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Theory and Design of Audio Rooms-Reformulation of Sabine Foroula

Dr. Ismail Abbas
Senior Lecturer, MTC, Cairo University

Abstract:- Sabine's semi-imperial formula, sometimes referred to as Sabine's theory, proposed a century ago, remains the main formula for calculating RT reverberation time in audio rooms, in addition to a rough estimate of sound volume in audio rooms. We prove that Sabine's formula for the reverberation time TR is fairly accurate but fails in the computation of the non-uniform sound energy density field whereas the proposed techniques can do it with high precision and speed.

Recently, some essays have appeared in new papers based on sound scattering theory which applies digital resolution techniques to resolve the reverberation time TR of sound diffusion PDEs and the sound density field distribution in the sound space of a 3D room, but unfortunately without significant success. The numerical solution of the chains of matrix B has been used successfully to solve the Poisson and Laplace PDE as well as the heat diffusion equation. Here we use the B chain techniques as a real break with the problem of the time dependent sound field in 3D geometric space. We predict the design of audio rooms in two examples of cubic rooms. It has proven effective to provide a reformulation of the Sabin reverberation time equation in addition to calculating the non-uniform field of sound energy density with high precision and speed.

I. INTRODUCTION

The quality of sound in audio rooms is determined by four main factors,

1-an appropriate reverberation time.TR
2 - an appropriate sound level or sound intensity I.
3-uniform sound distribution.
4-low noise / signal ratio.

Sabine's semi-imperial formula sometimes referred to as Sabine's theory, proposed a century ago, remains the main formula for calculating RT reverberation time in audio rooms. In addition it is also an approximate basis for calculating the intensity of sound W / m ^ 2 in sound rooms assumed to be uniform and is generally expressed in decibels.

The reverberation time TR seconds for empty given by Sabines' formula, can be expressed a as,
TR = 53,46 V / C A S,. . sec. . . . .

Assuming that the speed of sound in air C, at NPT is 330 m / s, Sabines' formula for TR (60 db) simply denoted TR , reduces to [1,2],
TR = 0.161 V / A S,. . sec. . . . . (1)

V is the volume of the room in m ^ 3, A is its total interior area in m ^ 2 and s is the average

sound absorption coefficient S .
S (av) = (A1 S1 + A2 S2 +.. An Sn) / (A1 + A2 +... An). . . (2)
For sound rooms populated with N humans, the denominator of equation 2 is simply changed to [As + N (humans) * 0.25] Sabine units.

i-The appropriate or recommended TR for large cathedrals and mosques is between 2 and 2.5 seconds,

ii-2 seconds is an optimal reverberation time for a concert hall,

iii-One second is an optimal reverberation time for an amphitheater conference room.
iv-0.3 to 0.5 seconds is standard for recording studios.

TR Below 0.3 seconds is an acoustically dead room while TR above 2.5 seconds is a boring echogenic room.

On the other hand, the Sabine formula for the uniform sound intensity I in audio rooms is given by, [1,2]

I = SUM sound power sources P in watts / (A s + N (humans * 0.25) . . . Watt / m ^ 2 . . . . (3),
or,

I = SUM sound power sources P in watts * RT / 0.161 V . . . . Watt / m ^ 2 ... (3)

The practical unit for I is the decibel (db),

I in decibel = 10 Log (base10) I / I (0). . . . (4)

Where I (0) is the hearing threshold for a normal, healthy human ear = 10 ^ -12 watt / m ^ 2 or zero decibels and the pain threshold is 1 watt / m ^ 2
it is 120 db.

The 40 to 70 db range is quite audible and comfortable for the human ear.

However, during this investigation we found that Sabine's formula for reverberation time is fairly accurate while Sabine's formula for intensity considers the sound field to be uniform throughout the audio room and cannot take into account the spatial variations of the sound field near the source and does not justify its hypothesis of uniformity of sound intensity over the space of the room.

Recently, some trials have appeared in new papers called Sound Diffusion Theory which applies the digital heat PD diffusion solution to solve reverberation time TR and sound energy density field distribution in 3-D space.

