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Portfolio Optimisation for Two Investors

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Abstract:- There are different types of investors based on risk aversiveness and this significantly impacts their investment decisions. A common goal of any type of investor is to maximize the returns by minimizing the risk. This study tries to solve this problem as a non-linear programming problem and aims to optimize the portfolio weights. A diversified portfolio is built for aggressive and defensive investors and a modified version of the Markowitz model with different strategies is used to analyze the returns. A peculiar observation on the analysis reveals that the aggressive investor maximizes the reward to risk ratio but due to the presence of higher risk-bearing assets in his portfolio the investor needs a higher CVaR to be prepared for the downfall. The regression model helps to understand the relation between absolute portfolio returns and absolute market returns. It identifies the higher certainty and predictability of the defensive investor which becomes crucial since this becomes an important trade-off against returns. We also find that this fact holds since the aggressive investor holds a higher margin as compared to a defensive investor due to the high exposure to uncertainty in the investment. This study fulfills the objectives for the two investors in the Indian market where the asset classes are allocated and defined as per qualitative security and fundamental analysis.

Keywords:- *Expected Return, Risk, Standard Deviation, Beta, Risk-free rate, Market Risk Premium, Portfolio Optimisation, Tail Risk.*

I. INTRODUCTION

India is the second-largest economy in the world with 1.39 billion people (India Population, 2021). But surprisingly only a meager 4% of the people invest in equity (Balwani, Mazumdar, & Acharya, 2021). This trend is now changing with more and more people investing in equity and debt. Mutual funds have become a new avenue of investing which has replaced the orthodox avenues of gold and fixed deposits. It has grown at a staggering 20% CAGR in the last 4 years (Mutual fund industry has potential to grow exponentially, sa .., 2021). These mutual funds invest in a wide range of equity and debt instruments and use multiple strategies for selecting their assets. What most of these portfolio managers lack is the incorporation of quantitative techniques and operations research into their respective investing activities.

The operations research used in the paper is using the Solver- excel feature. The problem has been formulated as a linear programming problem whereby the problem is maximizing the reward-to-risk ratio. This helps the investor in optimizing the portfolio weights of the respective assets selected using quantitative techniques. But the problem has been solved using the Generalised Reduced Gradient method in solver which helps accommodate for non-linear objectives and constraints. As per the Random Walk Down Wall Street, short-term as well as medium-term stock price changes are not predictable based on the historical data and appear random. This is why it becomes necessary to incorporate uncertainty into the portfolio construction process. This also accommodates for non-linearity which is not prevalent in simplex.

We have constructed two portfolios for the two different types of investors based on non-linear objectives, a Monte-Carlo simulation that randomly assigns values 1274 times based on a normal distribution to calculate the tail risk, and prepared a minimum amount of margin the investors should hold to nullify the expected shortfalls through CVaR. We attempt to optimize portfolio weights and prepare a comprehensive view of the portfolio.

II. LITERATURE REVIEW

Earlier, there didn't exist a quantitative approach to portfolio construction but the seminal study done by Markowitz, 1952 talks about how the portfolio management process is divided into two steps, observing the past and preparing a portfolio for future returns. This model deals with the second step. It prepares a relationship between expected average returns and variances of individual assets and their covariance. The underlying assumption being that a rational investor would like to maximize expected returns and minimize risk. The author goes on to say that a diversified portfolio is always preferable and just because an individual asset has a higher expected average return doesn't mean that the portfolio has a lower risk variance. Young, 1996 identified that instead of using mean-variance or standard deviation we use minimum returns as a measure of risk, and through linear programming, the objective can be satisfied which is to minimize the maximum loss thereby reducing the "risk" while having a constraint of a minimum acceptable return. The gap Young identified was that the Markowitz model was based on having a normal distribution, whereas this model could give logical advantages when the returns of the assets are not normally distributed. The factors that contribute to the minimum expected return for a portfolio is a function of 5- the risk-free rate, the market price of dollar risk, the variance of the individual asset, and covariance with the other existing assets and covariance with other assets included concurrently. (Lintner, 1965). The optimal portfolio allocation is constructed using the Markowitz model but to ensure whether the portfolio is outperforming or

underperforming against an index can be depicted through the Capital Asset Pricing Model. Fama and French, talk about how it can be used to measure the amount of risk and the relationship between risk and return. This model is based on the assumption that the market factor is the only factor that affects an asset. It is calculated by measuring the relative risk between an asset and the market, the risk-free rate, and the excess returns produced by the market over the risk-free rate.

