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Operations Research Techniques Applied to the Financial Markets

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Abstract:- Operations Research and Financial markets share a relation like Tom and Jerry, both do well when they are alone, but when they are together, things are even better. In this paper, we have reviewed the application of operations research in financial markets. Historically, various Operations research techniques like Game theory, network analysis, Markov chain, neural networks have been implemented to solve the irrationalities arising from the financial markets. Our research paper attempted to outperform the local index using the weights resulting from the linear programming problem under certain constraints. In addition to this, we have run a simple Monte Carlo simulation with the geometric Brownian motion (GBM) model to forecastthe future price movement of a multinational company.

I. INTRODUCTION

Operation research(OR) originated during Second World War in the U.K. and was used for military strategy. Throughout World War II, a gaggle of researchers from arithmetic, statistics, physical and social sciences were entrusted with studying various military operations.

The demand for such a study arose from the military policies and choices made, which were crucial and high-priced, requiring researchers and scientists to provide quantitative intelligenceusing research and knowledge-based technical methods to arrive at decisions (Nikhila C, 2020). The critical application of OR is that it eases decision-making in the business where the division/allotment of resources is of utmost importance, i.e., finance, staffing, and other business resources.

In its past decade, OR has solved multiple research situations for the military, the government, and industries. The prime issue of all underdeveloped and upcoming countries has been to eliminate the shortage of food. With the explosion of population, which resulted in a scarcity of food, all countries were facing problems allocating land for various crops under climatic conditions and available facilities (LISA C. TOWNES, July 30, 2021). For over four decades, OR has been applied to financial markets, which has helped develop new theories for finance, and in turn, there has been tremendous growth in OR techniques (Kaur & Singh, 2014). Alternatively, it has been used in solving equity, debt, foreign exchange, derivative markets. While solving financial problems, the aims are usually to maximize profits or minimize costs with the best possible result. With OR techniques, reaching optimality becomes more straightforward as there are various methods to choose from to solve a financial problem.

II. LITERATURE REVIEW

2.1 Portfolio Optimization

For investors and asset managers, knowing what proportion of the capital needs to be allocated to any particular asset can statistically build or break the portfolio. Balancing the portfolio can be illustrated by terms and parameters such as time, risk, markets, and technologies (Cooper, Edgett, & Kleinschmidt, 2000). The optimal portfolio is the Max Sharpe Ratio Portfolio. Portfolio optimization is predicated on Modern Portfolio Theory (MPT). The main objective of the MPT is to achieve the highest return with the lowest possible risk. To achieve this, assetsin a portfolio should be selected after considering how they perform relative to each other, i.e.,they must have a low correlation. MPT assumes diversification of assets to avoid a crash whena particular asset or asset class underperforms to provide an optimal portfolio.

The classical MPT is valid only when the expected return is multivariate, and the investor is risk averted and prefers low risk. This approach has drawn many criticisms mainly because of its complexity. It is extremely tough to find an optimal portfolio when the number of assets islarge. Additionally, even when this practice is achieved, implementing the replica portfolio is near to impossible. Markowitz's standard formulation considers variance to measure risk volatility and completely disregards the investor's risk appetite. The main argument is that the variance cannot be considered a good measure of risk since high returns might increase the variance. Better risk measures are based on the chances of the value of losses. The developments in risk theory recommend quantile-based measures that are well-suited functionsto quantify risk. The most popular suggestions are the Treynor and Sortino ratios, defined as the expected return to the portfolio beta and the portfolio semi deviation, respectively (Taras & Taras, 2016). Another drawback of the MPT is its use of estimates and assumptions. Amongst the basic assumptions of MPT, all of them are considered impractical and oversimplified for an investor looking to trade long term. This model completely disregards intermediary fees like brokerage, tax, commission fees, and other transactional fees. Another fault with this model is that it estimates findings using historical data.

Now, the chosen subset should replicate the given time period as accurately as possible. Another weakness of the model is that it does not consider the possibility of unanticipated events like political tensions, unseen policy changes, or even a financial crisis. Thus, this model should not be the sole quantitative tool to rely on for investment planning. Several attempts have created less complex portfolio selection problems by linearizing the quadratic objective function (Henriques & Duarte Neves, 2019). In the finance domain, especially in Portfolio management Linear Programming is used to optimize portfolio returns. One can either maximize returns by putting risk constraints or minimize risk by applying return constraints. Nevertheless, the introduction of fundamental features involving the use of integer variables may significantly increase problem complexity and make LP solvable models more competitive with quadratic models. No risk measure can be a linear function of the portfolio shares to guarantee that the portfolio takes advantage of diversification. However, a risk measure can be LP computable in discrete random variables when realizations define returns under the specified scenarios. This applies, in particular, to the mean absolute deviation from the mean (Mansini, Wlodzimierz, & Grazia, 2013). Conventional multi-objective models usually address practical portfolio selection problems in which all coefficients and parameters are given beforehand. Nevertheless, in real-world situations, information regarding asset returns, risk, and liquidity are often incomplete, and the markets in which the assets are traded exhibit extreme volatility (Henriques & Duarte Neves, 2019). Optimization and investment communities have faced a long-standing challenge to devise an efficient algorithm to select a small number of assets from the variety in the asset pool and work out an optimal portfolio objective (Jianjun & Duan, 2013).