Chiara [3] considers that the sound energy in the room is composed of two components, that is to say a component of ray close to the source and a reverberation component reflected diffusively by the walls,

d / dt (partial) U (r, t) = D Nabla2 U (r, t) + P (r, t) ... . (5)

with the boundary conditions B C on the limits of the domain of U and the initial conditions IC of U namely U (0, r).

Where D is the thermal / sound diffusion coefficient of the room and U is the sound energy per unit volume J / m ^ 3 and P is the sound source located in the sound room at the corresponding free nodes.

Chiara and similar articles [3,4,5] use the FDM or the classical FEM which is a long and complicated procedure because it requires a grid of hundreds of thousands of free nodes for the uniqueness, the precision and the stability of the solution. the convergence of states is quite slow and the computation time is extremely long.

The time-dependent numerical analytical solution in 4D space (x, y, z, t) is almost impossible even using classical Fourier transform techniques.

Here is the digital solution of B-Matrix chains as a real breakthrough that is the subject of this article

II. THEORY

In previous papers [6,7], we have successfully applied matrix chains B to solve the Poisson or Laplace diffusion equation in addition to the heat diffusion equation PD (Eq 5) to Eq 7 in references 3,4. Given by. The stochastic transition matrix B is well defined for a given spatial domain and does not depend on BC or IC [7], The techniques of the matrix chain B proposed are also well defined [5,6,7] and are based on the statistical recurrence formula [6,7],

Ui, j, k (N + 1) = B (Ui, j, k N + b + P). . . .. . (6)
P is the source term in J / s placed at the free node indicated and b is the 3D boundary condition vector arranged in the correct order.
It follows that,

U (r, t) = (B ^ 0 + B + B ^ 2 + B ^ 3 + ... B ^ N). (b + P) + B ^ N .U (r, 0). . . . . (7)
For a sufficiently large number of iterations N, E satisfies the equivalence relation,

E = (I-B) ^ - 1.. . . (8)

Equation 8 helps if we are looking for the equilibrium solution in a permanent equilibrium regime independent of time,
Obviously the time t is given by N dt where dt is the time step or the time jump. and Ndt = t is the time of the evolution.

In this article, we use the same B-chain theory to consider the sound field in empty rooms with only absorbent walls that can be extended. in the case of rooms inhabited by humans thanks to the simple correction factor explained above !.
We apply the same equation (6,7) but with a major difference:
The boundary conditions are not frozen in time as in the case of Dirichlet conditions but they are oscillating absorbent walls varying in time.
We can show that the equation. 6 changes to,
U (i, IT) = B (I, k, IT) * U0 (k) + B (i, j, IT) * BC (j, 1). . . . . . . (9)
Which corresponds to the superposition of the two terms proposed by Chiara [3]
The integer i in equation 8 corresponds to the 3D matrix vector b in equation 9 arranged in the correct order.

Here, the transition matrix B itself is multiplied by the absorption coefficient at the frontier S. Likewise, the vector BC b in Eq, 9 is multiplied by (S) ^ N, where N is the number iterations in order to take into account the time dependence of conditions on the limits.

In order not to worry too much about the details of the theory, let's jump right into 3D illustrative applications and numerical results.

III. APPLICATIONS AND NUMERICAL RESULTS

Case A- the sound source is suddenly cut off, i.e. P = 0 at t = 0

Here two cases of 3D geometric space will be considered:

Case B - the sound source exists permanently over time.
We start with case A.
Case III-A
zero sound source

In figure I, we consider the simplest 3D case of a cube of 8 equidistant free nodes and 8 time dependent boundary conditions.

In figure II, we consider a slightly more complicated case of a 3D cube of 27 free nodes equally spaced and dependent on time conditions at the limits.