With the rise of technology, newer strategies were introduced to make the seminal and empirical studies more comprehensive. Becker and Reinganum talk about how unique strategies such as alpha and factor investing and how they can benefit investors. With the help of big data, investors can leverage this to build quantitative models that can help them become more efficient and achieve their investment outcomes. But due to the rise of technology and markets becoming more volatile, Sukharev has concluded by saying that the portfolio has to be dynamic because a change in the way the asset behaves or the addition of assets will change the entire portfolio performance. The report also states how sometimes two assets can have the same risk to reward ratio and this is where qualitative techniques need to take over. Treynor and Black in 1973**,** spoke about how there can be a balance between the experiential and judgmental knowledge of the security analyst and complex quantitative methods like the Markowitz Model. He goes on to say that if two portfolio managers with the same information and market returns, one manager can do better than the other through the appraisal ratio. Hence, the judgment of each investor is very important. A modern perspective to the previous theories about the psychologies of two portfolio managers can be displayed through the study done by (Seetharaman, Niranjan, Patwa, & Kejriwal, 2017). It depicts how each investor has its objectives and hence needs to allocate their assets accordingly. It also shows how an investor chooses a particular asset and how they are more favorable to choose one asset over another. A short-term investor would select assets based on announcements whereas a long-term investor will choose assets that don't require active involvement. The different avenues for low-risk investors are fixed deposits, savings bank account, government securities. For moderate risk investors- debentures, mutual funds, ETFs. For High-risk investors- Stock Market, commodity market, crypto market, Forex market. For traditional investors- Real estate, gold, and silver. For Emerging investors- Virtual real estate, hedge funds, art, game cards, etc. In terms of the method of portfolio allocation, Graham and Dodd enlightened with the benefits of dollar-cost averaging for individual, defensive and passive investors. This strategy is to buy an equal amount of investments at regular intervals, this way the investor will be able to buy a higher number of shares when the market is down and average the cost price.

A modern perspective to the Dollar-cost averaging strategy is displayed by Mukherjea through Coffee Can Investing in which he displays how investors should go beyond the traditional avenues of investing and select assets based on their growth and sit tight on it without actively modifying the portfolio. But whatever happens in the past may not follow in the future which is when the Stochastic portfolio theory by Fernholz comes in. It is a dynamic framework to judge the behavior of a portfolio and the structure of the equity market relative to a benchmark. The SPT provides the investor with strategies based on the size of companies to invest in a certain period that can outperform the benchmark with the probability of 1. Diverse, intrinsically volatile, and rank-based models are all a part of SPT. In such volatile environments, it also becomes important to hedge the positions in the portfolio. A commonly used method of buying put options against a long position can get very expensive since the premiums are very high. A new hedging strategy was proposed that aims at having a target risk level and if the portfolio goes beyond the target risk level, amounts of it must shift towards the risk-free asset. (Happersberger, Lohre, & Nolte, 2020)**.**

Rockafellar (2000) talks about how it becomes important to protect the positions in the portfolio as a whole. An innovative approach towards optimization through minimizing the conditional value at risk also known as expected shortfall and how VaR is redundant. VaR represents the maximum loss for a given confidence level which is usually 95% whereas CVaR is the average loss if the returns lay in the remaining 5%. The method of calculating this is by quantifying the amount of tail risk of an investment portfolio and take the weighted average of the extreme losses beyond the point where VaR is cut-off. This method talks about how a portfolio should include unexpected events during construction. Gaivoronski (2005) has found that in some cases optimization of VaR and CVaR may lead to quite different portfolios. But there are exceptions to diversification. Demirer and Lien (2005) have worked on measuring the correlation between various sectors within a market and the movements of the market in either direction. The paper found thsat the sectoral correlation is higher in the market uptrend. Although in the context of the Chinese market, the finance sector had a strong correlation with a downtrend market. Whereas in the USA there was a downside correlation between the sectoral indices and the market. Meric and Ratner (2008) have found in their paper that global diversification in the same sector during a bull market is better than different sectors within the same market. It was also found that global diversification would provide a better return in the bear market since all the sectors in the local markets are positively correlated. Mangram (2013) shows how we can incorporate the theories of Markowitz and many other seminal studies using Microsoft excel. The studies and analysis conducted by the earlier authors were done in the 1950s and 1960s hence the complex calculations would lead to a lot of tedious work but the use of software, frequency trading, algorithmic trading makes the portfolio balancing process automatic.

What we're going to present through this report is how an individual investor must allocate her portfolio by optimizing the weights which minimize the risk and maximize the returns, this will be done with the help of Non-Linear Programming. What we have identified is that

there haven't been attempts to construct a portfolio especially in the Indian market which includes a modified version of the Markowitz model with different strategies, the incorporation of minimum expected return through CAPM, uncertainty, and risk management. We are also going to fill in the gaps from the **Seetharaman Report** where we will construct two different types of portfolios- Aggressive and defensive since all investors are not the same and don't have the same goals. These two portfolios will use different strategies based on the risk-aversiveness of the investor. We will also incorporate the risk-management techniques from the **Happersberger Report** into a quantitative portfolio where investments are based on Dollar-cost averaging at regular intervals. Through these specialized portfolios, we will be able to fill a gap in the past reports.