2.2 Option Pricing

In the financial market, the term asset refers to any object accessible in the market whose value is precisely known at the present but prone to change in the future. Typical examples of assets are stocks, bonds, and currencies. A financial option, or otherwise referred to as an option, is a contract tied to one or more assets and involves two parties, specifically a writer and a holder. Since the action taken on a particular asset is to sell or buy, options can be divided into two categories depending on the option holder's right. The first being Call option, which is the term used to describe the option where its holder has the right to buy the underlying asset at the strike price from the option writer, and the second being the Put option which is the term used to describe the option where its holder has the right to sell the underlying asset at the strike price to the option writer (Rujeerapaoboon, 2012).

Option pricing is the core content of modern finance (Chen, 2011). Option pricing theory calculates the value of an options contract by allocating a premium based on the calculated chance that the contract will expire in the money. The traditional method of valuing European-style options is that of using the Black-Scholes model. This method relies on many assumptions that sometimes fail to hold out in fundamental markets, so over time, OR techniques have been used to price complex options that do not have an analytical solution (Board, Sutcliffe, & Ziemba, Applying Operations Research Techniques To Financial Markets, 1999). Monte Carlo simulation is used to create possible trajectories for options until they reach maturity by manyanalysts. They can then discount the cash flows from the option for each path, weighted by their risk-neutral probabilities (inferred from prices by assuming that investors have risk- neutral linear utility functions) back to the present using the risk-free rate, to compute the average present value across all the sample paths to obtain the current price of the option (Joy,Boyle, & Seng Tan, 1996). Empirical studies suggest that, even though the Black Scholes pricing version affords correct charges for at-the-cash alternatives (the modern fee of the underlying asset is near the fee at which the choice may be exercised), a few styles arise in alternatives charges, along with the "volatility smile." A smile occurs while the implied volatility for out-the-cash alternatives (specifically puts) exceeds that for at-the-cash alternatives. However, OR techniques may permit the smile while pricing options (Board, Sutcliffe, & Ziemba, Applying Operations Research Techniques To Financial Markets, 1999). Other OR techniques that can be used to valuation options include Linear Programming, Dynamic programming, Quadratic Programming, Non- Linear Programming, and Neural networks. Given a contemporary set of prices for European style put and call options on the same underlying asset, (Rubinstein 1994) used quadratic programming to calculate the value of risk-neutral probabilities and made a binomial with the help of these probabilities tree is consistent with the observed options. For incomplete markets, when Black-Scholes is inapplicable, (Ritchken 1985) used linear programming to compute higher and lower bounds on option prices.

2.3 High-Frequency Trading

The World of High-Frequency Trading. In the past three decades, the average time scale over which the high-frequency trading firms like Citadel, Tower Research process a trade has gone from minutes to milliseconds. "Ultra-low latency" is even considered to be under one millisecond! (Moallemi & Sağlam, 2013). In the book- "Real-Time Risk: What Investors Should Know About FinTech, High-Frequency Trading, and Flash Crashes,"- Aldridge and Krawciw estimated that in 2016 High-Frequency Trading on average initiated 10–40% of trading volume in equities and 10–15% of volume in foreign exchange and commodities. (Aldridge & Krawciw, Real-Time Risk: What Investors Should Know About FinTech, High- Frequency Trading, and Flash Crashes, 2017). So much so is their influence that Algorithmic and High-Frequency traders were both found to have contributed to volatility in the Flash Crash of May 6, 2010, when high-frequency liquidity providers rapidly withdrew from the market (Kirilenko, Kyle, Tuzun, & Samadi, 2017). A $4.1 billion trade on the NYSE resulted in a loss of more than 1,000 points, which rose back to its previous values in just

15 minutes, hence thename FLASH CRASH.

High-Frequency Trading firms battle across the globe to be "milliseconds" faster. In the movie"*The Hummingbird Project*,"- They tried to run an optic fiber cable line directly from Kansas to the Wall Street Data Bank in New Jersey, cutting through mountains and under rivers to gain a millisecond of info advantage-the speed of a Hummingbird's wingbeat-that could potentiallyearn them Billions (Nguyen, 2019). It has been stated that a one-millisecond advantage can be worth $100 million to a major brokerage firm (Martin, 2007). The Sharpe Ratio is an often- used metric to evaluate the risk-adjusted return (Excess Return divided by Standard Deviation) proposed by Nobel Prize winner William Sharpe (Sharpe, 1994). It measures the return for each unit of risk. Higher the Sharpe Ratio higher is the risk-adjusted return of the stock/portfolio. The Sharpe Ratio of EUR/USD trading strategies held positions for 10 seconds scored Sharpe ratios well over the five thousand marks! Held positions for one minute-Over the 1800 mark (Aldridge, HuffPost, 2010). For some context-The Sharpe Ratio of the S&P 500has averaged between 2 and 3 in the last ten years. It peaked at a Sharpe Ratio of 4 in 2018 during the previous eight years (Portfolio Lab).