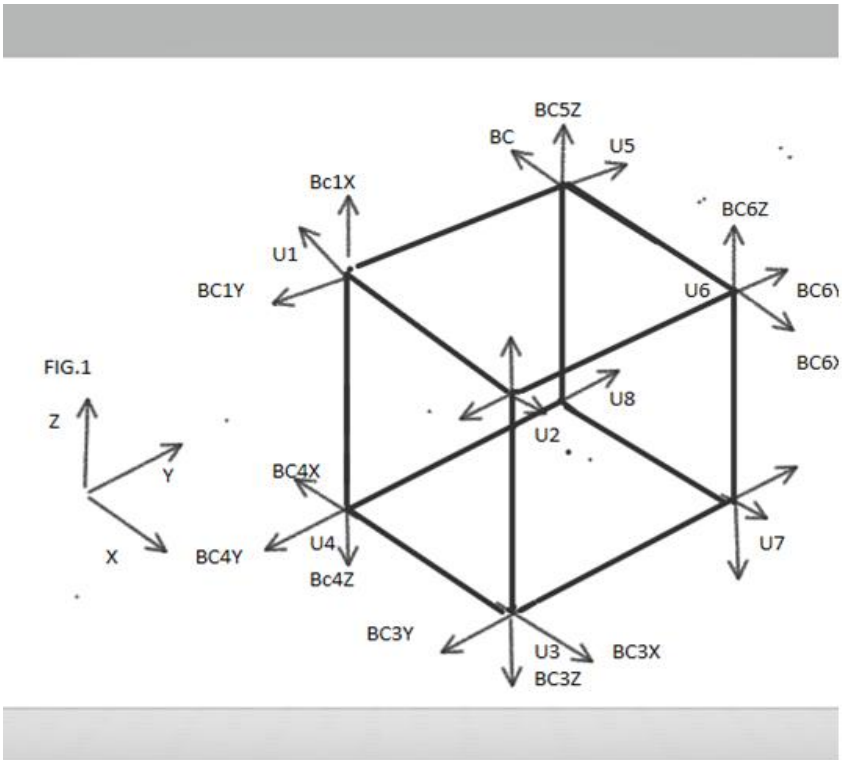

Fig 1-3D case of a cube of 8 equidistant free nodes and 8 time dependent boundary conditions.

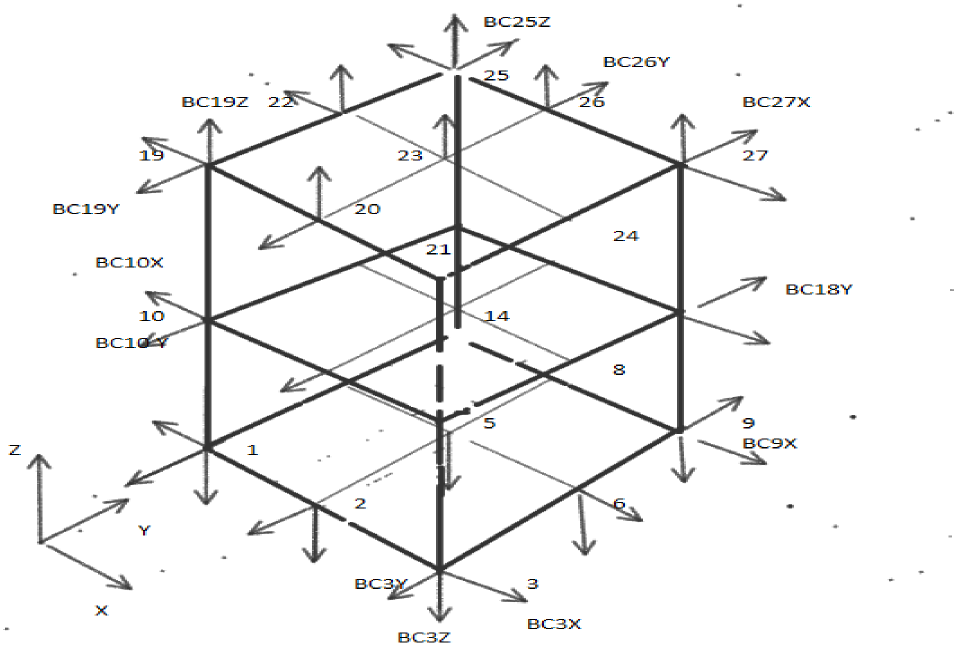

Fig 2 -3D cube of 27 free nodes equally spaced and 27 boundary conditions dependent on time

In figure I, we consider the simplest 3D case of a cube of 8 equidistant free nodes and 8 time dependent boundary conditions.

In Fig. II we consider a slightly more complicated case of a 3D cube of 27 free nodes equally spaced and dependent on time conditions to the limits.