III. METHODOLOGY

As discussed in the above literature review about the past works of authors and researchers, portfolio optimization is a vast study since it needs to inculcate the various factors relating to risk, uncertainty, performance, and returns. The portfolio has been solved using non-linear programming since returns and risks are non-linear. Using the in-built GRG Non-Linear programming software in Microsoft excel, we derive our optimal portfolio weights while fulfilling the constraints. To calculate and quantify the exposure of uncertainty in a portfolio, a monte Carlo simulation is used. It is based on randomly generating numbers based on a probability distribution- normal, uniform, Bernoulli. We have used it based on normal distribution since it is the most common method for quantifying risks and returns. Based on this and the CVaR calculations we derive our margin against expected shortfall. Based on these methods, the optimal portfolio with the incorporation of uncertainty will be formulated as follows.

IV. ANALYSIS AND FINDINGS

All investors in the market are not the same and they have different characteristics such as Risk aversion, amount of investment, investment goals, and many other aspects. The two investor classes are Defensive and Aggressive. A defensive investor is termed as someone who has the highest amount of risk aversion which is that she chooses certainty over uncertainty which acts as a trade-off against the amount of return earned. A defensive investor invests passively in the market hence we have applied the concept of dollar-cost averaging into the portfolio where a fixed amount of money is invested every month.

The assets chosen are all leaders in their industries and are currently in the maturity phase of the company life cycle and hence possess a very low amount of risk. An aggressive investor, on the other hand, chooses to maximize returns over minimizing risk. Hence the assets chosen are in the growth phase of the company life cycle, high beta, and invests in alternative asset classes such as cryptocurrency. Based on this information we aim to prepare optimal portfolio weights for the two types of investors.
(Becker & Reinganum, 2018)

Defensive Investor

For a Defensive investor, we have first built a portfolio comprising of 10 assets- *HDFC bank, Britannia, NMDC, HUL, Infosys, TCS, PFC, Glenmark, GAIL, and Risk-free Asset*. These assets are chosen on a qualitative analysis of the assets.

Problem Formulation using Solver in Excel

Maximizing the reward-to-risk ratio

Objective Function: Maximise Z = $\frac{x_1}{x_2}$

Where x_1 and x_2 are given as the daily average returns of the portfolio and the portfolio risk respectively.

In Portfolio returns,

$Let\ w_1 -> HDFC\ Bank,\ w_2 -> Britannia,\ w_3 -> NMDC,\ w_4 -> HUL,\ w_5 -> Infosys, w_6 -> TCS,\ w_7 -> PFC,\ w_8 -> Glenmark,\ w_9 -> GAIL,\ w_{10} -> Risk - free\ Asset$

$$x_1 = w_1 r_1 + w_2 r_2 + w_3 r_3 \ldots\ldots\ldots w_{10} w_{10}$$

$$x_1 = 0.08\% * w_1 + 0.08\% * w_2 + 0.07\% * w_3 + 0.1\% * w_4 + +0.12\% * w_5 + 0.15\% * w_6 + 0.06\% * w_7 + (-0.01\%) * w_8 + 0.06\% * w_9 + (-0.01\%) * w_{10}$$

Portfolio Risk is calculated by solving the portfolio variance-covariance matrix based on the respective portfolio weights.

Subject to Constraints:

1. $x_2 \leq 1\%$

Daily Risk below 1%

For a defensive investor, the protection of the investment is the most important criterion. For the portfolio, we have assumed that the risk limit permitted daily is 1%.

2. $\sum w = 100\%$

Total Portfolio weights as 100%

The portfolio constructed does not permit the use of leverage or margin since the amount of risk involved increases and is not ideal for a defensive investor since they are the most risk-averse.

3. $w_1;\ w_2 + w_4;\ w_4;\ w_5 + w_6;\ w_7 + w_9;\ w_3;\ w_8 \leq 25\%$

Sectoral allocation is capped at 25%

A defensive investor aims to diversify not only between assets but also between sectors and risk factors. But she doesn't want to overexpose to a particular sector regardless of how lucrative it may seem.

Table 1- Sectoral Allocation of a Defensive Investor

Sectoral Allocation		
Sectors	**Weights**	**Stocks**
Banking	25%	w1
FMCG	25%	w2 + w4
IT	25%	w5 + w6
PSUs	7.62%	w7 + w9
Metal	7.38%	w3
Pharma	2.50%	w8
Risk-free	7.50%	w10

Total Portfolio weights= 100%
Source: The authors

4. $w_{10} = 7.5\%$
Fixed allocation in the Risk-free asset as 7.5%

A constant amount of the portfolio is allocated to the Risk-free asset. This helps the defensive investor to protect the investment at all times since the Govt bond has the least amount of default risk.

5. $w_8 \geq 2.5\%$
Minimum exposure to the Pharma sector 2.5%.