High-Frequency Trading is a highly automated process. Problems tend to re-occur, possibly many times per day, spreading the costs of developing an OR solution over many transactions. This scale and repetition make creating OR models more attractive for such purposes than forsmall or one-time decisions. Appropriately designed OR models, in conjunction with powerful computers, allows for trading to happen in milliseconds (Board, Sutcliffe, & Ziemba, Applying Operations Research Techniques to Financial Markets, 2003).

Financial market problems are numerical, with well-defined boundaries and objectives, clear and stable relationships between variables, and excellent data that make it well suited to OR analysis. In the paper "Behaviour Based Learning in Identifying High-Frequency Trading Strategies,"- traders' behavior was characterized by the reward functions most likely to have given rise to the observed trading actions. Trading decisions were modeled as a Markov Decision Process (MDP) and an optimal decision policy observations to find the reward function-Inverse Reinforcement Learning (IRL). An IRL algorithm based on linear programming resulted in 90% classification accuracy in distinguishing high-frequency tradingfrom other trading strategies in experiments on a simulated E-Mini S&P 500 futures market. These results suggest that high-frequency trading strategies can be accurately identified and profiled based on observations of individual trading actions (Yang et al., 2012). Game Theory has also been implemented in High-Frequency Trading. In the paper- "Quantum Prisoner'sDilemma and High-Frequency Trading on the Quantum Cloud"-

High-Frequency Trading has been taken as an instance of the famous Prisoner's Dilemma.There are two players.

Player I and Player II

They represent the market's mindset, buying and selling, using the two strategies, Buy or Sell. Player I : (Sell, Buy) (Buy, Buy) (Sell, Sell) (Buy, Sell)

Player II : (Buy, Sell) (Buy, Buy) (Sell, Sell) (Sell, Buy)
The Dilemma in High-Frequency Trading, in this case, is that the game will reach the Nash equilibrium Sell, Sell, which would be devastating for the markets, like was the case in the Flash Crash in 2010 (Khan & Bao, 2021).

2.4 Regulating Banking Exposure to Market Risks

In the past, regulators took the help of simple models to calculate the capital adequacy of banks. The rapidly developing world also means the internationalization and universalization of banking operations. This also led to more complex models to measure the capital adequacy of banks. This change took place because of the increasing market risk. Presently, there are mainlythree approaches to regulating the market risk of banks. Each approach is determined by how well it fulfils the aim of regulation. The three approaches are the building block approach, the internal models approach, the pre-commitment approach (Stephanou, 1996). Different capital requirements are determined for each of the four major market risk categories in the building block approach and then aggregated (Hill, 2003). An internal model is an approach where theloss to the bank's portfolio is calculated with a specified probability over a specified holding period of time (Capital.com).The pre-commitment approach is where banks pre-commit a fixed capital amount to protect their maximum trading loss exposure over a regulatory period. Out of all the approaches experts have analyzed, the internal models remain the most reliable and market-friendly approach (Stephanou, 1996).

III. OBJECTIVES

1) To use a Simple Linear Programming Model with a simplex solution and backtest the resulting weights in a “real world” situation to check the performance of the portfolio and beatthe index returns.
2) To Apply a Simple Monte Carlo Simulation Model to predict the future stock price Movement using 100,000 different simulations.

a. Monte Carlo Stock Simulation - Geometric Brownian Motion

How does one predict future stock prices?
Broadly there are two approaches-Technical Analysis and Fundamental Analysis.

Technical Analysis is based on the assumption of future stock prices following specific patternsseen in the past (Fama & French, 1995). Fundamental analysis assumes that a particular company's stock has an "intrinsic value" derived from forecasting its future earnings potential. Fundamental analysis suggests that the stock, therefore, is either overvalued or undervalued and, at some point in the future, will trade at its actual "intrinsic value" (Fama & French, 1995). Monte Carlo Simulation with the Geometric

Brownian Motion follows the Random Walk Theory. The Random Walk Theory is entirely different from both the Fundamental Analysis and Technical Analysis. Random Walk Theory suggests that past patterns are in no way indicative of future performances. The stock is bought or sold only if and when a particular trend has developed - Technical Analysis. Fundamental analysis is also not dependable due to the possibility of misconstruction and misinterpretation of financial information. There may also be situations where the stock may never reach its "fair intrinsic value" due to external market forces-Fundamental Analysis (Smith, 2020). Random Walk Theory assumes that movement in the stock is indeed random. It follows a completely unpredictable path, and the only way to outperform the market is to assume additional risk (Reddy & Clinton, 2016). The Monte Carlo Simulation builds a model of millions of simulations and "random trials" (read: possibilities) of stock prices with specific predefined input parameters. It produces a distribution of outcomes that can be analyzed to create various targets, stop loss levels (Harper, 2020)

(Sengupta, 2004) has specified the following Geometric Brownian Motion assumptions:

1) Going Concern Company
2) The stock follows the Markov process, which means it follows a random walk and isconsistent with the weakest form of the Efficient Market Hypothesis.
3) Proportional Returns are log-normally distributed.
4) The compounded returns follow a Normal distribution.