The numerical results obtained show that the temporal evolution of the sound energy intensity follows exactly an exponential decrease (Exp-Alpha) and that the energy intensity distribution is uniform for Figure I and non-uniform for Figure II.

For simplicity, we have assumed a uniform spatial distribution of the initial conditions U (r, 0) = constant.

The temporal evolution of the total sound energy in the room follows exactly an exponential decrease U (t) = U (0) Exp (-ALPHA .t) and the numerical values for Alpha vs the absorption coefficient S are calculated in order to find an expression for the reverberation time TR as presented in Table I.

Table I. numerical values for Alpha vs absorption coefficien

Absorption coefficient S	1.0	0.9	0,8	0.7	0.6	0.5	0.4	
Log Alpha	-0.69	-0.8	- 0.92	- 1.05	- 1.2	-1.39	---	. . . . For Fig I
Log Alpha	-0.36	-0.46	-0.58	-0.68	----	------	---	. . . . For Fig II

An in-depth study of Table I shows that,
i-Alpha is inversely proportional to S,
ii- For a given 3D cube of length L with nxnxn free nodes or square of Length L with nxn free nodes, the distance between two
nodes is h = L / (n + 1), Alpha is inversely proportional to (n + 1) ^ 2 .

For example, in Figure I, we have Alpha x (3) ^ 2 = -6.3 for all S. Similarly for Figure II, we have Alpha x (4) ^ 2 = -6.3 for all S.
iii-We can draw the conclusion that for all cubes (or squares) the exponential decrease Alpha multiplied by S gives a constant value of -6.3

A simple manipulation of i, ii, iii gives the following general formula,

Considering the above facts and the fact that log(e) TR corresponds to Log(e)10^-6 = -13.8 and the time dt= L/c
Then TR= -13.8 x-6.3xL/330 S =0.263 L/S for enegy density.

And assuming a ratio of 2/3 to transfer from sound energy to sound intensity
reverberation time,
TR=2/3x,263xL/S =0.176 L / S . . . . (10) . . . . . for any cube
Comparing Eq, 10 with Sabine Eq 1 where TR=0.161 L/S shows that Sabine equation for relaxation time is fairly accurate.

Case III-B
Sound source continues to work permanently in time.

Here, we implement a sound energy source term of 100 J / s at free node 5 and calculate the resulting sound host distribution for Figure II and calculate the result non-uniform steady-state energy density distribution in 3D space.

Numerical results for the absorption coefficient S = 0.8 are shown in Figures 3,4,5.