Due to qualitative analysis, it is assumed that the pharma sector will provide the defensive investor with healthy returns for a limited amount of risk. But according to historical data, we have identified that the pharma sector possesses a high amount of risk for a lower amount of reward. But looking into the future, we have allocated a minimum amount of 2.5% to the pharma sector.

6. $w_1, w_2, w_3 \ldots\ldots\ldots w_{10} \geq 0\%$
No Short Selling constraint

Short selling usually comprises a higher amount of risk as compared to a long position and hence it is not in the best interest of a defensive investor to short an asset.

Qualitative methods of portfolio construction

Dollar-cost Averaging

We have used the method of dollar-cost averaging as proposed by Benjamin Graham for a defensive investor whereby a fixed amount of money is allocated at the beginning of the month which in this case we have taken as INR 50,000. This helps the investor average out the cost by buying more when the price is low.

Portfolio Weights

By solving the problem for the constraints above, we have arrived at a solution where the: -

Average Daily Returns= 0.0885%
Portfolio Risk= 0.99%
Reward-to-Risk Ratio= 0.089

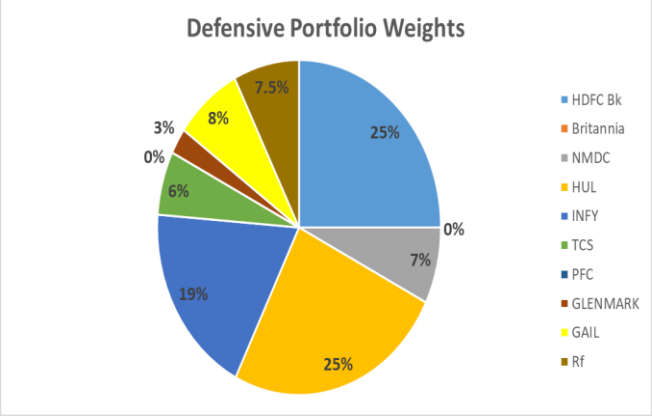
Whereby, the reward-to-risk ratio is maximized.

Table 2- Portfolio weights for a Defensive Investor

Assets	Daily Returns	Weights
HDFC Bk	0.08%	25%
Britannia	0.08%	0%
NMDC	0.07%	7%
HUL	0.10%	25%
INFY	0.12%	19%
TCS	0.15%	6%
PFC	0.06%	0%
GLENMARK	-0.01%	3%
GAIL	0.06%	8%
Rf	-0.01%	7.5%
		100%

Source: The authors

Figure 1- Defensive Portfolio Weights



Source: The authors

As we can observe from above, we have not allocated any amount to Britannia and PFC due to their low rewards or high covariance to other assets.

Portfolio Performance

We shall now compare our portfolio with a benchmark which in this case we will take BSE Sensex. Only then, we will identify the surplus amount of return and the benefit of diversifying between assets using quantitative methods.

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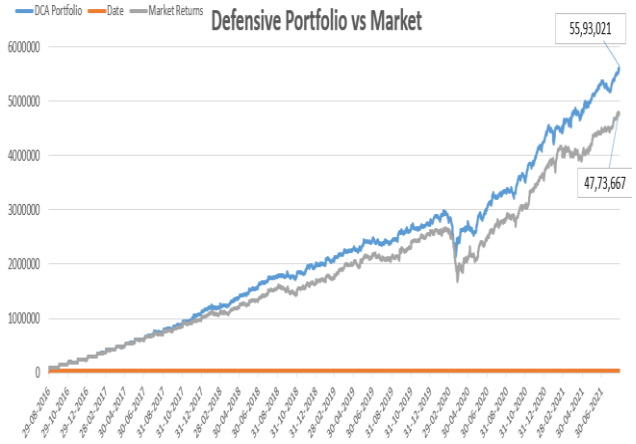
Figure 2- Defensive Portfolio vs Market (Sensex) Performance



Source: The authors

As we can see, the portfolio over the past 5 years, if we had allocated the portfolio based on the above weights, we would've gained an *excess of approximately INR 8 lakhs more* using the dollar-cost averaging method of a *monthly investment of INR 50 thousand* which cumulates to a *total investment of INR 30.5 lakhs.*

Capital Asset Pricing Model

This model assumes that the market factor is the only factor that affects an asset, in this case, portfolio. It also helps us to calculate the minimum expected rate of return required for the investment to make sense.