The Geometric Brownian Motion (GBM) with the Monte Carlo Simulation incorporates the Random Walk theory. In this case, we aim to test this in actual circumstances with a simple model.

CASE:

We have performed a simple Monte Carlo Simulation with the GBM process on Reliance Industries Ltd from 2005 to 2015. We tested the results on the actual stock movement of Reliance Industries from 2015 to September 1, 2021, using monthly stock prices and returns. (NSE: RELIANCE).

Inputs into the Python Model: (Refer to Appendix)

1. Annualized Historical Returns/ Drift Factor: 20%
2. Annualized Standard Deviation/Volatility: 50%
3. Average Indian Government 10 Year Yields as Risk-Free Rate: 6%
4. Number of years: 6 years
5. Simulations: 100,000
6. Starting Value: Reliance Industries' initial stock price indexed to 100.

AIM:

To Test the stock movement of Reliance Industries from 2015 to September 1, 2021, with Monte Carlo GBM simulations.

IV. RESULT

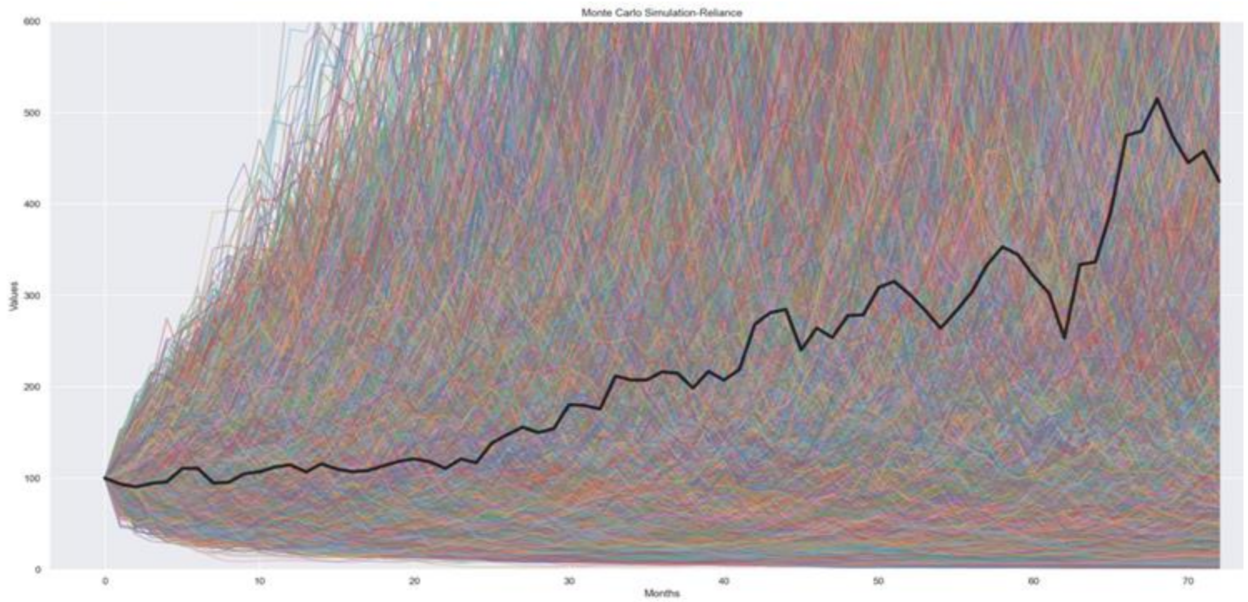

*Figure 1-**Monte Carlo Simulation***

1) The Dark Black line represents the actual monthly stock movement of Reliance Industries from 2015 to 1st September 2021 when indexed to 100.
2) The multicoloured lines represent the 100,000 simulations of the stock for the sameperiod.
3) The X-axis represents the 72 months (6 years*12 months) period for the simulations.

ANALYSIS

YEAR 1	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
Reliance Stock Price Year 1 (Indexed to 100)	100	93.47719	90.24858	94.22017	95.82081	110.5402	110.7336	94.60187	95.30927	104.7484	106.8927	112.1428
Average Stock Price of 100,000 simulations	100	101.47937	102.95937	104.38245	105.8735	107.52897	109.16304	110.84954	112.63069	114.37111	116.16409	117.89122
Difference in % (Actual-Simulated)		-8.56%	-14.08%	-10.79%	-10.49%	2.72%	1.42%	-17.17%	-18.17%	-9.19%	-8.67%	-5.13%
Average Error	-8.92%											

YEAR 2	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
Reliance Stock Price Year 2	114.4419	106.843	115.525	109.7447	106.9696	108.2315	113.3407	118.3774	121.0241	117.8134	110.5656	120.8789
Average Stock Price of 100,000 simulations	119.73164	121.57274	123.46033	125.32771	127.20275	129.12395	131.10147	133.05806	135.09733	137.10854	139.23287	141.34521
Difference in % (Actual-Simulated)	-4.62%	-13.79%	-6.87%	-14.20%	-18.91%	-19.30%	-15.67%	-12.40%	-11.63%	-16.38%	-25.93%	-16.93%
Average Error	-14.72%											