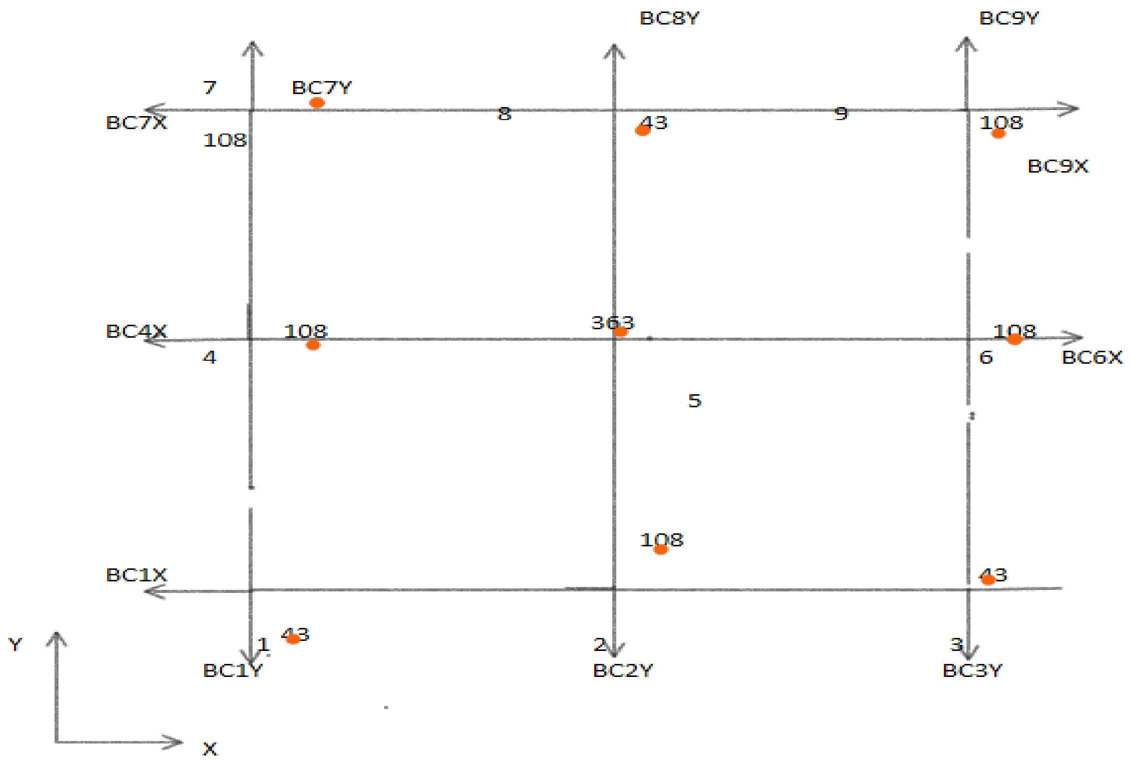

Fig 3. Non-uniform sound energy distribution for Fig II, first level free nodes 1-9

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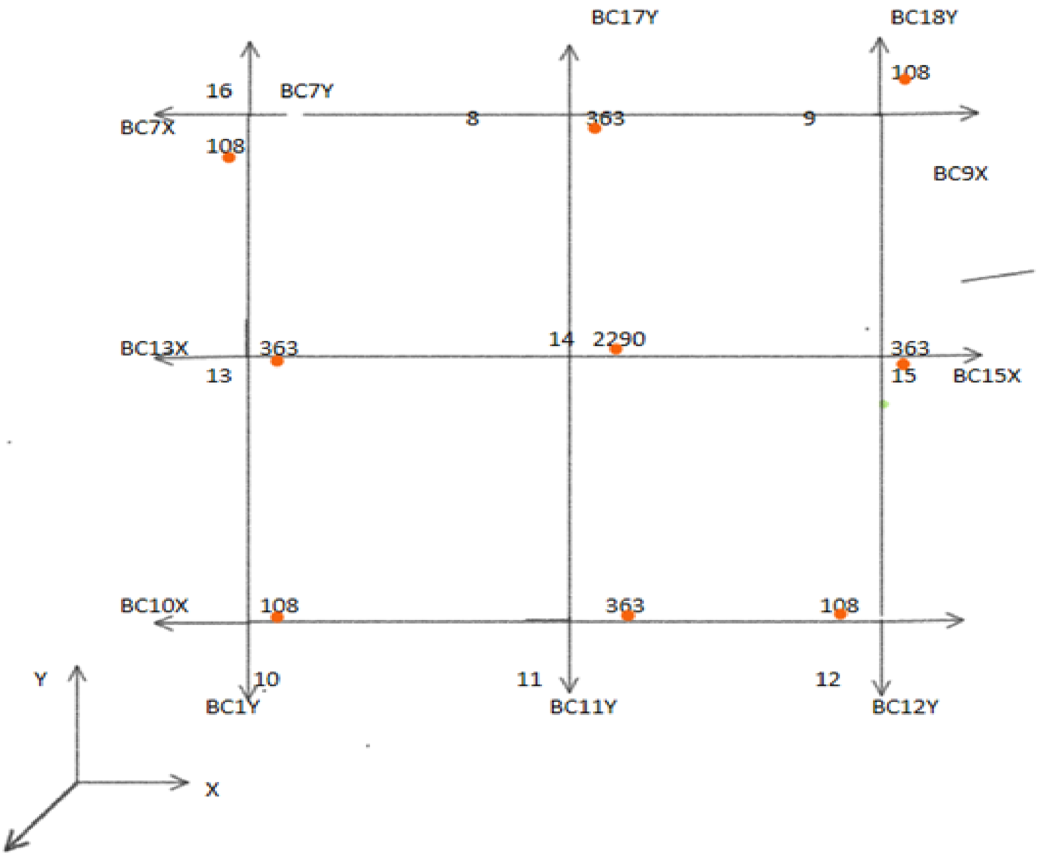