Table 3- CAPM of a Defensive Portfolio

CAPM	
Average Returns	0.09%
Daily Risk-Free Rate	0.02%
Excess Portfolio Daily Returns	0.07%
Standard Deviation	0.99%
Beta	**0.00382**
Alpha	**0.09%**
Sharpe Ratio	**0.0720**
Treynor Ratio	**0.19**
Excess Market Daily Returns	0.05%
Minimum Expected Return	**0.02%**

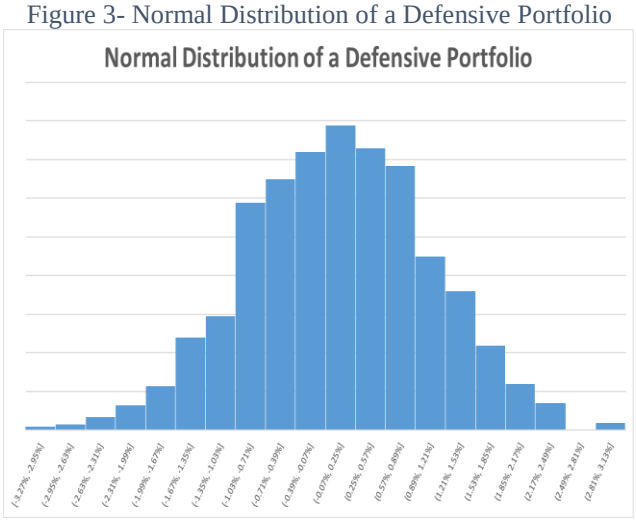
Source: The authors

The Beta, which is the relative risk between the portfolio and the benchmark shows that the portfolio has a very small amount of risk *(0.004)* as compared to the market. Alpha talks about how the portfolio has outperformed or underperformed as compared to its benchmark index, a *positive alpha of 0.09%* shows that the portfolio has outperformed the benchmark index by having an *excess return of 0.09% daily*. The Sharpe Ratio is a reward-to-risk ratio whereby the excess returns of the portfolio are divided by the standard deviation of the portfolio, a *Sharpe ratio of 0.07* shows a healthy reward-to-risk. The Treynor Ratio is a reward-to-volatility ratio, whereby the excess return of the portfolio is divided by the beta. Using all these portfolio metrics, we can deduce that the defensive portfolio has outperformed the benchmark index.

Probability

We also take into account that the market environment is extremely uncertain and apply the concept of "what happens in the past may not happen in the future" by conducting a *Monte Carlo simulation using a normal distribution*. This form of simulation incorporates uncertainty into the portfolio using a probability distribution, in this case, Normal distribution, to predict future random outcomes. Incorporating uncertainty into a portfolio is quintessential since *probability is a measure of risk that can impact the returns of a portfolio.*

Figure 3- Normal Distribution of a Defensive Portfolio



Source: The authors

In this, we have attempted to predict the portfolio returns for the next 1236 days or approximately 5 financial years using the empirical rule under normal distribution which states that the mean lies in the middle of the bell curve and for one standard deviation down and up, we can tell with 68% certainty. For 95% certainty, we can tell with two standard deviations down and up and with 99% certainty, three standard deviations down and up. A Normal or Gaussian distribution provides a stochastic model which is continuous and hence makes the quantitative model cohesive. Using the mean, 0.088%, and standard deviation of 0.98%, we have predicted that with *95% certainty that our portfolio returns will lie between -1.87% and 2.05%.* (IBM , 2020)

Protection against Uncertainty

The normal distribution can help us predict only with 95% but the conditional value at risk gives us the amount of uncertainty when the returns lie beyond the point where the value at risk and normal distributions are cut off. For a defensive investor the conditional value at risk is:

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Table 4- CVaR for a Defensive Portfolio

Conditional Value at Risk					
Total count	1233	Rank			Rank
var(95)	-1.21%	61.65		Cvar(95)	-2.15%
var(99)	-2.48%	12.33		Cvar(99)	-4.48%
Margin Amt 95%	-37010.7			CVaR Amt 95%	-65,683.19
Margin Amt 99%	**-75532.9**			**CVaR Amt 99%**	**-1,36,509.21**

Source: The authors

CVaR gives us the expected shortfall beyond the normal distribution. As an investor who wants to protect herself, we assume that the investor will keep a certain amount of the initial investment aside as a measure of safety to satisfy the expected shortfall.

The margin amount under Value at risk which takes into account only with 99% confidence level gives us the expected shortfall as INR 75,532 but the Conditional Value at risk with 99% confidence level gives us the *expected shortfall of INR 1,36,509*. Hence, the investor should keep this amount of money aside as a measure to protect against such *black swan events.*
(Hull, 2007)

Aggressive Investor

For an Aggressive Investor, the assets selected are *Apollo Hospitals, Tata Consumer Products, Bitcoin, ICICI Bank, Adani Ports, JSW Steel, Tata Elxsi, Birlasoft, Deepak Nitrite, and HCL Tech*. The aggressive investor will relatively have lesser constraints as compared to a defensive investor since they are less risk-averse. These companies are selected on a qualitative basis due to their exceptional performance over the last 5 years and are expected to continue performing well in the future as well. Parameters such as High Beta, Low Payout ratio (Growth stocks), Companies still in the growth phase of the life cycle an.