Table 1-Year 1 & Year 2

YEAR 3	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
Reliance Stock Price Year 3	116.7246	138.2614	147.5138	155.8114	149.725	154.1139	180.3803	179.3743	175.6958	211.6831	207.3408	207.2283
Average Stock Price of 100,000 simulations	143.5209	145.75558	147.99641	150.44764	152.86379	155.20444	157.62497	159.95621	162.37435	164.73723	167.14137	169.7285
Difference in % (Actual-Simulated)	-22.96%	-5.42%	-0.33%	3.44%	-2.10%	-0.71%	12.62%	10.83%	7.58%	22.18%	19.39%	18.10%
Average Error	5.22%											

YEAR 4	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
Reliance Stock Price Year 4	216.2842	214.7655	198.5999	216.7342	207.2958	218.7928	268.4853	281.0833	284.7733	240.2445	264.3086	253.8272
Average Stock Price of 100,000 simulations	172.48532	175.14112	177.6702	180.24596	182.8966	185.71318	188.39168	191.32267	194.02252	197.02426	199.9082	202.85659
Difference in % (Actual-Simulated)	20.25%	18.45%	10.54%	16.84%	11.77%	15.12%	29.83%	31.93%	31.87%	17.99%	24.37%	20.08%
Average Error	20.75%											

Table 2-Year 3 & Year 4

YEAR 5	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
Reliance Stock Price Year 5	277.8008	278.6837	308.6109	315.3004	301.1178	283.6753	264.0143	282.6453	303.2634	333.3336	353.0921	344.6469
Average Stock Price of 100,000 simulations	206.10202	208.77721	212.13091	215.46826	218.79877	222.44345	225.78307	229.65523	233.07317	236.32369	239.6467	243.26736
Difference in % (Actual-Simulated)	25.81%	25.08%	31.26%	31.66%	27.34%	21.59%	14.48%	18.75%	23.14%	29.10%	32.13%	29.42%
Average Error	25.81%											

YEAR 6	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12	Month 13	Month 14	Month 15	Month 16	Month 17	Month 18	Month 19	Month 20	Month 21
Reliance Stock Price Year 6	321.3374	302.4438	253.5257	333.7092	336.5048	391.5855	474.9993	479.9197	515.3596	473.8766	445.1142	457.9154	424.8513	481.0960982	462.0210371	460.0374293	498.27972	486.8277572	470.9582739	522.5246178	552.6869799
Average Stock Price of 100,000 simulations	247.18388	250.99309	254.82187	258.86696	263.16631	267.19258	271.40341	275.49607	279.75416	284.06379	288.12075	292.61455	297.179								
Difference in % (Actual-Simulated)	23.08%	17.01%	-0.51%	22.43%	21.79%	31.77%	42.86%	42.60%	45.72%	40.06%	35.27%	36.10%									
Average Error	29.85%																				

*Table 3-**Year 5 & Year 6***

2) Reliance Industries grew at a CAGR of 34% approximately from 2015 to 1st September 2021. Monte Carlo Simulations Average CAGR-20% approximately.

3) Reliance Industries ending value is 25% more than the 100,000 simulations averageending value.

POSSIBLE REFINEMENTS:

This is an extremely simple Monte Carlo Simulation with the GBM model. This model can be further refined by increasing the number of simulations, removing certain extreme outliers from those simulations, and using confidence interval levels instead of simple averages to test the Errors.Instead of entirely removing the outliers, minimum weights can be assigned to reduce the impact on the overall error.

2.1 Linear Programming Problems in Finance

Most financial decisions are made directing towards maximizing profit with a minimum risk factor. Operations techniques play an essential role in analyzing different financial problems, essentially equity, debt, design securities, foreign exchange, risk evaluations, regulations of capital reserves, devising pricing equations, and analyzing market data.

In financial problems, well-defined boundaries and objectives result in a clear, stable relationship between variables and the available data, making it suitable to work within OR techniques. (Kaur & Singh, 2014)

AIM:

To outperform the local index(Nifty) from a portfolio of 10 different companies with a fixed budget using data of the stock prices from 2005-2015.

CASE:

In this case, the objective is to acquire an optimal portfolio following the constraints with respect to the annualized returns and annualized standard deviations (risk). The investor has abudget of 1 million Rupees. The portfolio has ten different companies give with their annualized returns and annualized standard deviations.

Sr. No.	Company	Annualised return	Annualised Standard Deviation/Risk
1	Reliance Industries Ltd	20%	50%
2	Tata Consultancy Services Limited	25%	49%
3	HDFC Ltd	26%	33%
4	Infosys Ltd	16%	32%
5	Hindustan Unilever Ltd	22%	30%
6	Bayer CropScience Ltd	44%	33%
7	HCL Technologies Ltd	29%	41%
8	ITC Ltd	27%	29%
9	Dr Reddy's Laboratories Ltd	24%	30%
10	Natco Pharma Ltd	28%	44%

*Table 4-**Linear Programming Inputs***

The following constraints are to be followed while allocating funds:

1) The total money to be invested in the entire portfolio must be 1 million Rupees.
2) The average annualized return to annualized std deviation ratio should be greater thanor equal to 0.5
3) No companies should get an investment of more than ¼ of the entire budget i.e.,250,000.
4) Average annualized standard deviation should be less than or equal to 32%.

