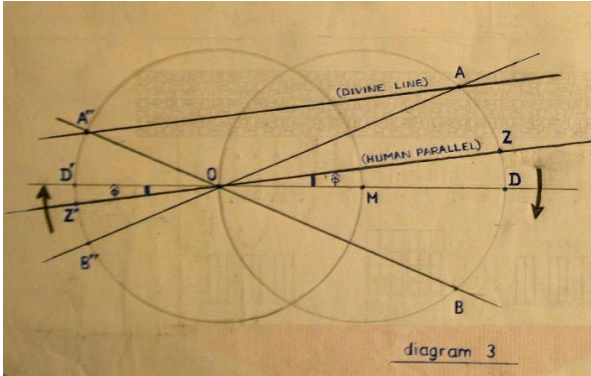


Diagram 3: Third stage of trisection

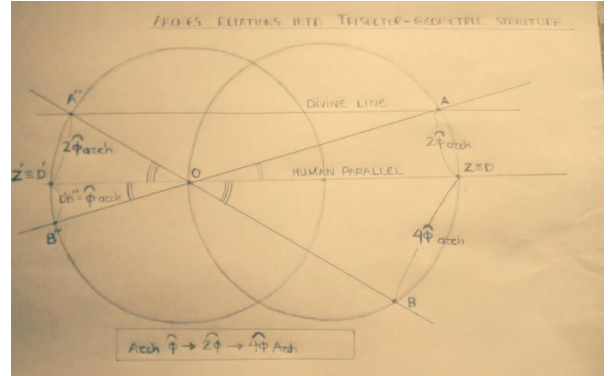


Diagram 5: Final geometric structure

GEOMETRICAL PROOF OF DIAGRAM 3

We draw the line AA". In this stage you will allow me, to call AA' as the DIVINE LINE, because of its crucial importance. Then from center O we draw a parallel line ZZ'//AA". This line will be called THE HUMAN PARALLEL. As we see, now we have to rotate the angles A"OB" and AOB, at an angle phi=angle D'OZ'. So point Z' now will meet point D' and Z point will meet point D. So the whole geometric structure will rotate at angle phi.

IV. CONCLUSION

This newly arrived simple Euclidean solution, presented in this article, puts an end to speculations or rumors and now a new Euclidean solution is possible.

Finally after 2500 years this gigantic unsolved problem has become solved by a very simple solution. In cases like this, appears that to make the simple simpler, in order to fully understand it, seems like the most difficult accomplishment for all history of human notion and intellectuality.

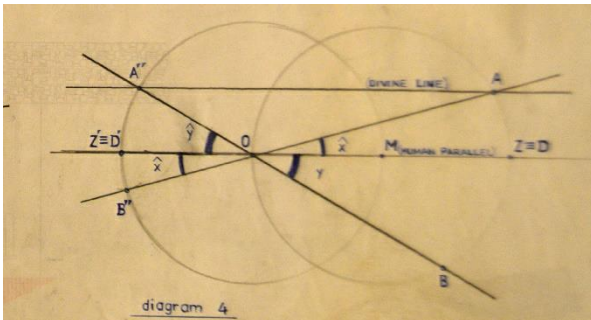


Diagram 4: Fourth stage of trisection

GEOMETRICAL PROOF OF DIAGRAM 4

This very geometric structure that leads to trisection, from now on it will be known by the name of "TRISECTOR"!

Now the solution has reached the end and we have the following equalities.

- a) AA"//ZZ'
- b) arch A"Z'=1/2* arch ZB (because angles A"OD' & DOB are vertical between them and also angle A"OB" is the central angle and ZOB the inscribed angle of it. So arch ZB =2*archA"D'.
- The same is applied for arches Z'B" and AZ. So arch AZ=2*archZ'B")
- c) arch A"Z'=arch AZ (they both are arches between parallel chords of equal circles)

AND FINALLY:

arch AB= arch AZ+ arch ZB=>
 arch AB= arch A"Z'+ arch ZB=>
 arch AB= arch A"Z' +2*arch AZ'=>

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