Problem Formulation using Solver excel

Maximizing the reward-to-risk ratio

Objective Function: Maximise Z= $\frac{x_1}{x_2}$

Where x_1 and x_2 are given as the daily average returns of the portfolio and the portfolio risk respectively.
In Portfolio returns,

$$Let\ w_1 -> Apollo\ Hospitals,\ w_2 -> Tata\ Consumer\ Products,\ w_3 -> Bitcoin,\ w_4 -> ICICI\ Bank,\ w_5 -> Adani\ Ports,\ w_6 -> JSW\ Steel,\ w_7 -> Tata\ Elxsi,\ w_8 -> Birlasoft,\ w_9 -> Deepak\ Nitrite,\ w_{10} -> HCL\ Tech$$

$$x_1 = w_1r_1 + w_2r_2 + w_3r_3 \ldots\ldots\ldots w_{10}r_{10}$$

$$x_1 = 0.14\% * w_1 + 0.17\% * w_2 + 0.22\% * w_3 + 0.12\% * w_4 + +0.11\% * w_5 + 0.14\% * w_6 + 0.18\% * w_7 + 0.18\% * w_8 + 0.28\% * w_9 + 0.11\% * w_{10}$$

Portfolio Risk is calculated by solving the portfolio variance-covariance matrix based on the respective portfolio weights.
Subject to constraints:

1. $x_1 \geq 0.25\%$
Minimum daily portfolio returns of at least 0.25%

An aggressive investor would be more focused on returns as compared to risk. The aggressive investor has therefore considered based on their own goals that she wants a minimum daily return of 0.25%.

2. $\sum w \geq 125\%$
Minimum leverage offered to the investor as 25%

An aggressive investor is also given the advantage of using leverage which is to borrow money to magnify returns. The investor can get minimum leverage of 25% and the investor wants to seize this opportunity to maximize returns.

3. $w_3 \leq 12.5\%$
The maximum allocation of up to 12.5% to Bitcoin

Even though we have assumed that an aggressive investor is more focused on maximizing returns than minimizing risk but she must cap the investment in cryptocurrency due to extremely high volatility.

Portfolio Weights

By solving the problem for the constraints above, we have arrived at a solution where the: -

Average Daily Returns= 0.25%
Portfolio Risk= 1.88%%
Reward-to-Risk Ratio= 0.133

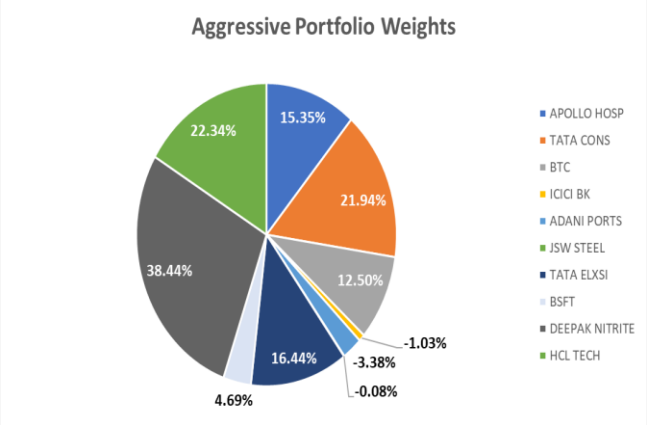
Table 5- Portfolio Weights for an Aggressive Investor

Assets	Daily Returns	Weights
APOLLO HOSP	0.14%	15.35%
TATA CONS	0.17%	21.94%
BTC	0.22%	12.50%
ICICI BK	0.12%	-1.03%
ADANI PORTS	0.11%	-3.38%
JSW STEEL	0.14%	-0.08%
TATA ELXSI	0.18%	16.44%
BSFT	0.18%	4.69%
DEEPAK NITRITE	0.28%	38.44%
HCL TECH	0.11%	22.34%
		127%

Source: The authors

Historical data of assets collected from (Yahoo Finance, 2021)

Figure 4- Aggressive Portfolio Weights

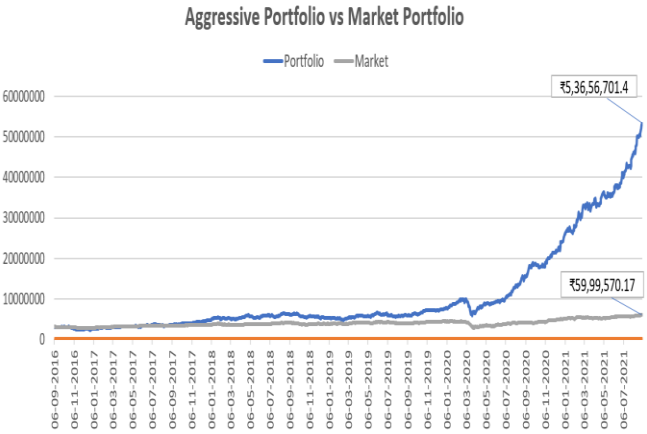


Source: The authors (Zerodha, 2021)

As we can observe from the portfolio weights above, we allocated the most in Deepak Nitrite (38%) and HCL Tech (22%) due to their high daily returns or low correlation with other assets in the portfolio. JSW Steel and Adani Ports have negative weights which indicate that we have shorted these stocks. Shorting takes place by selling quantities of the asset by borrowing it that we don't possess and bet that the price of the asset when we buy it back will be less than the amount, we paid thereby making a profit. An investor takes a long position when she expects the price will go up and a short position when she expects the price to go down.