Objective Function: To Maximize

$$Z = 0.20x_1 + 0.25x_2 + 0.26x_3 + 0.16x_4 + 0.22x_5 + 0.44x_6 + 0.29x_7 + 0.27x_8 + 0.24x_9 + 0.28x_{10}$$

subject to

A $x_1 + x_2 + x_3 + x_4 + 0x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} = 10{,}00{,}000$

B $$\frac{0.20x_1 + 0.25x_2 + 0.26x_3 + 0.16x_4 + 0.22x_5 + 0.44x_6 + 0.29x_7 + 0.27x_8 + 0.24x_9 + 0.28x_{10} +}{0.5x_1 + 0.49x_2 + 0.33x_3 + 0.32x_4 + 0.3x_5 + 0.33x_6 + 0.41x_7 + 0.29x_8 + 0.30x_9 + 0.44x_{10}} \geq 0.5$$

$$\Rightarrow -0.05x_1 + 0.05x_2 + 0.095x_3 + 0.07x_5 + 0.235x_6 + 0.085x_7 + 0.125x_8 + 0.09x_9 + 0.06x_{10} \geq 0$$

C $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \leq 2{,}50{,}000$

D $$\frac{0.5x_1 + 0.49x_2 + 0.33x_3 + 0.32x_4 + 0.3x_5 + 0.33x_6 + 0.41x_7 + 0.29x_8 + 0.30x_9 + 0.44x_{10}}{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}} \leq 32\%$$

=> 0.18x1+0.17x2+0.01x3-0.02x5+0.01x6+0.09x7-0.03x8-0.02x9+0.17x10 <= 0

Also

$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \geq 0$

*Figure 2-**Linear Programming Constraints***

(Solved using solver on excel)

ANALYSIS:

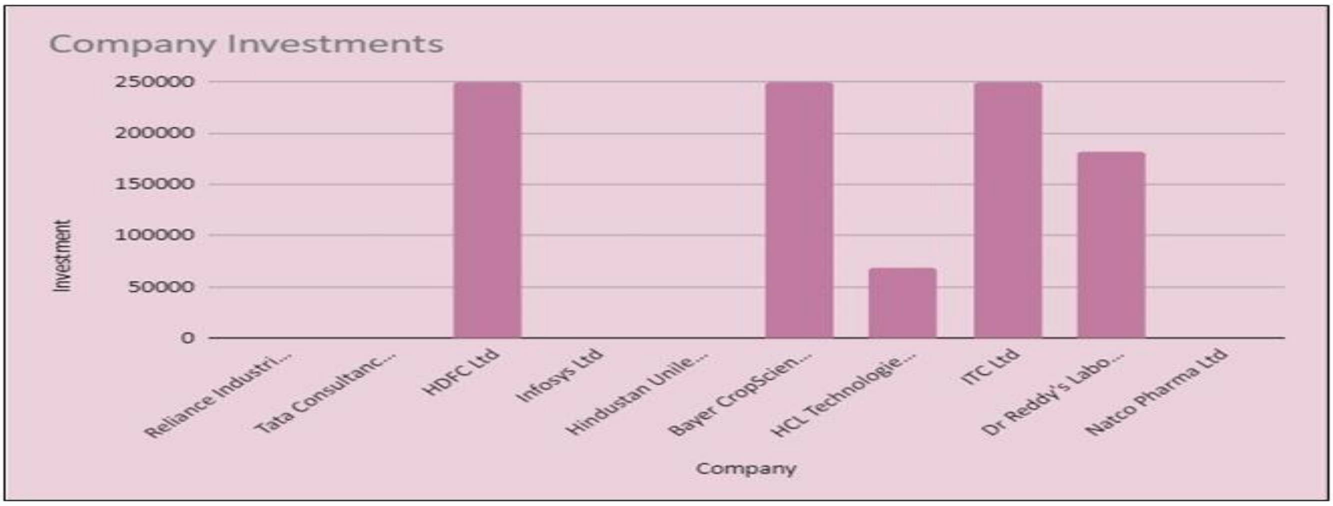

*Figure 3-**Company Investments***

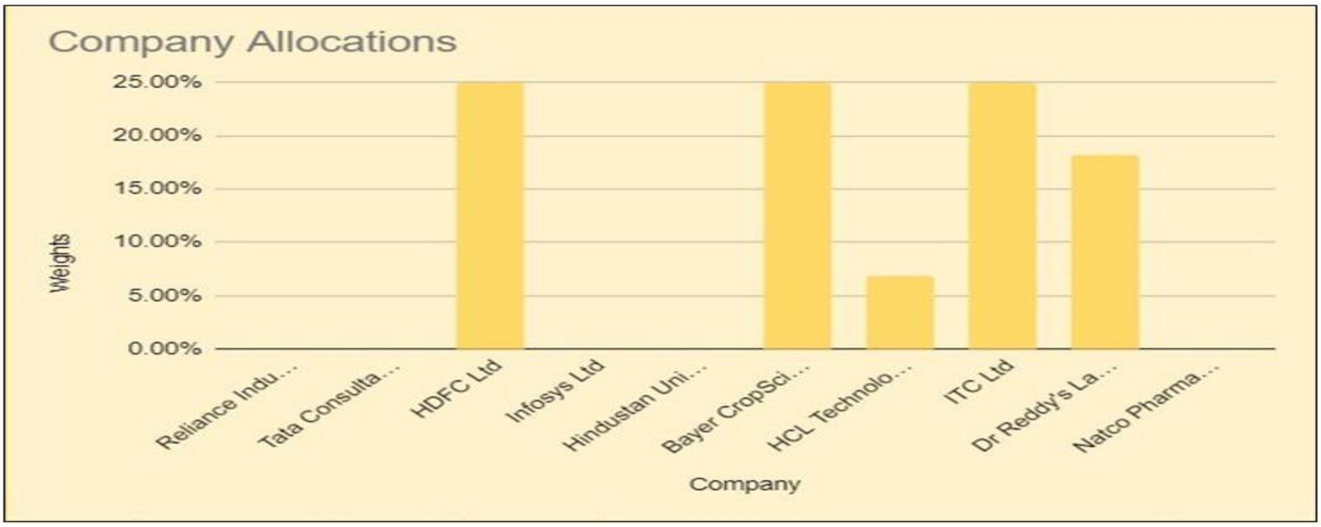

*Figure 4-**Company Weightages(Allocations)***

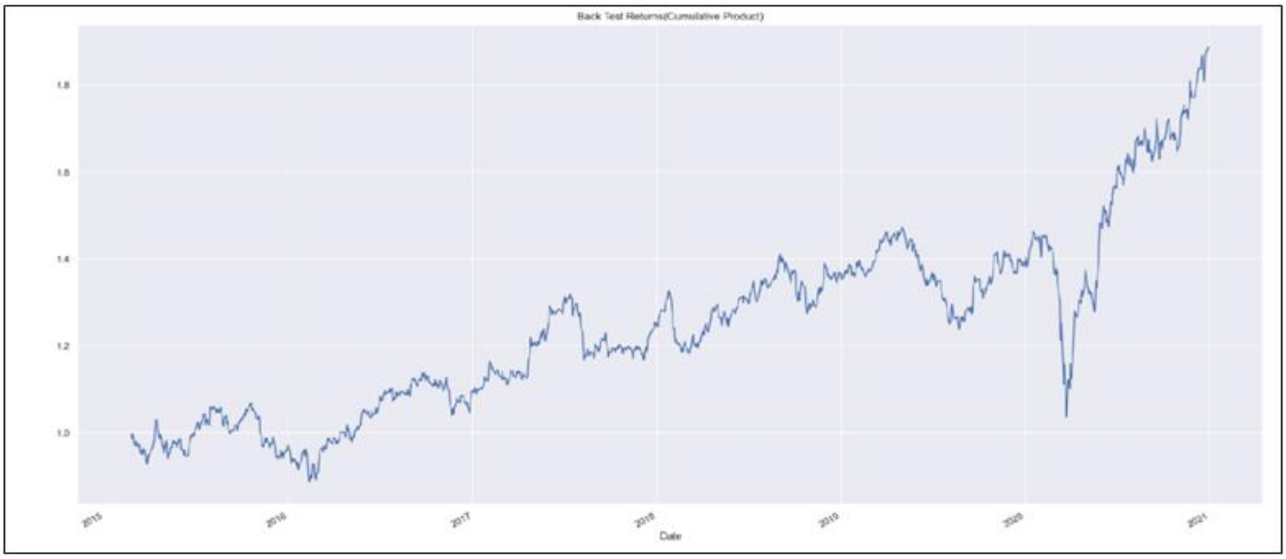

*Figure 5-**Backtesting Results***

Back Testing Inputs

1. **Estimation Windows = 36 months**
2. **Daily Historical Adjusted Close Returns of the Companies.**

Nifty 50 (Local Index)

3. **Annualized Returns (1/1/2015-1/1/2021)- 9.2% approximately (NOTE: 252 daysper year considered)**
4. **Annualized Volatility (1/1/2015-1/1/2021) - 18% approximately (NOTE: 252 daysper year considered)**

LPP Portfolio

5. **Annualized Returns (1/1/2015-1/1/2021) - 11.8% approximately (NOTE: 252 daysper year considered)**
6. **Annualized Volatility (1/1/2015-1/1/2021) - 16.3% approximately (NOTE: 252 days per year considered)**

V. LIMITATIONS OF OR IN FINANCE

It is indeed acceptable that OR techniques are instrumental in financial markets and are a greatadvantage to those who apply them. However, there are some limitations to OR techniques being used. There is a necessity for a computer most of the time. Some of the factors taken in the research are enormous, and establishing relationships between these requires computers to handle them. The financial market is very dynamic; incorporating the data into OR models is time-consuming and costly. A better solution than OR techniques may be available at this stage. The implementation of the research should be done correctly, and it takes a variety of factors and complexities (Gilb, 1988). Sometimes, due to psychological factors, resistance may be offered, which may not have any bearing on the problem and the solution. In addition, the OR job, a specialist’s job, might not be aware of the business problems.