Fig 4 - Non-uniform sound energy distribution for Fig II, second level free nodes 10-18

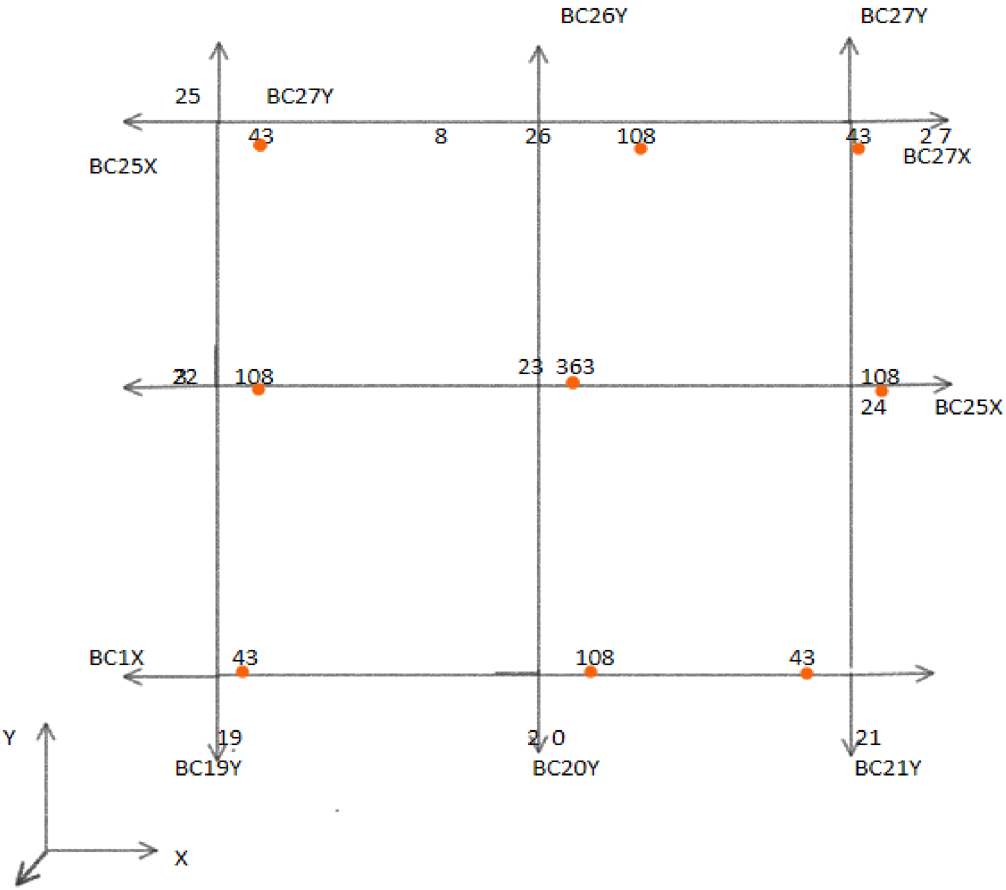

Fig 5 - Non-uniform sound energy distribution for Fig II, third level free nodes 19-27

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IV. CONCLUSION

The application of matrix B chains in the numerical solution of sound diffusion theory is promising. The B-chain techniques previously used in the Numerical Solution of Poisson, Laplace PDE as well as PDE heat diffusion can be extended to solve the reverberation time TR and the sound energy density field distribution in 3-D i.e, U (r, t), space The prediction of TR and U (r, t) is of paramount importance in the theory and design of audio rooms of different sizes.

In this paper, we conveniently use time dependent boundary conditions instead of the fixed Dirichlet boundary conditions used in heat diffusion problems, Here we use matrix B chain techniques as a true breakthrough with the problem of 3D geometric space theory and design in two examples of cubic audio rooms where it turned out to find an alternative formulation of the Sabin TR reverberation time equation which is fairly precise, Although Sabine's formula fails in calculating the non-uniform sound energy density field, the proposed techniques can do so with high accuracy and speed.

N.B. All calculations in this article were produced with the author's double precision algorithm to ensure maximum precision, followed by ref. 9 for example

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