Performance

Figure 5- Aggressive Portfolio vs Market (Sensex) Performance



Source: The authors

As we can observe from above, the aggressive portfolio has outperformed the benchmark index (Sensex) many times over. We have assumed the same amount of investment as the defensive investor of *INR 30.5 lakhs* except for the aggressive investor we have invested the amount in the beginning. Due to the compounding effect, the investment after 5 years increases much more than it would have under dollar-cost averaging but with a higher risk associated. As we can observe from the chart above, the portfolio returns have grown exponentially.

Capital Asset Pricing Model

Table 6- CAPM of an Aggressive Portfolio

CAPM	
Average Returns	0.25%
Daily Risk Free Rate	0.02%
Excess Daily Returns	0.23%
Standard Deviation	1.87%
Beta	**0.07**
Alpha	**0.23%**
Sharpe Ratio	**0.12**
Treynor Ratio	**0.03**
Excess Market Daily Returns	0.04%
Minimum Expected Return	**0.02%**

Source: The authors

As we can observe from the CAPM performance indicators above, the aggressive portfolio performs better than the benchmark index and also the portfolio for the defensive portfolio. It has a comparatively higher beta of *0.07* than the defensive portfolio but is still relatively lower than market standards. It has an *alpha of 0.23%* and a reward-to-risk *Sharpe Ratio of 0.12* both much higher than the defensive portfolio but the reward-to-volatility *Treynor ratio of 0.03* is way less than that 0.19 of the defensive portfolios. This shows that although the portfolio produces a greater reward than its standard deviation of returns, it underperforms based on the relative volatility of the benchmark index. Nevertheless, the minimum expected return of the portfolio is *0.02%* is much lower than what the portfolio is *giving a return of 0.25%.*

Probability

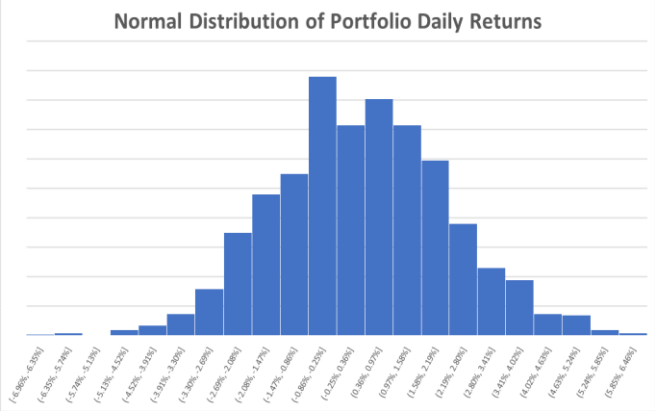
Table 7- Sensitivity Analysis of the 2 major stocks in an Aggressive Portfolio (HCL Tech and Deepak Nitrite)

Sensitivity Analysis of the 2 major stocks						
		Deepak Nitrite				
HCL TECH	**0.25%**	0.18%	0.23%	0.28%	0.33%	0.38%
	0.01%	0.19%	0.21%	0.23%	0.25%	0.27%
	0.06%	0.20%	**0.22%**	**0.24%**	**0.26%**	0.28%
	0.11%	0.21%	**0.23%**	**0.25%**	**0.27%**	0.29%
	0.16%	0.22%	**0.24%**	**0.26%**	**0.28%**	0.30%
	0.21%	0.23%	0.25%	0.27%	0.29%	0.31%

Source: The authors

For an aggressive investor, it is necessary to conduct a sensitivity analysis since the respective portfolio weights for the most allocated assets are *38% and 22%*. As identifiable, they contribute approximately half of the portfolio hence the contributions of the daily returns and performance of *Deepak Nitrite and HCL Tech* play a major role in the portfolio's performance. The above data table shows the sensitivity of the returns of these stocks and how they can change the daily returns of the entire portfolio.

Figure 6- Normal Distribution of an Aggressive Portfolio



Source: The authors

As illustrated earlier, a normal distribution helps with inculcating uncertainty in the portfolio. For an aggressive investor's portfolio, we can say with *95% certainty that our returns will lie between -3.49% and 3.99%.* As we can observe that since the standard deviation is higher than that of the defensive portfolio, the range between the returns is much higher.