Similarly, a person not familiar with the OR working will fail to understand the complex working of the job, in this case, the manager. Because of this, there is always a significant gap between these two. The management itself will offer much resistance due to conventional thinking (Hillier, Frederick S, 2010)

VI. CONCLUSION

Quite a few OR techniques have been used in financial markets, but the most widely used is Mathematical Programming (linear, quadratic, nonlinear, integer, goal, DEA, and dynamic). Apart from this, Monte Carlo Simulation is widely used to ascertain the value of derivatives and securities. In some cases, it has also been used to test trading rules and examine the risks of loan portfolios. Over the years, OR techniques have permitted traders to make better financial decisions in lesser time. In the future, as the availability of data and computing power continue to improve, the application of OR Techniques in the Financial Markets will only increase. Faster computers and the convenience of immense amounts of data give rise to the development of efficient and methodical algorithms that would provide a base to solve even more complicated problems (Board, Sutcliffe, & Ziemba, Applying Operations Research Techniques To Financial Markets, 1999).

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APPENDIX

1)
*Def Backtest_Weighting_Scheme(returns,estimation_window=60 ,weighting=Weight_ Equally_Weighted_Portfolio,verbose=False,**kwargs):*

"""
Backtests a given weighting scheme, given some parameters: returns : Asset returns to use to build the portfolio
estimation_window: The window(in months) to use to estimate parameters.
It must be a function that takes "returns", and a variable number of keyword-valuearguments.
"""
n_periods = returns.shape[0]# return windows
windows = [(start, start+estimation_window) for start in range(n_periods-estimation_window)]
*weights = [weighting(returns.iloc[win[0]:win[1]], **kwargs) for win in windows]# convert List of weights to DataFrame*
weights = pd.DataFrame(weights, index=returns.iloc[estimation_window:].index, columns=returns.columns)
*returns1 = (weights * returns).sum(axis="columns", min_count=1) #mincount isto generate NAs if all inputs are NAs*
returns2=(1+returns1).cumprod()
returns2.plot(figsize=(24,12), title="Back Test Returns(Cumulative Product)")return returns1

2)
Def Geometric_Brownian_Motion_with_Monte_Carlo_Simulations(annualised_expected_ return_mu,annualised_volatility_sigma,riskfree_rate,n_years=10, simulations=1001,steps_per_year=12,starting_value=100,prices=True):
“”” INPUTS:
annualised_expected_return_mu:Input the Annualised expected returns of the stock.NOTE this is also the drift factor i.e the market drift i.e this will remain
constant till time "t" so dont take an extraordinary year for thestock where that were very impressive returns
annualised_volatility_sigma:Input the Annualised volatility of the stock. riskfree-rate:Input the 10 Year Government Treasury Bond Rate n_years:The number of years for the analysis
simulations:Input the number of MONTE CARLO simulations for the stock.Keep abare minimum of 1000.For actual analysis can use around 50000. steps_per_years:Input the steps per year.It is like periods_per_year.
“””
 # Derive per-step Model Parameters from User Specificationsdt = 1/steps_per_year
*n_steps = int(n_years*steps_per_year) + 1*
 # the standard way ...
*# rets_plus_1 = np.random.normal(loc=mu*dt+1, scale=sigma*np.sqrt(dt),size=(n_steps, n_scenarios))*
 # without discretization error ...
*rets_plus_1 = np.random.normal(loc=(1+annualised_expected_return_mu)**dt,scale=(annualised_volatility_sigma*np.sqrt(dt)), size=(n_steps,simulations))*
rets_plus_1[0] = 1
*ret_val = starting_value*pd.DataFrame(rets_plus_1).cumprod() if prices elserets_plus_1-1*
ax=ret_val.plot(legend=False,figsize=(24,12),alpha=0.5,linewidth=2,title="Monte Carlo Geometric Brownian Motion",fontsize=12,ylabel="Values",xlabel="Time Period")
ax1=ax.axhline(y=starting_value,ls=":",color="black")
plt.plot(0,starting_value,marker='o',color='darkred',alpha=1) b=ret_val.pct_change().dropna()
last_value=ret_val.iloc[-1:] f = plt.figure() f.set_figwidth(24) f.set_figheight(12)
mean=ret_val.mean(axis=1)
print("The Minimum Values per Year/Month")
print(ret_val.min(axis=1))
#ret_val.min(axis=1).plot.bar(figsize=(24,12),title="Minimum Values") print("The Maximum Values per Year/Month")
print(ret_val.max(axis=1))
#ret_val.max(axis=1).plot.bar(figsize=(24,12),title="Maximum Values") print("The Average CAGR in % for all Simulations")
*xyz=((mean.iloc[-1]/mean.iloc[0])**(1/n_years))-1*
*print(xyz*100)*
print("The Mean Values per Year/Month") print(mean)
plt.plot(mean,color="magenta")
plt.plot(ret_val.min(axis=1),color="green")
plt.plot(ret_val.max(axis=1),color="blue")
plt.legend(["Mean Values","Minimum Values","Maximum Values"])plt.title("Mean Values-Minimum Values-Maximum Values") plt.axhline(100,color="goldenrod")
return ret_val