Protection against uncertainty

Table 8- CVaR of an Aggressive Portfolio

Conditional Value at Risk					
Total count	1274	Rank			Rank
var(95)	-0.02640312	63.7		Cvar(95)	-0.035152929
var(99)	-0.039781289	12.74		Cvar(99)	-0.049376838
Margin Amt 95%	-80529.51449			CVaR Amt 95%	-107216.4328
Margin Amt 99%	**-121332.9309**			**CVaR Amt 99%**	**-150599.356**

Source: The authors

Similarly, for an aggressive investor, the risk involved is higher hence the expected shortfall is higher. Therefore, an aggressive investor must keep a certain amount of *INR 1.5 lakhs* aside from the investment of *INR 30.5 lakhs* aside to protect themselves from a tail risk and a black swan event.

Regression Analysis comparing the Defensive Portfolio vs the Aggressive Portfolio

Regression is a commonly used analysis tool that looks from a linear perspective to the portfolio returns. It is a tool meant for forecasting future returns for multiple variables.

$$Y = a + bX + \epsilon$$

Where:
Y – Dependent variable
X – Independent variable
a – Intercept
b – Slope
ϵ - Residual (error)

In our analysis, the dependent variable is the absolute portfolio returns and the independent variable is absolute market returns. The regression model aims to find the correlation between the portfolio returns and the market return. R square, coefficient of determination, is a statistical measure of fit that indicates how much variation of the dependent variable is explained by the independent variable. It is generally used to interpret how much percent of the portfolio movements can be explained by the movements in the benchmark index. An ideal R square value should be over 90%. However, a low value, less than 70%, indicates that the portfolio does not generally follow the movements of the index. The aggressive portfolio having an *R square of 78.43%* indicates that 78% of the portfolio returns can be explained by the market returns. While the defensive portfolio's *R square is 92.93%* which indicates that nearly 93% of the defensive portfolio could be explained by the market returns. The market sentiments are better reflected in the defensive portfolio than aggressive. *It can also be predicted that the defensive portfolio is more predictable as compared to the aggressive portfolio which abides with our first assumption that the defensive investor is more risk-averse than the aggressive investor.*

V. CONCLUSION

This study has been able to apply the two different portfolio strategies of the investors and incorporate them into allocating portfolio weights. We have done this through the Markowitz model and with modifications devised various strategies.

From the analysis, we can identify that the aggressive investor maximizes the reward-to-risk ratio but the absolute risk and reward are different as compared to the defensive investor. Also, through the regression analysis, we can identify the predominant fact that the defensive investor chooses certainty of returns with a trade-off of higher returns. We have also been able to identify that due to the higher risk inherently and collectively present in the assets, the aggressive investor needs to hold a higher CVaR amount as an expected shortfall as compared to the defensive investor.

This study is most likely the first attempt to construct two different portfolio strategies, especially in the Indian market. While this model, cannot be followed blindly, investors can certainly draw inspiration from it and use the strategies provided to fit their needs.

LIMITATIONS

There is no one-shoe-fits-all strategy when it comes to investing. The following limitations are:

1. No two investors are alike. Placing them in two categories of defensive and aggressive is like putting square pegs in round holes. Most investors lie in between this grey phase. For example, a young woman with a monthly salary of Rs.1,00,000 who has no other responsibilities wishes to invest in the market, she can aggressively invest around 40-50% of his money in the market. In another situation, there is a middle-aged father of two with a monthly salary of Rs. 50,000 who also wishes to invest aggressively in the market, there is a major contrast in the portfolio management strategy on a

predominantly qualitative level which cannot be incorporated through the model.
2. Most investors would like to expose themselves to more than 10 assets. It becomes very difficult to calculate the portfolio risk on this basis. Hence, Markowitz may not apply to such scenarios of portfolio allocation.
3. The Markowitz model assumes that assets are normally distributed which may not be the case in the real world. Further, it assumes that the inputs (variance and correlations) are static and hence do not change over time, however in reality they are time-varying.
4. The market can rarely be predicted due to the magnitude of changes that occur in the world around. Any positive or negative change in the political, social, or global news along with ever-changing laws and policies regarding the functioning of the industries can build volatility in the market.

Nevertheless, the research provides an excellent overview of the allocation and investing strategies that have commonly been identified in the market.

RECOMMENDATIONS

This study focuses on two types of portfolios based on the risk aversiveness of the investor. However further studies can be undertaken by identifying the third category of investor. This will help researchers get better insights on portfolio maximizations techniques based on the risk appetite. The three categories are aggressive, moderate, and conservative investors. Moderate investors are a mixture of the two extremes and earn lesser than the aggressive when the market does well but does not suffer when it falls. This helps them get the better of both. Further studies can also be undertaken based on the BSE 500 index returns. The modified version of the Markowitz model with different strategies for the same time could also be analyzed for various other financial markets across the globe.

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